Republic of Rwanda
Ministry of Education

RTB | RWANDA TVET BOARD

GENAM402

Applied Mathematics II

Apply Mathematics

RQF Level: 4 **Competence** Learning Hours: 100

Credits: 10

Sector: ICT and Multimedia

Trade: Software Programming and Embedded Systems

Module Type: General

Issue Date: September 2023

CURRICULUM: GENAM4002-TVET CERTIFICATE 4 – A

1200

<table>
<tr><td>Purpose statement</td><td colspan="5">This module describes the knowledge, skills and attitudes required to apply fundamentals mathematical analysis and statistics in modelling different real-life problems. At the end of this module, the learner will be able to analyze functions, apply integrals, apply differential equations in modelling real life problems, apply theory of sequences and series and apply bivariate statistics in prediction.</td></tr>
<tr><td rowspan="6">Delivery modality</td><td colspan="2">Training delivery</td><td>100%</td><td colspan="2">Assessment</td><td>Total 100%</td></tr>
<tr><td colspan="2">Theoretical content</td><td>70%</td><td rowspan="4">Formative assessment</td><td>25%</td><td rowspan="4">50%</td></tr>
<tr><td colspan="2">Practical work:</td><td rowspan="3">30%</td><td rowspan="3">25%</td></tr>
<tr><td>• Group project and presentation</td><td>10%</td></tr>
<tr><td>• Individual project /Work</td><td>20%</td></tr>
<tr><td colspan="3"></td><td colspan="2">Summative Assessment</td><td>50%</td></tr>
</table>

Elements of Competency and Performance Criteria

Elements of competency	Performance criteria
1 Apply theory of Transcendental functions, sequences and series	1.1 Transcendental functions and different properties are properly established based on their definition as used in Mathematics
	1.2 Limits are correctly defined and calculated based on their rules of calculation in Calculus
	1.3 Asymptotes are correctly defined based on limits calculation
	1.4 Differentiation is correctly performed based on rules of differentiation
	1.5 Applications are correctly defined and solved based on theory of optimization
	1.6 Arithmetic, Harmonic and geometric sequences are correctly defined based on their types
	1.7 Sums of series are accurately calculated based on the type of the series
	1.8 Convergence of sequences and series is properly studied based on theory of limits calculation
	1.9 Sequences are correctly applied in modelling real life problems in accordance with the type of problem
	1.10 Taylor and Mac Laurin Series are properly defined based on the theory of series expansion
	1.11 Application of Mac Laurin and Taylor series to known number calculation is correctly done based on the type of the number

	1.12 Application of Taylor and Mac Laurin series in limits calculation is perfectly carried out based on the type of the function
2 Apply integration	2.1 Differentials of functions are correctly defined based on the theory of infinitesimal approximation
	2.2 Application of differentials of functions is correctly done based on the type of problem
	2.3 Anti-derivatives off functions are calculated based on elementary table of integrals
	2.4 Techniques of integration are properly applied based on the type of the function
	2.5 Reduction formulas are established based on the theory of sequences
	2.6 Definite integrals are calculated based on fundamental theorem of calculus
	2.7 Improper integrals are calculated based on discontinuity aspects
	2.8 Definite integrals are correctly applied in area, arc length, and volume of solids calculation based on the sketched situation
3 Apply ordinary differential equations	3.1 First order differential equations are properly defined based on their type
	3.2 First order differential equations are correctly solved based on the type of equation
	3.3 Second order differential equations with constant coefficients are properly defined in accordance with the coefficients
	3.4 Second order differential equations with constant coefficients are properly defined based on their form.
	3.5 Modelling with differential equations is correctly defined in line with the type of real-life problem
	3.6 Second order differential equations with constant coefficients are properly applied in solving real-life problem based on the type of the problem to be solved
4 Apply Bivariate statistics	4.1 Bivariate data and bivariate statistics are accurately defined based on the type of data
	4.2 Regression line and prediction are accurately obtained based on its theory of obtention
	4.3 Correlation coefficient is accurately calculated based on its theory of calculation
	4.4 Machine learning is properly applied to data analysis and prediction based on the type of problem

Course content

Learning outcomes	At the end of the module the learner will be able to: 1. **Apply theory of Transcendental functions, sequences and series** 2. **Apply integration** 3. **Apply Ordinary differential equations** 4. **Apply Bivariate statistics**
Learning outcome 1: Apply theory of Transcendental functions, sequences and series	**Learning hours: 25**

Indicative content

- **Application of Trigonometric functions and their inverses**
  - ✓ Trigonometric functions
    - Definition,
    - Domain and range
    - Parity
    - Periodicity
    - Curve sketching
  - ✓ Limits and continuity
    - Calculation using tables
    - Calculation using graphs
    - Calculation using limits rules
    - The squeezing theorem
    - Indeterminate cases
    - Continuity
  - ✓ Asymptotes
    - Definition
    - Finding asymptotes
  - ✓ Differentiation
    - The tangent problem
    - Derivative from first principles

- ✓ Rules of differentiation
  - Sum and power rules
  - Product and quotient rules
  - Chain rule
  - Implicit differentiation
  - Differentiation of parametric defined functions
  - Critical and extremum points
  - Concavity and point of inflexion
  - Application to calculation of limit (hospital rule)
- ✓ Successive derivatives
- ✓ Curves sketching using variation table
- ✓ Inverse trigonometric functions
  - Domain and range
  - Properties and identities
  - Parity
  - Curve sketching
- ✓ Limits and continuity
  - Calculation of limits
  - Asymptotes
  - Continuity
- ✓ Differentiation
  - Differentiation from first principle
  - Rules of differentiation
- ✓ Successive derivatives
- ✓ Curves sketching using variation table

- **Application of Exponential and Logarithmic functions**
  - ✓ Exponential functions
    - Definition and properties
    - Domain and range
    - Composition and inverses of functions
  - ✓ Limits and asymptotes
    - Calculation of limits
    - Different indetermination cases
    - Limits and continuity

- Asymptotes

✓ Differentiation
- Definition
- Derivative from first principles
- Rules of differentiation
- Implicit differentiation
- Differentiation of parametrically defined functions
- Logarithmic differentiation

✓ Application of derivatives
- Critical and extremum points
- Concavity and inflexion points
- Tangents and normal lines equations
- Investment and Finance problems
- Maximization problems
- Machine Learning and Feature Engineering problems

✓ Successive derivatives

✓ Curves sketching from tables of variations

✓ Logarithmic functions
- Definition and properties
- Domain and range
- Composition and inverses of functions

✓ Limits and asymptotes
- Calculation of limits
- Different indetermination cases
- Limits and continuity
- Asymptotes

✓ Differentiation
- Definition
- Derivative from first principles
- Rules of differentiation
- Implicit differentiation
- Differentiation of parametrically defined functions
- Logarithmic differentiation

- ✓ Application of derivatives
  - Critical and extremum points
  - Concavity and inflexion points
  - Tangents and normal lines equations
  - Investment and Finance problems
- ✓ Successive derivatives
- ✓ Curves sketching from tables of variations

- **Application of Arithmetic and harmonic sequences**
  - ✓ Definition and properties
  - ✓ General term
  - ✓ Sum of terms
  - ✓ Mathematical induction
  - ✓ Convergence
    - Ration test
    - Root test
  - ✓ Recursive formula for arithmetic sequences
    - First order recurrence relations
    - Second order recurrence relations
  - ✓ Application in modelling and ICT
    - Application in finance
    - Convergence and limits
    - Exponential growth
    - Fibonacci sequence
    - Fibonacci sequence and photography

Application of Geometric sequences

- ✓ Definition and properties
- ✓ General term
- ✓ Sum of terms
- ✓ Mathematical induction
- ✓ Convergence
  - Ratio test
  - Root test

- ✓ Recursive formula for geometric sequences
- ✓ Application in modelling and ICT
  - Sequences, selections, and loops

- **Application of Series**
  - ✓ Definition and convergence
  - ✓ Sum and calculation of sums
    - Calculation of known sums
    - Partial fraction method
  - ✓ Taylor expansion and Taylor series
  - ✓ Mac Laurin expansion
  - ✓ Applications
    - Limits calculation
    - Integration
    - Known constants calculations

Resources required for the learning outcome	
Equipment	Computer, projector
Materials	Reference Books, Didactic materials such as manila paper, and geometric instruments
Tools	Hand-out notes slide and internet.
Facilitation techniques	• Brainstorming on variation tables • Practical exercises on limits and differentiation • Practical exercises on variation tables • Practical exercise on sequences and series • Practical exercises and expose on different applications of sequences and series • Course labs on applications of sequences and series • Trainer guided documentation of sequences and series • Course lab sheet on sequences • Group discussion on sequences and series • Pairing work on sequences • Curves sketching on manila papers using variation tables • Documentary Research
Formative assessment methods	• Written assessment • Practical assessment (Labs)

Learning outcome 2: Apply integration	Learning hours: 40

Indicative content

- **Calculation of differential of functions**
  - ✓ Definition and properties
  - ✓ Applications
    - The rate of change problems
    - Approximate value of small change in a quantity problem
    - Calculation of profit and loss with respect to business using graphs
    - Calculation of the rate of change of the temperature
- **Application of Table of elementary integrals and anti-derivatives of functions**
  - ✓ Elementary table of integration
  - ✓ Integration using elementary table of integration
    - Algebraic functions
    - Transcendental functions
- **Application of integration techniques**
  - ✓ Integration by substitution
  - ✓ Integration by parts
  - ✓ Generalization of elementary table of integration
  - ✓ Integration of rational functions
    - Completing square
    - Partial fractions
  - ✓ Trigonometric substitution
    - Integration of trigonometric functions
    - Integration of functions involving powers
    - Linearization
    - Reduction formula revisited
- **Application of definite integrals and fundamental theorem of calculus**
  - ✓ The fundamental theorem of calculus
    - Integration of continuous functions
    - The Charles' relation and its application
  - ✓ Improper integrals
    - Integrals with infinite boundaries
    - Integration of discontinuous functions on an interval

- ✓ Reduction formulas

- **Application of definite integrals to area, arc length and volume calculation**
  - ✓ Area and mean value
    - Area in rectangular coordinates
    - Area in polar coordinates
  - ✓ Arc length
  - ✓ Volume of solid of revolution
  - ✓ Surface of revolution
  - ✓ Tangent's slope problems
  - ✓ Velocity and acceleration problems

Resources required for the indicative content	
Equipment	Computer, projector
Materials	Reference books
Tools	Hand-out notes slide and internet.
Facilitation techniques	• Individual and group work on anti-derivatives and differentials • Practical exercise on integration and its applications • Trainer guided exercises and solutions • Documentary research
Formative assessment methods	• Written assessment
Learning outcome 3: Apply differential equations	**Learning hours: 20**

Indicative content

- **Application of First order differential equation**
  - ✓ Definition, and formation
  - ✓ Order and degree
  - ✓ Separable variable equations and solving
    - Finding the solution
  - ✓ Homogeneous equation and solving
    - Testing homogeneity
    - Solving using substitution
  - ✓ Linear equations and solving
    - Definition
    - Solving using integrating factor method
  - ✓ Bernoulli equation
    - Definition
    - Solving
- **Application of Second order differential equation with constant coefficients**
  - ✓ Definition

- ✓ Solving homogeneous equation
  - Auxiliary equation method
  - Numerical method (finite difference method)
- ✓ Solving non-homogeneous equations
  - Finding integral
  - Superposition principle
  - Variation of parameters method

- **Application of differential equations in modelling**
  - ✓ Definition of problems and parameters identification
  - ✓ Problem formulation
  - ✓ Problem solving
  - ✓ Some specific problems
    - Population growth,
    - Pooling problem
    - Carbon dating problem

Resources required for the indicative content	
Equipment	Computer, projector
Materials	Reference book, Didactic materials such as manila paper
Tools	Hand-out notes, Scientific calculator, Internet, Geometric instruments (Ruler, T-square)
Facilitation techniques	• Individual and group work • Practical exercise on types of differential equations • Trainer guided document on differential equations • Group discussion on different types of differential equations • Practical exercises on solving differential equations • Individual exercises on solving differential equations and applications • Documentary Research
Formative assessment methods	• Written assessment • Practical assessment (Labs)

Learning outcome 4: Apply Bivariate statistics	**Learning hours: 15**
Indicative content	

- **Review on univariate statistics**
  - ✓ Grouped data
  - ✓ Calculation of variance,
  - ✓ Standard deviation
  - ✓ coefficient of variation
- **Application of Bivariate statistics**
  - ✓ Definition and terminologies

- ✓ Scatter plot and data presentation
  - Non-grouped data
  - Grouped data
- ✓ Variance and covariance
- ✓ Correlation coefficient
  - Definition and properties
  - Calculation of correlation coefficient
  - Calculation using coefficient of determination
- ✓ Regression lines and prediction
  - Regression line using normal equations
  - Regression line using covariance and
- ✓ Use artificial intelligence and machine learning for data analysis
  - Data tuning
  - Model training
  - Model testing
  - Prediction

Resources required for the indicative content	
Equipment	Computer, projector
Materials	Reference books, Didactic materials such as manila paper ,Geometric instruments (Ruler, T-square)
Tools	Hand-out notes slide and internet.
Facilitation techniques	• Brainstorming on bivariate data • Demonstration and simulation on bivariate data • Individual and group work on manipulation of bivariate data • Practical exercise on presenting data • Trainer guided on bivariate statistics • Group discussion • Pairing work on manipulating bivariate data • Labs on application of bivariate statistics in prediction and ML • Documentary Research
Formative assessment methods	• Written assessment • Practical assessment (Labs)

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