



RQF LEVEL 5

TRADE:

MODULE CODE: PHYDM501

TEACHER'S GUIDE

**Module name: DYNAMICS AND
MECHANICAL WAVES**



MODULE NAME : PHYDM501 DYNAMICS AND MECHANICAL WAVES

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Acronyms

TV: Television

GEO: Geostationary Orbit

MEO: Medium Earth Orbit

LEO: Low Earth Orbit

HEO: Highly Elliptical Orbit

MRI: Magnetic Resonance Imaging

SHM: Simple harmonic motion

AU: Astronomical Unit

CM: Center of Mass

SI: International system of unit

REMA: Rwanda Environment Management Authority

CHP: Combined heating and power systems

REB: Rwanda Education Board

MIDMAR: The Ministry of Disaster Management and Refugee Affairs

INTRODUCTION

This module describes skills, knowledge and attitudes required to apply concepts of dynamics and mechanical waves. At the end of this module, the trainee will be able to describe motion in orbits, climate change and greenhouse effect, apply oscillations and mechanical waves.

It will help a trainee to carry out his/her specialized tasks that are useful in analyzing data, solving real life problems encountered in related fields. In a nutshell, the features depicted on above helps trainees identify the essential steps in solving problems and increases their skills as problem solvers.

MODULE CODE AND TITLE: PHYDM501 DYNAMICS AND MECHANICAL WAVES

Learning Units:

1. Describe motion in orbits
2. Apply Oscillations and mechanical waves
3. Describe Climate change and Greenhouse effect.

LEARNING UNIT 1: DESCRIBE MOTION IN ORBITS



Learning outcomes:

1. Describe universal Gravitation in orbits
2. Apply Kepler's laws in motion of celestial objects
3. Describe energy considerations in planetary, rocket and satellite motion.

Learning outcome 1.1. Clear description of Universal Gravitation inn orbits based on Kepler' laws



Duration: 5 hours



Learning outcome 1.1. Objectives:

By the end of the learning outcome, the trainee will be able to:

1. Define orbital motion and universal gravitation correctly.
2. State Newton's law of universal gravitation accurately.
3. Solve problems related to Newton's law of universal gravitation correctly.



Resources

Equipment	Tools	Materials
- Computer	- PhET simulations of universal Gravitation and orbit.	- markers
- Projector	- Whiteboard - Chalkboard	- chalks - Scientific calculator - Textbooks



Advance preparation: prepare and avail PhET simulations of universal Gravitation and orbital motion of planets.



Content 1: Definition of orbital motion and Universal gravitation

- **Orbital motion** is a motion that occurs when an object moving forward in space is at the same time pulled by gravity towards another object.

Orbital motion around the Earth involves giving something enough horizontal velocity so that by the time gravity pulls it down, it has travelled far enough to have Earth's surface curve away from it. As a result, it stays above the Earth surface.

An object in orbit around the Earth is essentially falling around the Earth but going so fast that it never hits it. That is, it continues its repeated movement in space around the Earth surface but never falls down.

- **An orbit** is the trajectory followed by a body under the influence of the gravity of another body. Any object in orbit is called a *satellite*. A satellite can be natural like the Earth and the moon as well as it can be artificial (man-made).



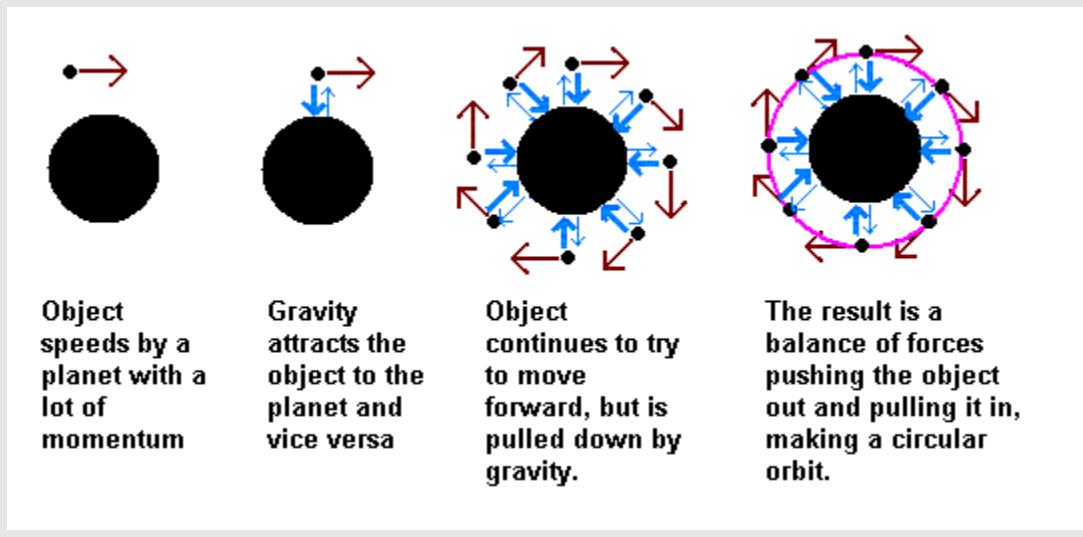
Figure1 : satellite in orbit

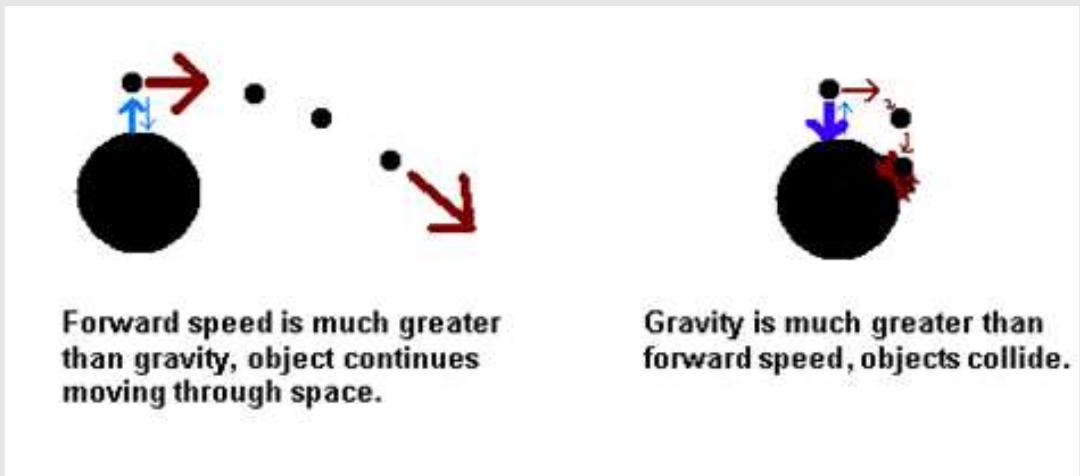
What causes an orbit to happen?

Orbits are the result of a perfect balance between the forward motion of a body in space such as a planet or moon and the pull of gravity on it from another body in space such as a large planet or star.

An object with a lot of speed goes forward and wants to keep going forward. However, the gravity of another body in space pulls it in.

The whole process is well explained in the figures below:





➤ Universal gravitation:

Universal gravitation refers to the acting force between any two bodies with masses.

All objects attract each other with a force of gravitational attraction. Gravity is universal.

This force of gravitational attraction is directly dependent upon the masses of both objects and inversely proportional to the square of the distance that separates their centers.

➤ What is gravity?

Gravity is a mysterious force that many people consider to make everything fall down towards the Earth. But after scientific research, it has turned out that all objects with mass have gravity. Some objects like the Earth and the Sun have a stronger gravity than others. Gravity is universal. How much gravity an object has depends on its mass. Gravity also depends on how close bodies are to each other. The closer the bodies are, the stronger the gravity.

➤ **Importance of gravity to human lives:**

- ⊕ Gravity is very important to our everyday lives because without the Earth's gravity we would fly right off it. If you kick a ball, it would fly off forever. So, without gravity, we would not play our favorite games including football, volleyball, basketball,...
- ⊕ We certainly can't live without gravity. Gravity also is important on a larger scale. It is the Sun's gravity that keeps the Earth in orbit around it and life on Earth needs the Sun's light and warmth to survive.
- ⊕ Gravity helps the Earth to stay at just the right distance from the Sun, so it is not too hot or too cold for the living things dwelling on it.



Theoretical learning Activity:

In groups of four, Brainstorm and try to find answers to the following questions:

- 1) Is there gravity in space?
- 2) What is an orbit?
- 3) What is a satellite?
- 4) What is gravity?
- 5) How can a spaceship leave orbit?
- 6) How does speed affect an orbit?
- 7) Are there orbits within orbits?
- 8) What could cause an orbit to fail?
- 9) Discuss other importance of gravity to human kind and other living things on the Earth.
- 10) Discuss how gravity can affect the orbital motion of a body.



Points to remember

- ⊕ Orbital motion is a motion that occurs when an object moving forward in space is at the same time pulled by gravity towards another object.
- ⊕ Gravity in mechanics is the universal force of attraction acting between any two bodies with mass.



Content 2: Description of Newton's law of gravitation

This is also called the universal law of gravitation or inverse square law. It states that *“the gravitational force of attraction between two bodies with masses m_1 and m_2 is directly proportional to the product of masses and inversely proportional to the square of their mean distance apart.”*



Mathematically:

$$F \propto \frac{m_1 m_2}{r^2}$$
$$F = \frac{G m_1 m_2}{r^2}$$

Where F = force in newton (N), m_1 and m_2 are masses in kg, r = mean distance between bodies in meters (m) $G = 6.7 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$ and it is called the universal gravitational constant

➤ Properties of Gravitational Force:

- It is always attractive in nature while electric and magnetic force can be attractive or repulsive.
- It is independent of the medium between the particles while electric and magnetic forces depend on the nature of the medium between the particles.
- It holds well over a wide range of distances. It is found true for interplanetary to interatomic distances.
- It is a central force, i.e. it acts along the line joining the centers of two interacting bodies.
- It is the weakest force in nature: As $F_{\text{nuclear}} > F_{\text{electromagnetic}} > F_{\text{gravitational}}$.
- It is an action reaction pair, i.e. the force with which one body (say, Earth) attracts the second body (say, moon) is equal to the force with which moon attracts the Earth. This is in accordance with Newton's third law of motion.

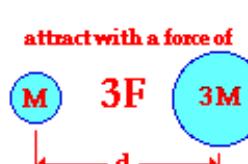
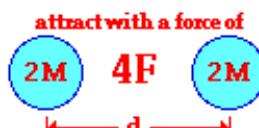
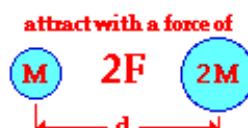
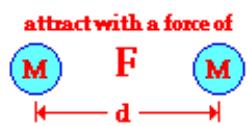


Theoretical learning Activity:

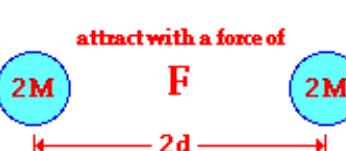
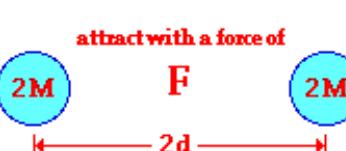
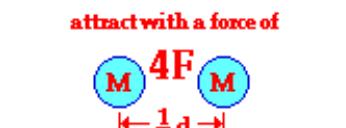
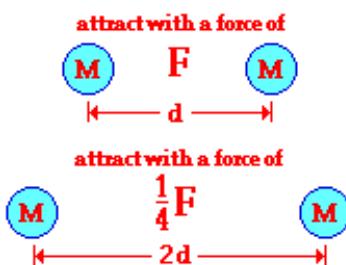
Thinking Proportionally About Newton's Equation

The proportionalities expressed by Newton's universal law of gravitation are represented graphically by the following illustrations. Observe how the force of gravity is directly proportional to the product of the two masses and inversely proportional to the square of the distance of separation. Work in groups of four to discuss the effect of changing the masses and the separation distance on gravitational force of attraction.

Effect of Mass on F_{grav}



Effect of Distance on F_{grav}



Points to remember

- ✓ Sir Isaac Newton established that it was the force of gravity between the Sun and the planets responsible for keeping planets in orbital motion along their elliptical path.
- ✓ Gravitational force is always attractive in nature while other forces can be attractive or repulsive.



Content 3: Measurement of Gravitational constant

The precise value of the Universal gravitational constant **G** was first determined experimentally by **Lord Henry Cavendish in 1798** after Newton's death. Cavendish determined the value of **G** using a torsion balance.

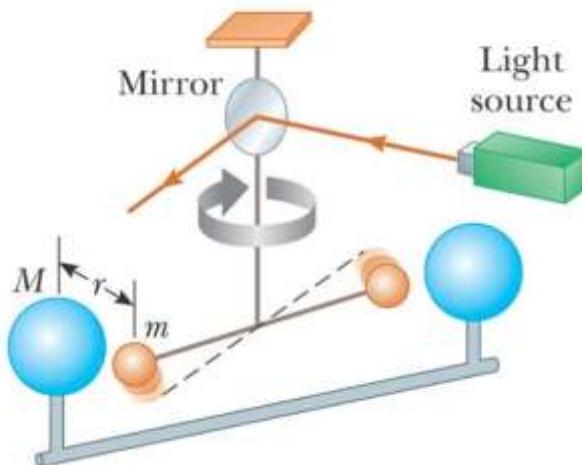
➤ Explanation of Cavendish experiment:

Cavendish's apparatus for experimentally determining the value of **G** involved a light source, a mirror, a rigid rod about 60cm long. Two small lead spheres were attached to the ends of the rod and the rod was suspended by a thin wire.

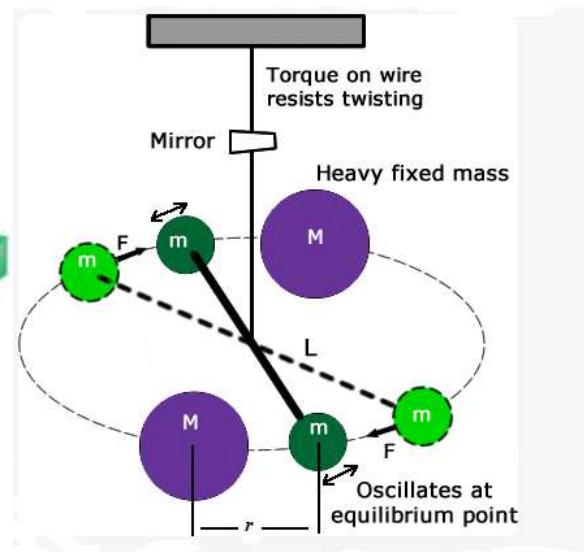
When the rod is twisted, the torsion of the wire begins to exert a torsional force that is proportional to the angle of rotation of the rod. The more twist of the wire, the more the system pushes *backwards* to restore itself towards the original position.

Cavendish had calibrated his instrument to determine the relationship between the angle of rotation and the amount of torsional force.

The gravitational constant was measured by Henry Cavendish using a setup like this:



(a)



(b)

Cavendish then brought two large lead spheres near the smaller spheres attached to the rod. Since all masses attract, the large spheres exerted a gravitational force upon the smaller spheres and twisted the rod a measurable amount. Once the torsional force balanced the gravitational force, the rod and spheres came to rest and Cavendish was able to determine the gravitational force of attraction between the masses and the value of **G** could be determined. Cavendish's measurements resulted in an experimentally determined value of **$6.75 \times 10^{-11} \text{ N m}^2/\text{kg}^2$** .

Today, the currently accepted value is $6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$.

- The value of **G** does not depend upon the nature and size of the bodies.
- It also does not depend upon the nature of the medium between the two bodies.
- As **G** is very small hence gravitational forces are very small, unless one (or both) of the masses is huge.



Theoretical learning Activity:

In groups of four,

- 1) Make an internet search on YouTube on Cavendish experiment to determine the gravitational constant.
- 2) Describe Cavendish's experiment in Measurement of Gravitational constant.



Points to remember

- ❖ The value of **G** is nowadays equal to $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ in S.I. units.
- ❖ The value of **G** does not depend upon the nature and size of the bodies.
- ❖ Don't confuse **G** with **g**: "Big G" and "little **g**" are totally different things.



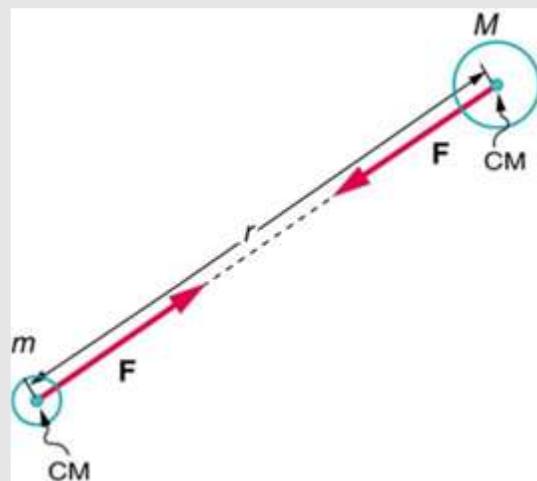
Content 4: Determination of Gravitational force of bodies moving in orbits

The Force of Gravity between any two bodies moving in orbit depends only upon the following factors:

- ❖ The **masses** of the two objects: More massive objects exert a **stronger** gravitational force between them.
- ❖ The **distance** between them: The force gets **stronger** as the two objects move closer together and the force gets **weaker** as the two objects move farther apart.

The gravitational force of attraction between bodies in orbit does not depend on their shapes, colors, or compositions.

In the figure below, let us consider a body of small mass **m** moving in orbit around massive body of mass **M**. The separation distance between them is **r**.



The force of gravitational attraction **F** between the two massive bodies in orbit is proportional to the product of their masses and inversely proportional to the square of the distance between their centers **r**.

$$F \propto \frac{Mm}{r^2} \Rightarrow F = G \frac{Mm}{r^2}$$

Where; **F** is the gravitational force in newton (N)

$$G = 6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2, \text{ m and M are the masses of the bodies in kilogram, kg}$$



Theoretical learning Activity:

In groups of four, brainstorm about the following problems:

- A planet and its satellite are separated by a distance of 3×10^6 km. The mass of the planet is 9.9×10^{22} metric tons and that of the satellite is 10^{27} metric tons. Given that the universal gravitational constant is $6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ and that 1 metric ton equals 1,000 kilograms, find the force of gravity between them.
- Find the mass of a planet given that the acceleration due to gravity at its surface is 6.003 m/s^2 , its radius is 2,400 km and the universal gravitational constant is $6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$.



Points to remember

The Force of Gravity between any two bodies moving in orbit depends only upon the following factors:

- ❖ The **masses** of the two objects: More massive objects exert a **stronger** gravitational force between them.
- ❖ The **distance** between them: The force gets **stronger** as the two objects move closer together and the force gets **weaker** as the two objects move farther apart.
- ❖ The gravitational force of attraction between bodies in orbit does not depend on their shapes, colors, or compositions.
- ❖ The gravitational force is given by: $F = G \frac{Mm}{r^2}$

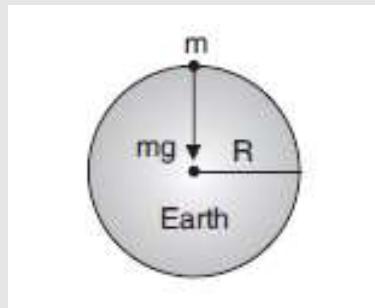


Content 5: Variation of acceleration due to gravity with height, h

➤ Acceleration due to gravity at the surface of the Earth

The force of attraction exerted by the Earth on a body near its surface is called gravitational pull or gravity. The acceleration produced on a body under the effect of gravity is called **acceleration due to gravity (g)**.

Consider a body of mass m lying on the surface of Earth. The mass and the radius of the Earth are M and R respectively.



The gravitational force on the body is given by:

$$F = G \frac{Mm}{R^2} \dots\dots (1)$$

Where; F is the gravitational force, $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$, m and M are the masses of the body and the Earth respectively. The Earth's mass is $5.9736 \times 10^{24} \text{ kg}$.

R is the radius of the Earth. The Radius of Earth = 6400 km. Or $R = 6.4 \times 10^6 \text{ m}$

If g is the acceleration due to gravity, then the force on the body due to Earth is given by

Force = mass \times acceleration due to gravity

$$\text{or } F = mg \dots\dots (2)$$

from equation (1) and (2), the two forces are equal. Then we have

$$mg = G \frac{Mm}{R^2}$$

$$\Rightarrow g = \frac{GM}{R^2}$$

Replacing the values of $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$, $M = 5.97 \times 10^{24} \text{ kg}$ and

$R = 6.4 \times 10^6 \text{ m}$, the average value of g is taken as 9.8 m/s^2 or 9.81 m/s^2 on the surface of the Earth at mean sea level.

From the expression $g = \frac{GM}{R^2}$ it is clear that its value depends upon the mass and radius of the planet and it is independent of the mass and shape of the body placed on the surface of the planet. This means that a given planet produces the same gravitational acceleration in a light as well as heavy body.

Note: The equation $g = \frac{GM}{R^2}$ can be expressed in terms of the density of the Earth as follows:

As mass = density times volume, for the Earth the mass of the Earth equals its density times the volume. That is, $M = \rho V$

Where the volume of the Earth is given by $V = \frac{4}{3}\pi R^3$

$$\text{Thus, } M = \rho V = \rho \left(\frac{4}{3}\pi R^3 \right) \Leftrightarrow M = \frac{4}{3}\pi \rho R^3$$

Therefore, the gravitational acceleration becomes:

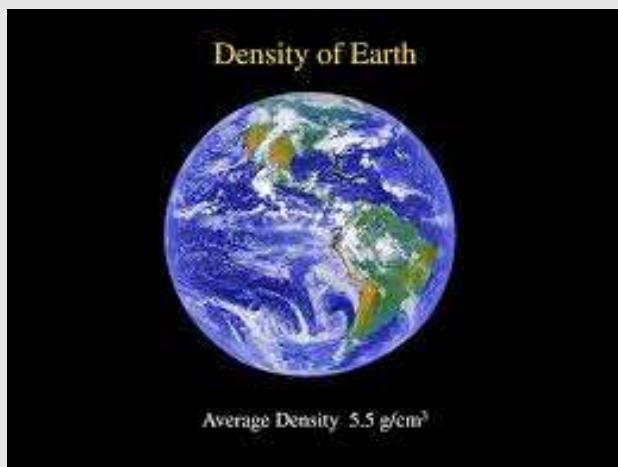
$$g = \frac{G \left(\frac{4}{3}\pi \rho R^3 \right)}{R^2}$$

$$\Leftrightarrow g = \frac{4}{3}\pi \rho G R$$

This equation shows clearly that the value of g depends also upon the density and the radius of the planet.

➤ **The density of the Earth:**

Density is a measure of how much matter (mass) there is in a given amount of space (volume). Density basically describes how tightly packed the tiny particles that make up any substance/material are. The higher the density, the more closely packed the particles (called atoms and molecules) are. The lower the density, the more spaced out the particles are.



The density of Earth is calculated by **dividing the planet's mass by its volume**.

That is, **density of earth** = $\frac{\text{mass of earth}}{\text{volume of the earth}}$

Where the volume of the Earth is given by: $V_e = \frac{4}{3}\pi R^3$ (the Earth has a spherical shape)

$$\rho_e = \frac{M_e}{4/3 \pi R_e^3}$$
$$\rho_e = \frac{5.98 \times 10^{24} \text{ kg}}{4/3 (22/7) (6.38 \times 10^6 \text{ m})^3}$$
$$\boxed{\rho_e = 5.52 \times 10^3 \text{ kg/m}^3}$$

➤ Factors that can affect the acceleration due to gravity on the Earth:

In general, the value of acceleration due to gravity vary due to the following factors:

- (a) Shape of the Earth: Earth is elliptical in shape. It is flattened at the poles and bulged out at the equator.
- (b) Height above the Earth surface,
- (c) Depth below the Earth surface and
- (d) Axial rotation of the Earth.

Solved examples:

Example1:

Acceleration due to gravity on moon is $(1/6)^{\text{th}}$ of the acceleration due to gravity on earth. If the ratio of densities of earth (ρ_m) and moon (ρ_e) is $\left(\frac{\rho_e}{\rho_m}\right) = \frac{5}{3}$, find the radius of moon R_m in terms of radius of the earth R_e .

Solution:

Acceleration due to gravity, $g = \frac{4}{3}\pi\rho GR \quad \therefore \quad g \propto R$

or $\frac{g_m}{g_e} = \frac{\rho_m}{\rho_e} \cdot \frac{R_m}{R_e} \quad [\text{As } \frac{g_m}{g_e} = \frac{1}{6} \text{ and } \left(\frac{\rho_e}{\rho_m}\right) = \frac{5}{3} \text{ (given)}]$
 \therefore

$$\frac{R_e}{R_m} = \left(\frac{g_e}{g_m}\right) \left(\frac{\rho_e}{\rho_m}\right) = \frac{1}{6} \times \frac{5}{3} \quad \therefore \quad R_m = \frac{5}{18}R_e$$

Example2:

The moon's radius is $(1/4)^{\text{th}}$ of that of earth and its mass is $1/80$ times that of the earth. If g represents the acceleration due to gravity on the surface of the earth, what is acceleration due to gravity on the surface of the moon?

Solution:

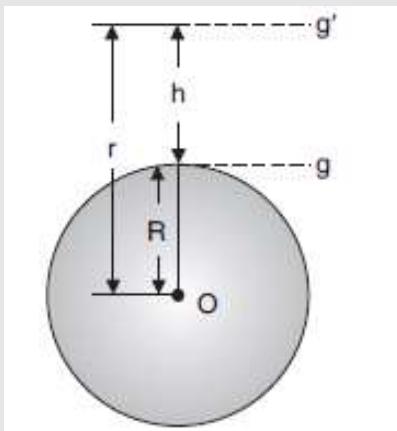
Acceleration due to gravity, $g = \frac{GM}{R^2}$

$$\frac{g_{\text{moon}}}{g_{\text{earth}}} = \frac{M_{\text{moon}}}{M_{\text{earth}}} \times \frac{R_{\text{earth}}^2}{R_{\text{moon}}^2} = \left(\frac{1}{80}\right) \left(\frac{4}{1}\right)^2$$

$$g_{\text{moon}} = g_{\text{earth}} \times \frac{16}{80} = \frac{g}{5}.$$

➤ The variation of acceleration due to gravity with height above the Earth surface

Consider a particle placed at a height h above the surface of the Earth where acceleration due to gravity is g' as shown in the figure below:



we already know that the acceleration due to gravity at the surface of the Earth is given by: $g = \frac{GM}{R^2}$ (3)

The acceleration due to gravity at height h from the surface of the Earth will be:

$$g' = \frac{GM}{(R+h)^2} \quad \dots \dots \dots \quad (4)$$

Now taking the ratio of equation (4) to (3) gives:

$$\frac{g'}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}} = \frac{R^2}{(R+h)^2} \Leftrightarrow g' = g \frac{R^2}{(R+h)^2}$$

From the figure, let $R + h = r$

Therefore, we have $g' = g \frac{R^2}{r^2}$

This equation shows that as we go above the surface of the Earth, the value of g' decreases because it is inversely proportional to the square of the height above its center.

This expression can be plotted on the graph as follow:

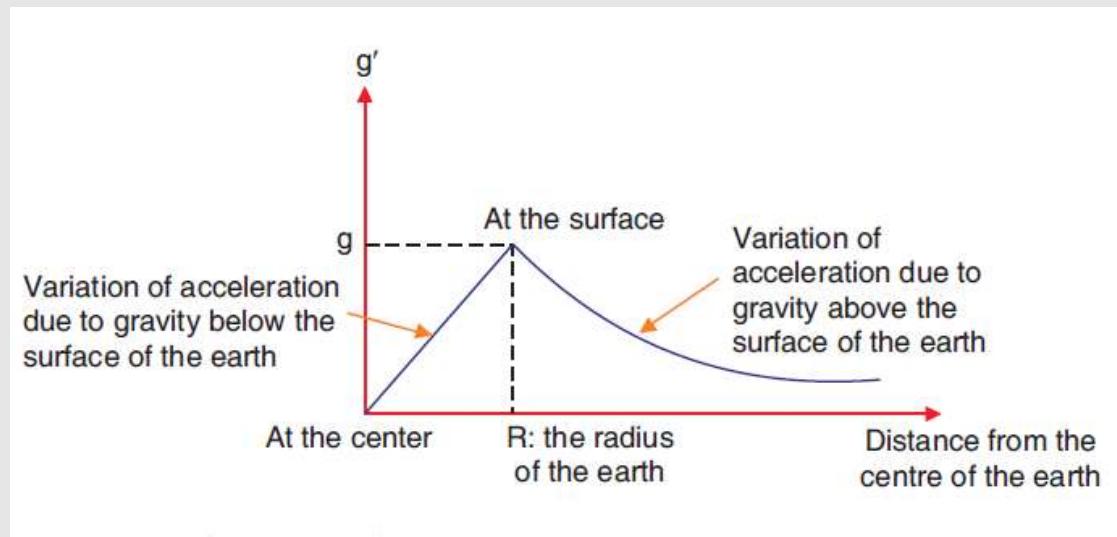


Figure: Curve of variation of acceleration due to gravity with height.

From the graph above, the value of gravitational acceleration "g" at the surface of the Earth is a constant and is approximately equal to 9.8 m/s^2 . But this value of "g" **decreases with an increase in altitude or height from the surface of the Earth.**

Example1:

The acceleration of a body due to the attraction of the earth (radius R) is g . Find the acceleration due to gravity at a distance $2R$ from the surface of the earth.

Solution:

$$\frac{g'}{g} = \left(\frac{R}{R+h} \right)^2 = \left(\frac{R}{R+2R} \right)^2 = \frac{1}{9} \Rightarrow g' = \frac{g}{9}$$

Example2:

Find the height of the point above the Earth's surface, at which acceleration due to gravity becomes 1% of its value at the surface is (Radius of the Earth is R)

Solution:

$$\text{Acceleration } \Rightarrow \frac{g}{100} = g \left(\frac{R}{R+h} \right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{10} \Rightarrow h = 9R$$

$$g' = g \left(\frac{R}{R+h} \right)^2$$



Theoretical learning Activity:

In groups of four, brainstorm about the following problems:

- 3) Derive the formula for the acceleration due to gravity at a certain height h above the Earth surface.
- 4) With a relevant graph, describe how the acceleration of gravity varies with altitude.

Points to remember

- ✓ The acceleration of gravity at any planet is given by $\mathbf{g} = \frac{GM}{R^2}$ or $\mathbf{g} = \frac{4}{3}\pi\rho GR$
- ✓ The value of \mathbf{g} depends upon the mass and radius and also depends also upon the density of the planet.
- ✓ The value of the gravitational acceleration "g" increases linearly from the center of the Earth, at the surface of the Earth is constant but this value decreases with an increase in altitude (or height) from the surface of the Earth.



Learning outcome 1.1 : Formative Assessment

1. Answer the Following by true or false :
 - a. Gravity is a force pulling together all matter. The more matter, the less gravity.
 - b. There is gravity everywhere. Gravity from the Sun reaches throughout the solar system and beyond, keeping the planets in their orbits.
 - c. An orbit is a regular and repeating path that an object in space takes around another one.
 - d. An object in an orbit is called a satellite. A satellite can be natural, like the moon, or human-made.
2. Choose the correct Answer from the Following options :
 - i. What apparatus did Henry Cavendish use in his experiment to determine the gravitational constant?
 - a) 1 bar, 1 small sphere and 1 large sphere
 - b) 1 bar, 2 small spheres and 2 large spheres

c) 2 bar, 1 small sphere and 2 large spheres
 d) 2 bar, 2 small spheres and 1 large sphere

ii. What material were the spheres made up of in Henry Cavendish's experiment?
 a) Lead
 b) Steel
 c) Iron
 d) Wood

iii. What are the dimensions of universal gravitational constant?
 a) $[M^2 L^3 T^2]$
 b) $[M^{-1} L^3 T^{-2}]$
 c) $[M^{-1} L^3 T^2]$
 d) $[M^1 L^3 T^{-2}]$

iv. Suppose that two objects attract each other with a gravitational force of 16 units. If the mass of both objects was tripled, and if the distance between the objects was doubled, then what would be the new force of attraction between the two objects?
(F = 24units, F = 32units, F = 36units)

v. Suppose that two objects attract each other with a gravitational force of 16 units. If the mass of object 1 was doubled, and if the distance between the objects was tripled, then what would be the new force of attraction between the two objects?
(F = 3.7units, F = 3.56units, F = 4.2units)

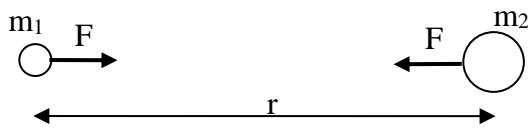
vi. Suppose that two objects attract each other with a gravitational force of 16 units. If the mass of both objects was doubled, and if the distance between the objects remained the same, then what would be the new force of attraction between the two objects?
(F = 62units, F = 56units, F = 64units, F = 65units)

3. (a) Using a relevant graph, explain how the acceleration of Gravity varies with altitude.
 (b) Find the height of the point above the Earth's surface, at which acceleration due to gravity becomes 2.1% of its value at the surface is (Radius of the Earth is R)

4. Newton's Universal Law of Gravitation (first stated by Newton): any two masses m_1 and m_2 exert an attractive gravitational force on each other according to

$$F = G \frac{m_1 m_2}{r^2}$$





Where: G = universal constant of gravitation = $6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2$

Answer the following questions:

1. Consider a space shuttle which has a mass of about $1.0 \times 10^5 \text{ kg}$ and circles the Earth at an altitude of about 200.0 km . Calculate the force of gravity that the space shuttle experiences. The mass of the Earth is $5.98 \times 10^{24} \text{ kg}$
2. What is the gravitational force between two identical $50,000 \text{ kg}$ asteroids whose centers of mass are separated by 1000 m ?
3. If you have a mass of 55 kg and you are standing 3 meters away from your car, which has a mass of 1234 kg , how strong is the force of gravity between you and the car?
4. The moon has a mass of $7.34 \times 10^{22} \text{ kg}$ and a radius of $1.74 \times 10^6 \text{ meters}$. If you have a mass of 66 kg , how strong is the force between you and the moon?
5. Romeo is a 68 kg astronaut. Juliet is a beautiful cosmonaut who is standing on the balcony of a $4.58 \times 10^5 \text{ kg}$ space station that is at rest and out of gas. Romeo is floating 25 meters away from the space station's center of mass, how strong is the force between Romeo and Juliet?

Learning Outcome 1.2: Apply Kepler's laws in motion of celestial objects



Duration: 5 hours



Learning outcome 1.2. Objectives:

By the end of the learning outcome, the trainee will be able to:

1. Define correctly planetary motion
2. Explain clearly the movement of the planets, stars and other celestial objects
3. Describe clearly Kepler's laws of planetary motion



Resources

Equipment	Tools	Materials
<ul style="list-style-type: none">- Computer- Projector	<ul style="list-style-type: none">-PhET simulations of planetary motion.- Whiteboard- Chalkboard	<ul style="list-style-type: none">- markers- chalks- Scientific calculator- Textbooks

Advanced preparation : Prepare PhET simulations on planetary motion.



Content 1: The motion of celestial objects in orbits

➤ Definition of Planetary motion

Planets are large natural bodies rotating around a star in definite orbits.

Planets may have either a terrestrial (rocky) surface or a gaseous surface. Gaseous planets are considerably larger than terrestrial planets.

Planets may have rings or other unique surface characteristics.

Movement of planets is based on revolution (orbit) around the Sun and rotation (turning) on the planet's axis

- **Planetary motion** is the motion of the planets around the Sun in elliptical orbits.
- **Movement of the planets, stars and other celestial objects**

The east to west daily motions of stars, planets and the Moon are caused by the rotation of the Earth on its axis. The Earth and all other planets revolve around the Sun on circular orbits. This produces the change in constellations observed from one time of year to the next. The figure below shows some of the planets rotating around the Sun and the time of revolution they take.



➤ Kepler's conclusion about Brahe's data.

Tycho Brahe (1546-1601)

Tycho Brahe was born from a rich Danish noble family was fascinated by astronomy but disappointed with the accuracy of tables of planetary motion at the time. He decided to dedicate his life and considerable resources to recording planetary positions ten times more accurately than the best previous work but he was limited by resources including effective Optical instruments.

Fortunately, the king of Denmark gave Tycho tremendous resources: an island with many families on it, and money to build an observatory. (One estimate is that this was 10% of the gross national product at the time!). So Tycho built vast instruments to set accurate sights on the stars and used multiple clocks and timekeepers.

He achieved his goal of measuring to one minute of arc. This was a tremendous feat before the invention of the telescope. His aim was to confirm his own picture of the universe, which was that the Earth was at rest, the Sun went around the Earth and the planets all went around the Sun.

Johannes Kepler (1571-1630)

Kepler was raised in the Greek geometric tradition so he believed God must have had some geometric reason for placing the six planets at the particular distances from the Sun that they occupied but he kept on wondering about this and made many trials to get the truth about it.

Kepler realized that Tycho's work could settle the question in one way or the other, so he went to work with Tycho in 1600. Tycho died the next year, Kepler stole the data and worked with it for nine years.

He reluctantly concluded that his geometric scheme was wrong. In its place, he found his three laws of planetary motion :

- I. The planets move in elliptical orbits with the sun at a focus.*
- II. In their orbits around the sun, the planets sweep out equal areas in equal times.*
- III. The squares of the times to complete one orbit are proportional to the cubes of the average distances from the sun.*

➤ **Kepler's laws :**

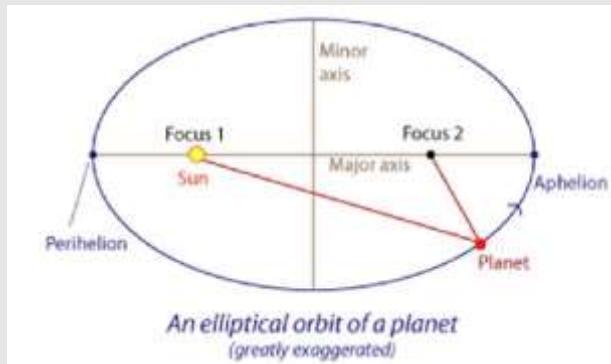
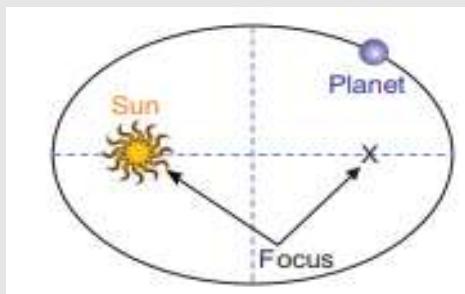
Johannes Kepler, after a life time study worked out three empirical laws which govern the motion of all planets and these are known as ***Kepler's laws of planetary motion***.

Statement of Kepler's laws:

✚ **Kepler's first law:** This law is known as the law of orbits.

It states that "***In their motion, planets describe ellipses about the Sun as one focus***".

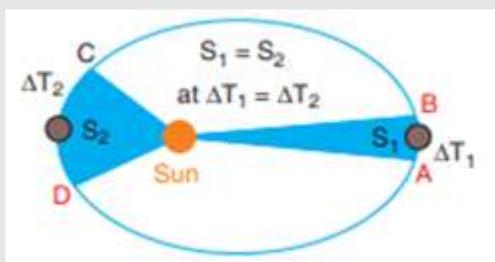
This can be illustrated in the figure below:



Note: An ellipse is a special curve in which the sum of the distances from every point on the curve to two other points is constant.

✚ **Kepler's second law:** This is called the law of areas.

It states that "***The line joining the Sun and the planet sweeps out equal areas in equal periods of time***".

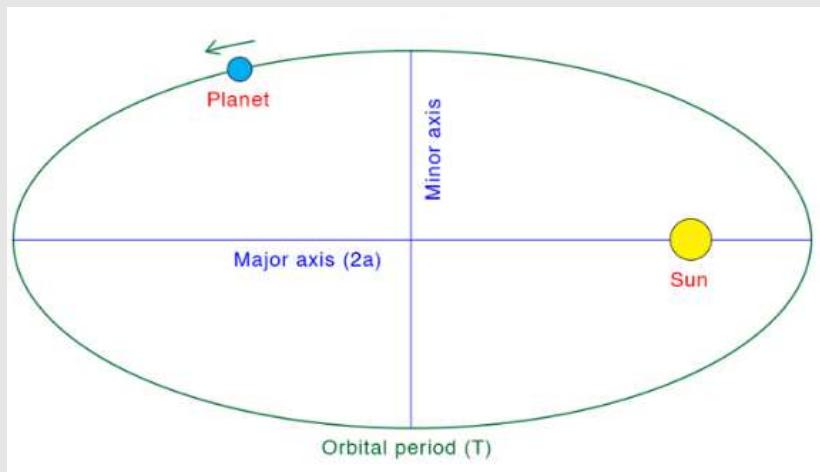


According to the law, if the time taken to move from A to B equals the time taken to move from C to D. Then: $S_1 = S_2$

➤ **Kepler's third Law:** This is known as the law of periods.

It states that "*The square of the period T of revolution of any planet is proportional to the cube of its mean distance R from the Sun*".

$$T^2 \propto R^3$$



Suppose we have any two planets whose periods of revolution are T_1 and T_2 . With their mean distance from the Sun being R_1 and R_2 respectively.

According to Kepler's 3rd law, the following relation holds:

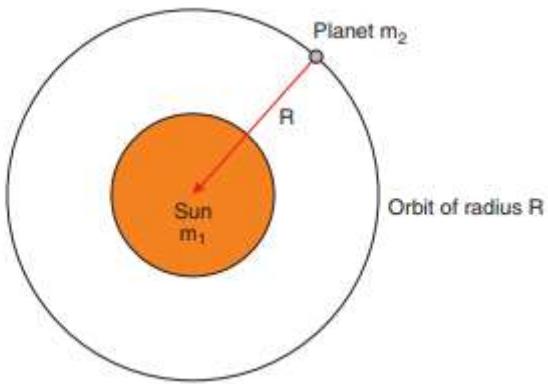
$$\frac{(T_1)^2}{(T_2)^2} = \frac{(R_1)^3}{(R_2)^3}$$

➤ Application of Kepler 's laws

Kepler's laws are used to predict the motion of satellites and launch Earth-observing satellites and observe their motion .

Assuming that a planet's orbit is circular (which is not exactly correct but is a good approximation in most cases), then the mean distance from the sun is constant –radius. Suppose, a planet of mass m_2 moving around the sun of mass m_1 . If the motion of the planet is circular, there are two types of forces: (a) Gravitational force of attraction F_1 between the sun and the planet,

Gravitational force of attraction of the sun and the planet:



$$F_1 = \frac{Gm_1m_2}{R^2}$$

(b) Centripetal force F_2 responsible for keeping the planet moving in a circular motion around the sun.

$$F_2 = \frac{m_2v^2}{R}$$

For the planet to move around the sun in orbit of constant radius: $F_1 = F_2$

$$\frac{m_2v^2}{R} = \frac{Gm_1m_2}{R^2}$$

$$v^2 = \frac{Gm_1}{R}$$

But linear velocity, $v = \omega R$ and $\omega = \frac{2\pi}{T}$ where ω is the angular velocity.

So,

$$\frac{4\pi^2R^2}{T^2} = \frac{Gm_1}{R}$$

$$R^3 = kT^2$$

The value of this constant k is $k = \frac{Gm_1}{4\pi^2}$ and M is the mass of the sun in this case.

This is true that $R^3 \propto T^2$

Remark: the formula: $R^3 = \frac{Gm_1}{4\pi^2} T^2$ is known as the Newton's version of Kepler's law. Where m_1 is the mass of the Sun.

Application example1: The distance of a planet from the sun is 5 times the distance between the earth and the sun. What is the time period of revolution of the planet?

Solution:

According to Kepler's law $(R_{\text{earth}})^3 \propto (T_{\text{earth}})^2$ and

$$(R_{\text{planet}})^3 \propto (T_{\text{planet}})^2 \text{ we have that } \frac{R_e^3}{R_p^3} = \frac{T_e^2}{T_p^2},$$

Making T_{planet} the subject gives:

$$T_{\text{planet}}^2 = T_{\text{earth}}^2 \cdot R_{\text{planet}}^3 / R_{\text{earth}}^3$$

But $R_{\text{planet}} = 5 R_{\text{earth}}$ which gives

$$5^3 R_{\text{earth}}^3 = R_{\text{planet}}^3$$

$$T_{\text{planet}}^2 = T_{\text{earth}}^2 \cdot (5)^3$$

But $T_{\text{earth}} = 1 \text{ year}$

$$T_{\text{planet}} = (5)^{\frac{3}{2}} \times T_{\text{Earth}}$$

$$T_{\text{planet}} = (5)^{\frac{3}{2}} \times 1 \text{ year} = 5^{\frac{3}{2}} \text{ years}$$



Theoretical learning Activity:

In groups of four, Brainstorm about the following question:

Describe Kepler's laws of planetary motion.



Practical learning Activity

In pair, perform the following problems about the application of Kepler's laws

- 5) Titan, the largest moon of Saturn has a mean orbital radius of 1.22×10^9 m. The orbital period of Titan is 15.95 days. Hyperion, another moon of Saturn, orbits at a mean radius of 1.48×10^9 m. Use Kepler's third law of planetary motion to predict the orbital period of Hyperion in days.
- 6) The planet Venus orbits the Sun with a mean orbital radius of 1.076×10^{11} m. The mass of the Sun is 1.99×10^{30} kg. Using Newton's version of Kepler's third law, calculate the orbital period of Venus.
- 7) Earth has an orbital period of 365 days and its mean distance from the Sun is 1.495×10^8 km. The planet Pluto's mean distance from the Sun is 5.896×10^9 km. Using Kepler's third law, calculate Pluto's orbital period in Earth days.

|



Points to remember



Kepler's first law: This law is known as the law of orbits.

It states that "*In their motion, planets describe ellipses about the Sun as one focus*".



Kepler's second law: This is called the law of areas.

It states that "*The line joining the Sun and the planet sweeps out equal areas in equal periods of time*".



Kepler's third Law: This is known as the law of periods.

It states that "*The square of the period T of revolution of any planet is proportional to the cube of its mean distance R from the Sun*".



Learning outcome 1.2 : Formative Assessment

1. From Kepler's law of orbit, we can infer that the Sun is located _____ of the planet's orbit.
 - a) at the centre
 - b) at one of the foci
 - c) at both foci
 - d) anywhere along the semi-minor axis

2. Kepler's laws of planetary motion replaced circular orbits with _____

- elliptical orbits
- parabolic orbits
- conical orbits
- hyperbolic orbits

3. Kepler's laws of planetary motion were proposed only for _____

- our sun
- any star in our galaxy
- any star in the universe
- stars of other solar systems

4. What is the value of universal gravitational constant?

- 6.022×10^{23}
- $6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$
- $1.602 \times 10^{-19} \text{ C}$
- 9.81 m/s^2

5. The value of universal gravitational constant changes in which of the following medium?

- Air
- Water
- Plasma
- The gravitational constant is independent of the medium

6. Our understanding of the elliptical motion of planets about the Sun spanned several years and included contributions from many scientists.

- Which scientist is credited with the collection of the data necessary to support the planet's elliptical motion?
- Which scientist is credited with the long and difficult task of analyzing the data?
- Which scientist is credited with the accurate explanation of the data?

7. Imagine that the star Altair has a system of three planets: Betazed, Charybdis and Deneb.

- Which of Kepler's Laws would you use to determine the length of a year on each planet?
- Assume that the orbits of Charybdis and Deneb have semi-major axes of 2 AU and 3.5 AU, respectively. Which planet travels more rapidly along its orbit?

8. Suppose a small planet is discovered that is 14 times as far from the sun as the Earth's distance from the sun ($1.5 \times 10^{11} \text{ m}$). Use Kepler's law of harmonies to predict the orbital period of such a planet. GIVEN: $T^2/R^3 = 2.97 \times 10^{-19} \text{ s}^2/\text{m}^3$

9. The average orbital distance of Mars is 1.52 times the average orbital distance of the Earth. Knowing that the Earth orbits the sun in approximately 365 days, use Kepler's law of harmonies to predict the time for Mars to orbit the sun.

Learning Outcome 1.3: Describe energy considerations in planetary, rocket and satellite motion



Duration: 5 hours



Learning outcome 1.3. Objectives:

By the end of the learning outcome, the trainee will be able to:

1. Describe correctly the terms gravitational field and gravitational potential as used in Physics.
2. Demonstrate clearly the expression of total mechanical energy of two object system.
3. Identify correctly the classifications of satellite orbit and spacecraft propulsion systems.
4. Explain clearly the three cosmic velocities.



Resources

Equipment	Tools	Materials
- Computer	- PhET simulations of satellite orbit and spacecraft propulsion.	- markers
- Projector	- Whiteboard - Chalkboard	- chalks - Scientific calculator - Textbooks

Advance preparations: prepare relevant tools to demonstrate mechanical energy



Content 2: Gravitational field

➤ Meaning of gravitational field

An approach to describing interactions between objects that are not in contact enables us to look at the gravitational interaction in a different way, using the concept of a **gravitational field** that exists at every point in space. When a particle of mass **m** is placed at a point where the gravitational acceleration is **g**, the particle experiences a force. $\mathbf{F} = \mathbf{mg}$. In other words, we imagine that the field exerts a force on the particle rather than considering a direct interaction between two particles.

In physics, a gravitational field is *a model used to explain the influences that a massive body extends into the space around itself, producing a force on another massive body*.

Thus, a gravitational field is used to explain gravitational phenomena, and is measured in newtons per kilogram (N/kg).



Figure : Gravitational field lines

➤ Mathematical treatment of gravitational field

The gravitational field is the gravitational force exerted per unit mass on a small mass at a point in the field.

Like force, it is a vector quantity: a point mass **M**, at the origin produces the gravitational field whose magnitude is given by:

$$g = -\frac{GM}{r^2}$$

The negative sign shows that at each point, the value of **g** is directed opposite to **r**. i.e. towards the central mass.

What is the gravitational field strength at the surface of Jupiter (mass 1.9×10^{27} kg, radius 7.1×10^7 m)?

How far from the centre of the Moon is the Earth-Moon neutral point, where the Earth and the Moon gravitational field strengths are equal in magnitude but opposite in direction?

$$M_E = 6.0 \times 10^{24} \text{ kg}$$

$$M_M = 7.4 \times 10^{22} \text{ kg}$$

The radius of Moon's orbit (assumed to be circular) is: 3.8×10^8 m.

➤ Gravitational potential energy

Meaning of gravitational potential energy

The gravitational potential V at a point is defined numerically as work done in taking a uniform mass from infinity to that point.

The potential at infinity is conventionally taken as *zero*. Consider a unit mass which is taken from infinity to point x , i.e. at a distance r from the center of the earth.

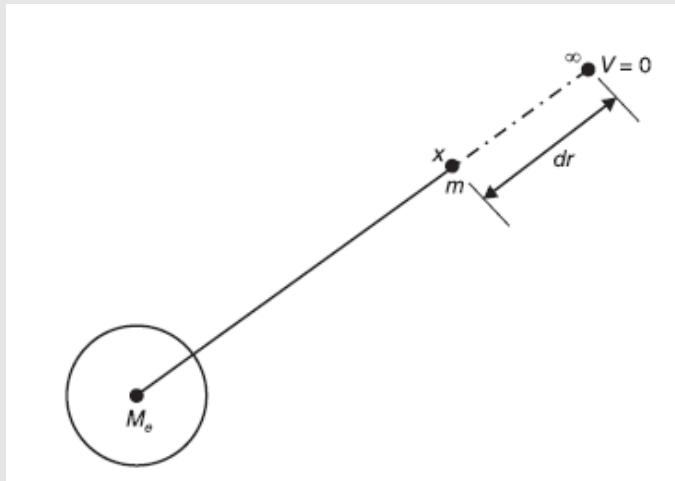


Figure 12. Gravitational potential at a point.

The force of attraction between the earth and the mass m is:

$$F = \frac{GM_e m}{r^2}$$

Where r is the distance of the mass from the centre of the earth. The work done by gravitational force in moving a mass through a distance dr towards the earth is:

$$dv = F dr = \frac{GM_e m}{r^2} dr$$

Since we are considering a unit mass, $m = 1 \text{ kg}$

$$\therefore dv = \frac{GM_e}{r^2} dr$$

So, the total work done in moving a unit mass from infinity to a point located at distance R from the center of the earth is obtained by integration:

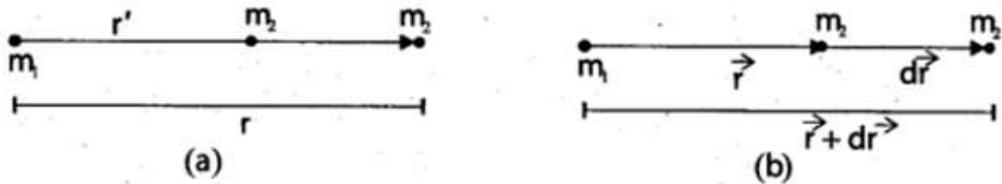
$$\int_0^V dv = \int_0^R \frac{GM_e}{r^2} dr$$

$$V = -\frac{GM_e}{R}$$

Notes: The negative sign indicates that potential at infinity is higher than potential at a point closer to the center of the earth.

Expression of gravitational potential energy associated with any pair of particles of masses, m_1 and m_2 .

Two masses m_1 and m_2 are initially separated by a distance r' . Assuming m_1 to be fixed in its position, work must be done on m_2 to move the distance from r' to r .



Two distant masses changing the linear distance

To move the mass m_2 through an infinitesimal displacement $d\vec{r}$ from \vec{r} to $\vec{r} + d\vec{r}$, work has to be done externally. This infinitesimal work is given by

$$dW = \vec{F}_{ext} \cdot d\vec{r} \quad \dots(1)$$

The work is done against the gravitational force, therefore

$$\vec{F}_{ext} = \frac{Gm_1m_2}{r^2} \vec{F}_G \quad \dots(2)$$

Substituting equation (2) in (1), we get

$$dW = \frac{Gm_1m_2}{r^2} \hat{r} \cdot d\vec{r} \quad \dots(3)$$

$$d\vec{r} = dr \hat{r} \Rightarrow dW = \frac{Gm_1m_2}{r^2} \hat{r} \cdot (dr \hat{r})$$

$\hat{r} \cdot \hat{r} = 1$ (Since both are unit vectors)

$$\therefore dW = \frac{Gm_1m_2}{r^2} dr \quad \dots(4)$$

Thus the total work for displacing the particle from r' to r is

$$W = \int_{r'}^r \frac{Gm_1m_2}{r^2} dr \quad \dots(5)$$

$$W = - \left(\frac{Gm_1m_2}{r} \right) \Big|_{r'}^r$$

$$W = - \frac{Gm_1m_2}{r} + \frac{Gm_1m_2}{r'} \quad \dots(6)$$

➤ Expression of total mechanical energy of two object system

- ✚ **Total Mechanical Energy:** It is defined as the sum of both potential and kinetic energy an object has or may have.
- ✚ **Potential Energy** is the stored energy of an object. Its formula can vary depending on situations.

For gravitational potential energy, the formula is $U = mgh$,

where **m** is mass, **g** is gravity, and **h** is height.

- ✚ **For elastic potential energy**, the formula is $U = \frac{1}{2}kx^2$,

where **k** is the spring constant and **x** is the compression or elongation of the spring.

If there are different forms of potential energy present in a system, the total potential energy is the sum of the different forms of potential energy.

- ✚ **Kinetic Energy** is the energy an object has due to motion.

Its formula is $K = \frac{1}{2}mv^2$ for translational motion.

For rotational motion, rotational kinetic energy is given by: $K = \frac{1}{2}I\omega^2$

where **I** is the moment of inertia, **ω** is the angular velocity which is the rate of rotation around a fixed point or axis

If there are different forms of kinetic energy present in a system, the total kinetic energy is the sum of the different forms of kinetic energy.

- ✚ **Calculating Total Mechanical Energy:** To calculate total mechanical energy, the following equation can be used:

$$\mathbf{E} = \mathbf{K} + \mathbf{U}$$

Where **K** is the Kinetic energy and **U** is the potential energy. Both kinetic energy and potential energy are measured in Joule (J).

Example1:

A child is sitting on a sled at the top of a 17m hill. The combined weight of the child and the sled is 42kg, and the child slides down the hill with an initial velocity of 18.263 m/s.

Determine the total mechanical energy?

Solution :

$$\begin{aligned}E &= K + U \\&= \frac{1}{2}mv^2 + mgh \\&= \frac{1}{2}(42 \text{ kg})(18.263 \text{ m/s})^2 + (42 \text{ kg})(9.81 \text{ m/s}^2)(17 \text{ m}) \\&\approx 7004 \text{ J} + 7004 \text{ J} \\E &\approx 14,008 \text{ J}\end{aligned}$$

The total mechanical energy for this system is equal to 14,008 Joule(J).

Example2 :

An 8kg block attached to a spring on a frictionless table oscillates in simple harmonic motion. At some instant, the distance from the block to the equilibrium point of the spring is 30cm (0.03m), the spring constant is 35 N/m, and the block has a velocity of 6.27×10^{-2} m/s. Determine the total mechanical energy at this instant?

Solution:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(8 \text{ kg})(0.0627^2 \text{ m/s})^2 = 0.01575 \text{ J}$$

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(35 \text{ N/m})(0.03 \text{ m})^2 = 0.01575 \text{ J.}$$

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(35 \text{ N/m})(0.03 \text{ m})^2 = 0.01575 \text{ J.}$$

Therefore, total mechanical energy is equal to $E = U + K = 0.01575 \text{ J} + 0.01575 \text{ J} = 0$.

The total mechanical energy for this system is equal to 0.0315 J.

➤ ENERGY CONSERVATION IN GRAVITATIONAL FIELDS

Conservation of energy tells us that the total energy of the system is conserved, and in this case, the sum of kinetic and potential energy must be constant. This means that every change in the kinetic energy of a system must be accompanied by an equal but opposite change in the potential energy.

Satellites have kinetic energy due to their motion and potential energy due to their position with respect to the center of the earth. A satellite of mass m in orbit round the earth moving at a velocity v has kinetic energy.

If the satellite is circling the earth at a radius r it has a centripetal force which balances with gravitational force of attraction with which the earth is attracting the satellite. If M_e is the mass of the earth;

Centripetal force = Gravitational force

$$\frac{mv^2}{r} = \frac{GM_e m}{r^2}$$

$$mv^2 = \frac{GM_e m}{r}$$

$$\frac{1}{2} mv^2 = \frac{GM_e m}{2r}$$

$$\therefore k.e. = \frac{GM_e m}{2r}$$

Assuming the zero potential energy in the earth's field at infinite, then at a distance r .

$$p.e. = -\frac{GM_e m}{r}$$

It is possible for a satellite at a point to possess both kinetic and potential energy. So, adding both expressions for kinetic energy (k.e) and potential energy (p.e) gives:

Total energy

$$E = k.e. + p.e.$$

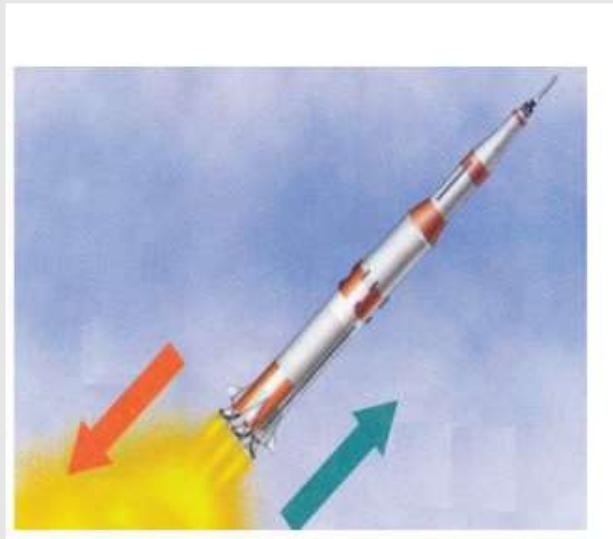
$$E = \frac{GM_e m}{2r} - \frac{GM_e m}{r}$$

$$E = -\frac{GM_e m}{2r}$$

- **Motion of rocket, satellite and spacecrafts**
- **Meaning of a rocket, satellite and spacecraft**

A rocket is a device that produces thrust by ejecting stored matter. A rocket moves forward when hot gas expelled from its engine pushes it in the opposite direction. In a rocket, a fuel is burned to make a hot gas and this hot gas is forced out of narrow nozzles in its back parts.

The motion of a rocket happens in the same way as a balloon moves when pumped and released into the air.



A rocket

- **A satellite** is an artificial or a natural body placed in orbit round the earth or another planet in order to collect information or for communication.

Communication satellites are satellites that are used specifically to communicate. Part of that communication will be the usual commands and signals we get from any satellite. The payload of the satellite consists of huge collection of powerful radio transmitters and a big dish or something like that, to enable it to talk to things on the ground. And we'll use them to transmit TV signals, to transmit radio signals, and in some cases, it might be to transmit internet signals. So, all of that gets turned into radio somehow and transmitted up into space and then bounced back down somewhere else.

A satellite doesn't fall straight down to the Earth because of its velocity. Throughout a satellite's orbit there is a perfect balance between the gravitational force due to the Earth, and the centripetal force necessary to maintain the orbit of the satellite.



A spacecraft is a vehicle or machine designed to fly in outer space.



Figure 2: Spacecraft

➤ Spacecraft Propulsion

Spacecraft Propulsion is characterized in general by its complete integration within the spacecraft (e.g. satellites). Its function is to provide forces and torques in (empty) space to:

- transfer the spacecraft: used for interplanetary travel
- Position the spacecraft: used for orbit control
- orient the spacecraft: used for altitude control

➤ Classification of satellites orbits and spacecrafts

⊕ Classification of spacecraft Propulsion Systems

Spacecraft propulsion can be classified according to the source of energy utilized for the ejection of propellant:

- **Chemical propulsion** uses heat energy produced by a chemical reaction to generate gases at high temperature and pressure in a combustion chamber. These hot gases are accelerated through a nozzle and ejected from the system at a high exit velocity to produce thrust force.
- **Electric propulsion** uses electric or electromagnetic energy to eject matter at high velocity to produce thrust force.
- **Nuclear propulsion** uses energy from a nuclear reactor to heat gases, which are then accelerated through a nozzle and ejected from the system at a high exit velocity to produce thrust force.

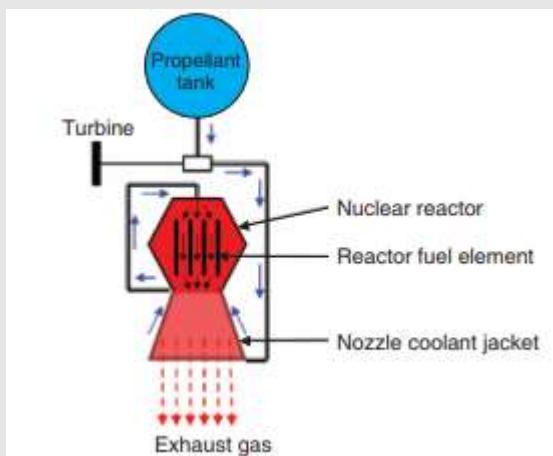


Figure 3: Figure: Nuclear propulsion

Notes: • While chemical and electric systems are used for the propulsion of today's spacecraft's, nuclear propulsion is still under study. Therefore, only chemical and electric propulsion will be dealt with in this module.

- **Movement of satellites in orbits**
- **Classifications of satellite orbits**

There are four classifications of satellite orbits that have been identified depending on the shape and diameter of each orbit:

- GEO (Geo-stationary earth orbit)
- MEO (medium earth orbit)
- LEO (Low earth orbit) and
- HEO (Highly elliptical orbit)

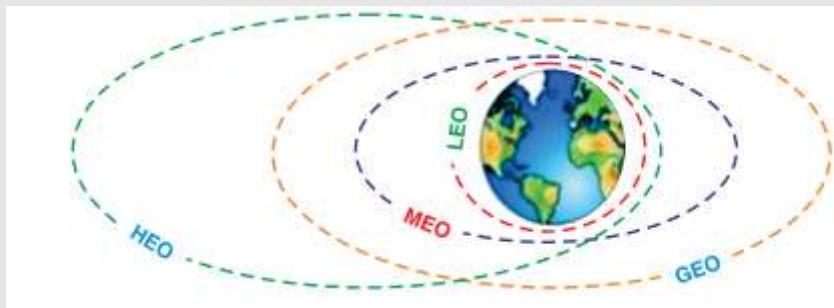


Figure 4: Classifications of satellite orbits

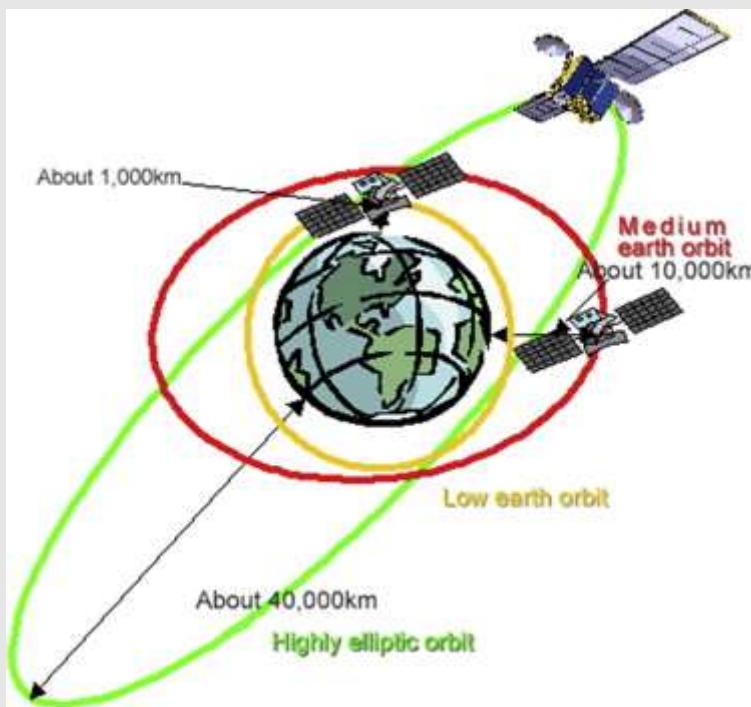


Figure 5: Satellite orbit and their approximate distances above the Earth surface

⊕ [GEO \(geostationary orbit\)](#)

A geostationary orbit or geosynchronous equatorial orbit (GEO) has a circular orbit 35,786 kilometers above the Earth's equator and following the direction of the Earth's rotation.

Most common geostationary satellites are either weather satellites or communication satellites relaying signals between two or more ground stations and satellites that broadcast signals to a large area on the planet. All radio and TV, whether satellite etc. are launched in this orbit.

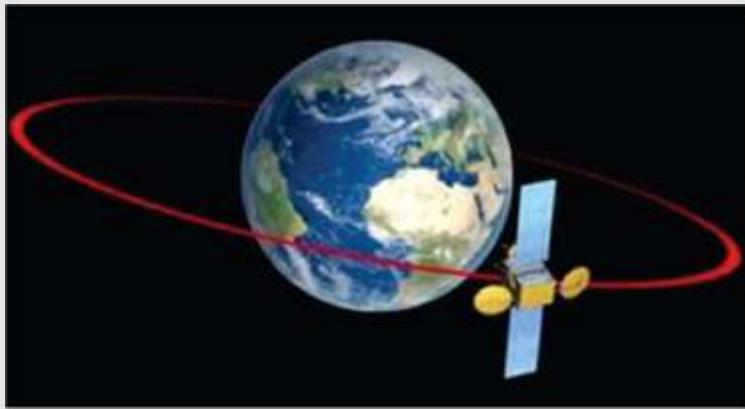


Figure 6: Geostationary orbit

Advantages of Geo-Stationary Earth Orbit

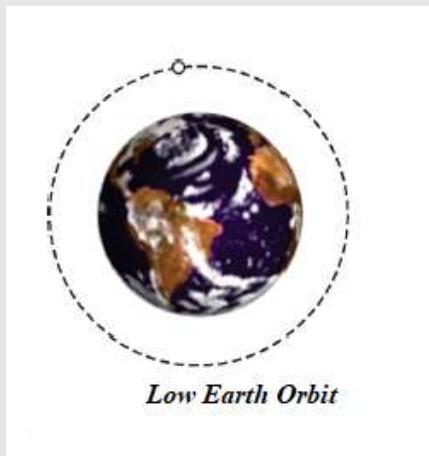
1. It is possible to cover almost all parts of the earth with just 3 geo satellites.
2. Antennas need not be adjusted every now and then, but can be fixed permanently.
3. The life-time of a GEO satellite is quite high usually around 15 years.

Disadvantages of Geo-Stationary Earth Orbit

1. Larger antennas are required for northern/southern regions of the earth.
2. High buildings in a city limit the transmission quality.
3. High transmission power is required.
4. These satellites cannot be used for small mobile phones.
5. Fixing a satellite at Geo stationary orbit is very expensive.

LEO (Low Earth Orbit)

Satellites in low Earth orbits are normally military reconnaissance satellites that can locate out tanks from 160 km above the Earth. They orbit the earth very quickly, one complete orbit normally taking 90 minutes.



Advantages of Low Earth Orbit

1. The antennas can have low transmission power of about 1 watt.
2. The delay of packets is relatively low.
3. Useful for smaller foot prints

Disadvantages of Low Earth Orbit

1. If global coverage is required, it requires at least 50-200 satellites in this orbit.
2. Special handover mechanisms are required.
3. These satellites involve complex design.
4. Very short life: Time of 5-8 years. Assuming 48 satellites with a life-time of 8 years each, a new satellite is needed every 2 months.
5. Data packets should be routed from satellite to satellite.

MEO (Medium Earth Orbit)

Medium Earth Orbit satellites move around the earth at a height of 6000- 20000 km above earth's surface. Their signal takes 50 to 150 milliseconds to make the round trip. MEO satellites cover more earth area than LEOs but have a higher latency. MEOS are often used in conjunction with GEO satellite systems.



Advantages of Medium Earth Orbit

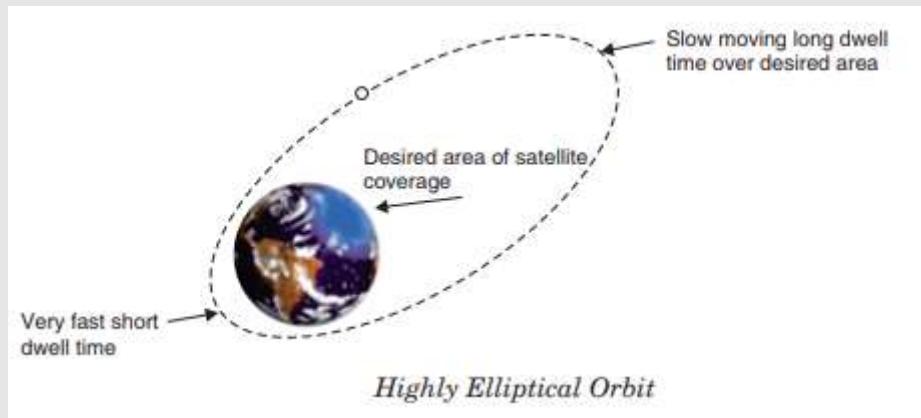
1. Compared to LEO system, MEO requires only a dozen satellites.
2. Simple in design.
3. Requires very few handovers.

Disadvantages of Medium Earth Orbit

1. Satellites require higher transmission power.
2. Special antennas are required.

⊕ HEO (Highly Elliptical Orbit)

A satellite in **elliptical orbit** follows an oval-shaped path. One part of the orbit is closest to the center of Earth (perigee) and another part is farthest away (apogee). A satellite in this type of orbit generally has an inclination angle of 64 degrees and takes about 12 hours to circle the planet. This type of orbit covers regions of high latitude for a large fraction of its orbital period.



➤ Orbital Velocity of Satellite:

Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth. For revolution of satellite around the earth, the gravitational pull provides the required centripetal force.

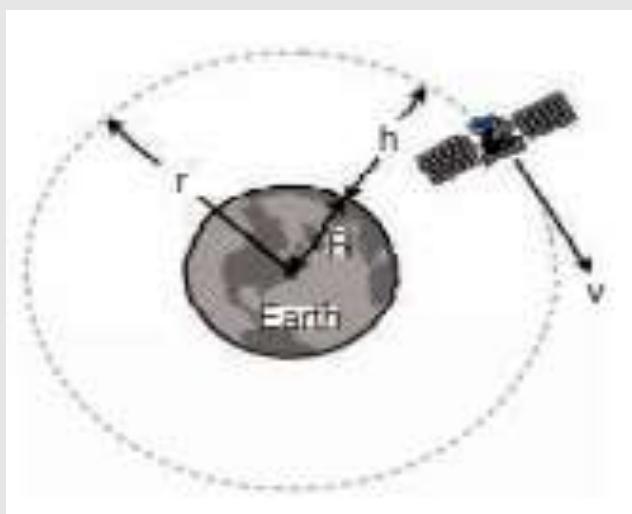


Figure 7: Orbital Velocity of Satellite.

$$\begin{aligned}
 \frac{mv^2}{r} &= \frac{GMm}{r^2} \\
 \Rightarrow v &= \sqrt{\frac{GM}{r}} \\
 v &= \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}
 \end{aligned}$$

[As $GM = gR^2$ and $r = R + h$]

EXAMPLE

Two satellites A and B go round a planet P in circular orbits having radii $4R$ and R respectively. If the speed of the satellite A is $3V$, what is the speed of the satellite B ?

Solution:

Orbital velocity of satellite

$$\begin{aligned}
 v &= \sqrt{\frac{GM}{r}} & \Rightarrow \frac{v_B}{v_A} = \sqrt{\frac{r_A}{r_B}} \\
 \Rightarrow \frac{v_B}{3V} &= \sqrt{\frac{4R}{R}} \Rightarrow v_B = 6V.
 \end{aligned}$$

EXAMPLE

A satellite is moving around the earth with speed v in a circular orbit of radius r . If the orbit radius is decreased by 1%, what is its speed?

Solution:

$$\text{Orbital velocity, } v = \sqrt{\frac{Gm}{r}}$$

$$\therefore v \propto \frac{1}{\sqrt{r}} \quad [\text{If } r \text{ decreases, then } v \text{ increases}]$$

$$\text{Percentage change in } v = \frac{1}{2} \text{ (percentage change in } r) = \frac{1}{2} (1\%) = 0.5\%$$

\therefore Orbital velocity increases by 0.5%.

➤ Time Period of Satellite

It is the time taken by satellite to go once around the earth.

$$\begin{aligned}
 & \therefore T = \frac{\text{circumference of the orbit}}{\text{orbit velocity}} \\
 \Rightarrow & T = \frac{2\pi r}{v} = 2\pi r \sqrt{\frac{r}{GM}} \quad [\text{As } v = \sqrt{\frac{GM}{r}}] \\
 \Rightarrow & T = 2\pi \sqrt{\frac{r^3}{GM}} = 2\pi \sqrt{\frac{(R+h)^3}{g R^2}} \quad [\text{As } GM = gR^2] \\
 \Rightarrow & T = 2\pi \sqrt{\frac{(R+h)^3}{gR^2}} = 2\pi \sqrt{\frac{R}{g} \left(1 + \frac{h}{R}\right)^{3/2}}
 \end{aligned}$$

Notes:

- From $T = 2\pi \sqrt{\frac{r^3}{GM}}$, it is clear that time period is independent of the mass of orbiting body and depends on the mass of central body and

radius of the orbit.

$$\begin{aligned}
 & \bullet \quad T = 2\pi \sqrt{\frac{r^3}{GM}} \\
 \Rightarrow & T^2 = \frac{4\pi^2}{GM} r^3 \quad \text{i.e.} \quad T^2 \propto r^3
 \end{aligned}$$

This is in accordance with Kepler's third law of planetary motion.

- Time period of nearby satellite,

$$\begin{aligned}
 \text{From} \quad T &= 2\pi \sqrt{\frac{r^3}{GM}} = 2\pi \sqrt{\frac{R^3}{gR^2}} = 2\pi \sqrt{\frac{R}{g}} \\
 & [\text{As } h = 0 \text{ and } GM = gR^2]
 \end{aligned}$$

For earth $R = 6400$ km and $g = 9.8$ m/s²

$$\Rightarrow T = 84.6 \text{ minute} \approx 1.4 \text{ h.}$$

- Time period of nearby satellite in terms of density of planet can be given as

$$\begin{aligned}
 T &= 2\pi \sqrt{\frac{r^3}{GM}} = 2\pi \sqrt{\frac{R^3}{GM}} = \frac{2\pi (R^3)^{1/2}}{\left[G \cdot \frac{4}{3} \pi R^3 \rho\right]^{1/2}} \\
 T &= \sqrt{\frac{3\pi}{G\rho}}
 \end{aligned}$$

EXAMPLE

A satellite is launched into a circular orbit of radius 'R' around earth while a second satellite is launched into an orbit of radius $1.02 R$. What is the percentage difference in the time periods of the two satellites?

Solution:

Orbital radius of second satellite is 2% more than the first satellite.

So from $T \propto (r)^{3/2}$, percentage increase in time period = $\frac{3}{2}$ (Percentage increase in orbital radius)

$$= \frac{3}{2} (2\%) = 3\%.$$

EXAMPLE

What is the periodic time of a satellite revolving above Earth's surface at a height equal to R , where R is the radius of Earth?

Solution:

$$\begin{aligned} T &= 2\pi \sqrt{\frac{(R+h)^3}{GM}} = 2\pi \sqrt{\frac{(R+R)^3}{gR^2}} \\ &= 2\pi \sqrt{\frac{8R}{g}} = 4\sqrt{2}\pi \sqrt{\frac{R}{g}} \quad [\text{As } h = R \text{ (given)}]. \end{aligned}$$

➤ Escape velocity of a rocket and satellite

What is escape velocity?

Escape velocity is the speed that an object needs to be traveling to break free of a planet or moon's gravity well and leave it without further propulsion.

For example, a spacecraft leaving the surface of Earth needs to be going 7 miles per second, or nearly 11200m/s to leave without falling back to the surface or falling into orbit.



Figure 8: A Delta II rocket blasting off. A large amount of energy is needed to achieve escape velocity.

Since escape velocity depends on the mass of the planet or moon that a spacecraft is blasting off of, a spacecraft leaving the moon's surface could go slower than one blasting off of the Earth, because the moon has less gravity than the Earth. On the other hand, the escape velocity for Jupiter would be many times that of Earth's because Jupiter is so huge and has so much gravity.

➤ **Description of three Cosmic velocities**

Definition:

Cosmic velocity is the initial velocity which a body must have to be able to overcome the gravity of another object.

We have: **The first cosmic velocity, the Second cosmic velocity (escape velocity) and the third cosmic velocity.**

➤ **The first cosmic velocity:**

As you know the satellites which were sent by a human are orbiting around the Earth. They had to be launched with a very high velocity, namely, with the first cosmic velocity.



Orbiting satellite

This velocity can be calculated using the gravitational force and the centripetal force of the satellite:

$$\frac{mv_1^2}{R_e} = \frac{GM_e m}{R_e^2}$$

$$v_1 = \sqrt{\frac{GM_e}{R_e}}$$

Where:

v = the value of the first cosmic velocity

M_e = the mass of the Earth

R_e = the radius of the Earth

G = the gravitational constant

We put the data into this formula and we obtain:

$$v_1 = \sqrt{\frac{6.67 \times 10^{-11} \times 6.2 \times 10^{24}}{6.4 \times 10^6}} = 7900 \text{ m/s}$$

Satellites must have extremely high velocity to orbit around the Earth. In fact, satellites go around the Earth at the height $h = 160$ km in order not to break into the atmosphere.

✚ Second cosmic velocity (escape velocity):

It is the speed needed to “break free” from the gravitational attraction of the Earth.

This value is calculated using the fact that as the body moves away from the Earth, the kinetic energy decreases and the potential energy increases. At infinity, both the energies are equal to zero, because, when the distance between the body and the Earth increases, the kinetic energy decreases and at infinity, it has the value of 0.

The value of the second cosmic velocity is calculated as follows:

$$\frac{1}{2}mv_2^2 = \frac{GM_e m}{R_e}$$

$$v_2 = \sqrt{\frac{2GM_e}{R_e}}$$

$$v_2 = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6.2 \times 10^{24}}{6.4 \times 10^6}} = 11200 \text{ m/s}$$

And finally, the practical curiosity.

$$v_2 = \sqrt{2 \times v_1}$$

We can also obtain the value of the second cosmic velocity by multiplying the value of the first cosmic velocity by the square root of two.

The third cosmic velocity:

The third cosmic velocity is the initial velocity which a body has to have to leave the Solar System and its value is: **$V_3 = 16.7 \text{ km/s}$** . At the surface of the Earth, this velocity is about 42 km/s. But due to its revolution, it is enough to launch the body with velocity 16.7 km/s in the direction of this movement.



Theoretical learning Activity:

In groups of four, brainstorm about the following problems:

- ✓ What is meant by gravitational field?
- ✓ What is meant by the term gravitational potential?
- ✓ What is a rocket? Briefly explain how rockets launch into space.
- ✓ Describe the classification of spacecraft propulsion.
- ✓ Explain the classifications of satellite orbit.

Points to remember

- ❖ *The gravitational potential* \mathbf{V} at a point is defined as work done in taking a uniform mass from infinity to that point.
- ❖ *Total Mechanical Energy* is defined as the sum of both potential and kinetic energy an object has or may have.
- ❖ *A rocket is* a device that produces thrust by ejecting stored matter.
- ❖ Classifications of satellite orbit: GEO, MEO, LEO and HEO.
- ❖ *Orbital velocity of a satellite* is the velocity required to put the satellite into its orbit around the earth.



Learning outcome 1.3: Formative Assessment

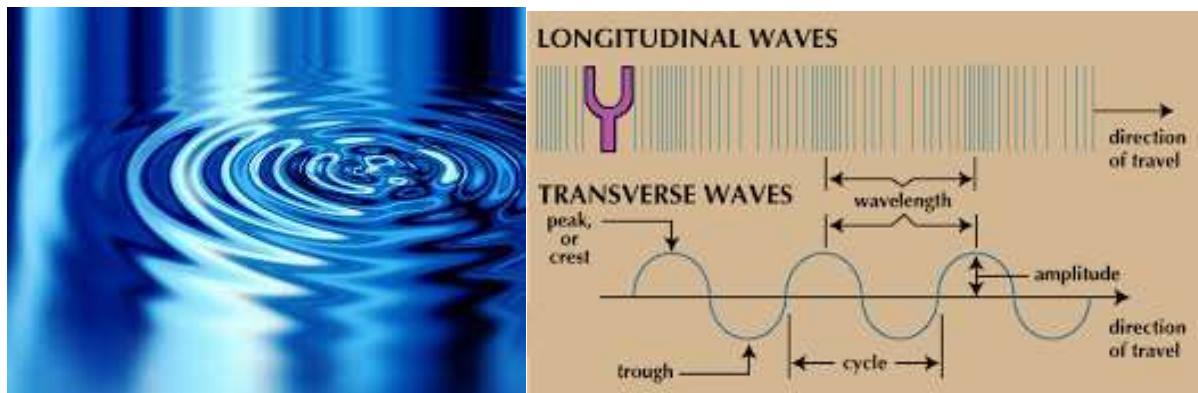
I. Choose the correct option:

1. The minimum velocity an object requires to break free from a celestial body's gravitational field is called the:
 - a. None of these answers is correct
 - b. Gravitational pull
 - c. Event horizon
 - d. Singularity
 - e. Escape velocity
2. The escape velocity is -----
(11.5km/s, 11.2km/s, 14km/s, 15.6km/s)

II. Answer the following questions:

1. The distance of Neptune and Saturn from sun are nearly 10^{13} and 10^{12} meters respectively. Assuming that they move in circular orbits, what will be their periodic times in the ratio?
2. A spherical planet far out in space has a mass M_0 and diameter D_0 . A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity which is equal to g . Derive the expression of g in terms of D .
3. At surface of earth, weight of a person is 72 N. What is his weight at height $R/2$ from surface of earth (R = radius of earth)?
4. Assuming earth to be a sphere of a uniform density, what is the value of gravitational acceleration in a mine 100 km below the earth's surface (Given $R = 6400$ km)?
5. If the gravitational force between two objects was proportional to $1/R$; where R is separation between them, then a particle in circular orbit under such a force would have its orbital speed v proportional to which value?
6. An earth satellite S has an orbital radius which is 4 times that of a communication satellite C . What is its period of revolution?

Learning unit2 : Apply oscillations and mechanical Waves



Learning Outcomes :

1. Describe oscillatory motion.
2. Describe effects of oscillations on systems.
3. Apply oscillatory motion in mechanical waves.

Learning Outcome 2.1: Describe oscillatory motion



Duration: 5hours



Learning outcome 2.1 Objectives :

By the end of the learning outcome, the trainees will be able to:

1. describe correctly simple harmonic motion.
2. describe clearly the energies of a simple harmonic oscillator
3. describe clearly Simple pendulum, Physical pendulum and Torsional pendulum.



Resources

Equipment	Tools	Materials
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-Computer - Projector	- PhET simulations of oscillatory motion - Simulation software - Chalkboard and chalks - Whiteboard and markers.	- Textbooks - Springs - Spherical balls - Stop watch
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Advance preparation:

- Demonstrate with PhET simulations of oscillatory motion on how to count the number of turns (complete oscillations) and measure frequency and period.
- Make necessary settings (simple pendulum) to determine the period of an oscillating system.



Content1. Oscillatory motion

➤ Definition of oscillatory motion

Oscillatory motion is defined as the to and fro motion of the body about its fixed position.

Oscillatory motion is a type of **periodic motion**. Examples of oscillatory motion are vibrating strings, swinging of the swing.

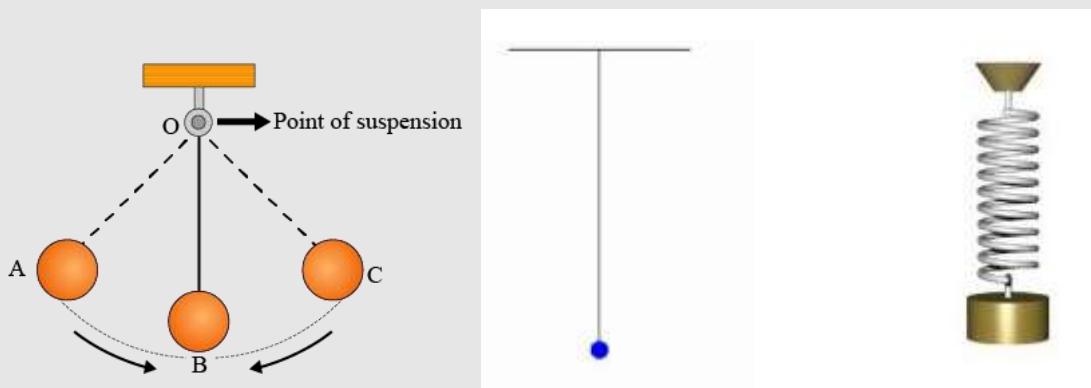
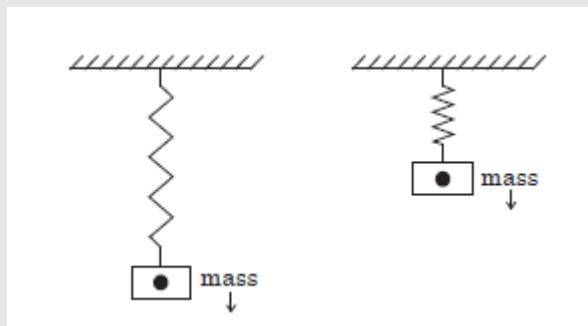


Figure 9: Examples of Oscillatory motion

➤ Motion of an object attached to a spring

Consider the system shown below with a spring attached to a support. The spring hangs in a relaxed, unstretched position. If you were to hold the bottom of the spring and pull downward, the spring would stretch. If you were to pull with just a little force, the spring would stretch just a little bit. And if you were to pull with a much greater force, the spring would stretch a much greater extent. Exactly what is the quantitative relationship between the amount of pulling force and the amount of stretch?

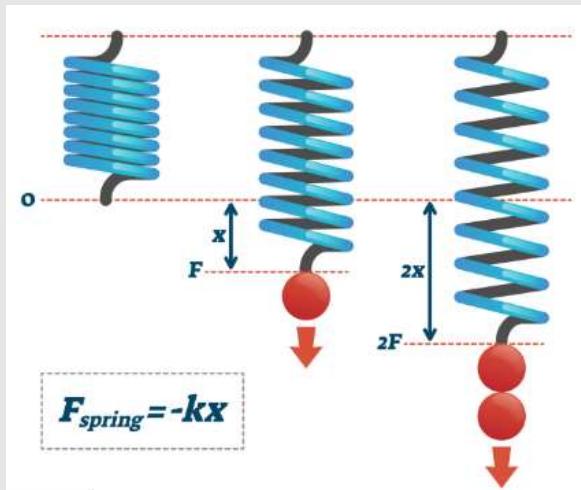


➤ Hooke's law

The relationship between the force applied to a spring and the amount of stretch was first discovered in 1678 by English scientist Robert Hooke.

This relationship states that, “*the amount that the spring extends is proportional to the amount of force with which it pulls*”.

Today this quantitative relationship between force and stretch is referred to as **Hooke's law**.



According to Hooke's law:

- The force F acting on an object attached to a spring is directly proportional to the displacement x from a fixed point and is always towards this point.

That is, $F \propto x \Rightarrow F = -kx$

where \mathbf{F} is the restoring force (force exerted upon the spring).

x is the amount that the spring stretches relative to its relaxed position, and \mathbf{k} is the proportionality constant, often referred to as **the spring constant**.

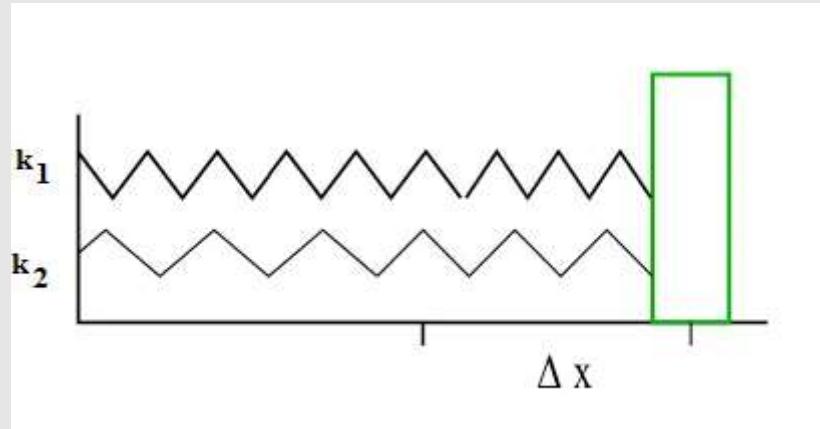
NOTE the following about the mathematical expression of Hooke's law:

- ✓ The spring constant \mathbf{k} is a positive constant whose value is dependent upon the spring which is being studied.
- ✓ A stiff spring would have a high spring constant. This is to say that it would take a relatively large amount of force to cause a little displacement.
- ✓ The units on the spring constant are Newton/meter (N/m).

- ✓ The negative sign in the above equation is an indication that the direction that the spring stretches is opposite the direction of the force which the spring exerts. For instance, when the spring was stretched below its relaxed position, x is *downward*. The spring responds to this stretching by exerting an *upward* force. That is, x and the F are in opposite directions.
- ✓ This equation works for a spring which is stretched vertically as well as for a spring that is stretched horizontally.

Springs in Parallel

Two springs are said to be in parallel when used in combination as in the figure below:



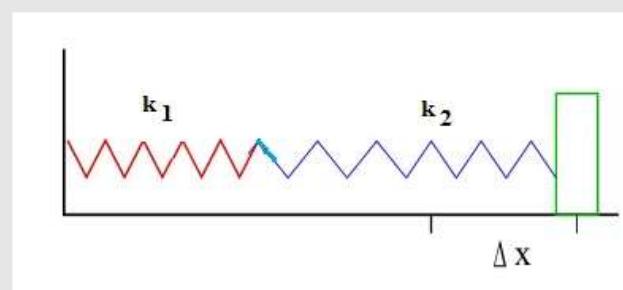
The two springs behave like one spring whose constant k is given by

$$k = k_1 + k_2$$

Therefore, Hooke's law takes the form : $F = -kx = -(k_1 + k_2)x$

Spring in Series

Two springs are said to be in series when used together as in the figure below:



The two springs behave like one spring whose constant k is given by:

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

2. What is the spring constant of a spring that needs a force of 3 N to be compressed from 40 cm to 35 cm?
3. What is the magnitude of the force required to stretch two springs of constants $k_1 = 100 \text{ N/m}$ and $k_2 = 200 \text{ N/m}$ by 6 cm if they are stretched together in parallel?
4. What is the magnitude of the force required to stretch two springs of constants 100 N/m and 200 N/m by 6 cm if they are connected in series?



Practical learning Activity:

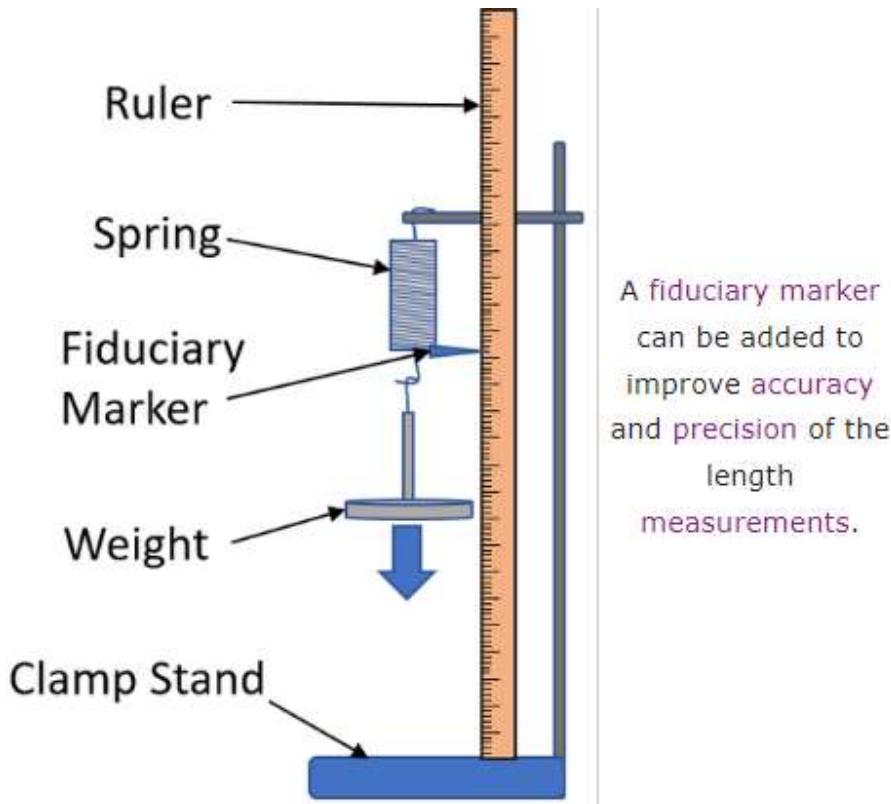
Work in pairs to perform the following task:

Aim

Investigate the relationship between the extension of a spring and the force applied to that spring.

Set up the equipment as shown in the diagram shown below.

1. Measure the original length of the spring using a ruler.
2. Attach a known weight (approximately 1N) to the spring.
3. Measure the new length of the spring.
4. Calculate the extension of the spring by subtracting the original length from the new length of the spring.
5. Repeat steps 2-4 up to around 6N.
6. Plot a scatter graph with the force of weight on the y-axis and the extension on the x-axis. The gradient of line of best fit will be the spring constant of the coil spring.



Points to Remember

- ✓ Oscillatory motion is defined as the to and fro motion of the body about its fixed position.
- ✓ Hooke's law states that: "the amount that the spring extends is proportional to the amount of force with which it pulls".

That is, $F = -kx$

- ✓ For springs used in parallel combination: $k = k_1 + k_2$
- ✓ For springs used in series combination: $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$



Content2. Simple harmonic motion (SHM)

➤ Definition of S.H.M

Simple harmonic motion (SHM) is a type of motion where the restoring force is directly proportional to the displacement and acts in the direction opposite to that of displacement.

- a. SHM is a repetitive movement back and forth through an equilibrium position (or central position).
- b. SHM is an oscillatory motion under a retarding force that is proportional to the amount of displacement from an equilibrium position.

This retarding force is given by Hooke's law which states that "**the force needed to extend or compress a spring by some distance x is proportional to that distance**".

That is $F \propto x$ or $F = -kx$

A restoring force is a force acting opposite to the displacement to bring the system back to the equilibrium position (rest position). It is only dependent on the displacement from the equilibrium position such as in Hooke's law.

➤ Conditions necessary for an oscillating system to be S.H.M.

The Conditions to produce simple harmonic motion include:

- ⊕ *The restoring force must be proportional to the displacement and act opposite to the direction of motion*
- ⊕ *There must be no drag forces or friction.*
- ⊕ *The frequency of oscillation does not depend on the amplitude.*
- ⊕ *The rate of change of velocity (acceleration) must be proportional to the displacement.*

➤ Definition of terms used in SHM

- ❖ **Time Period or Periodic Time T :** It is the time taken for the particle to complete one oscillation, that is, the time taken for the particle to move from its starting position and return to its original position.

This is generally denoted by the symbol T .

- ❖ **Frequency, f** means how many oscillations occur in one second.

Since the time period is the time taken for one oscillation, the frequency is expressed by; $f = \frac{1}{T}$

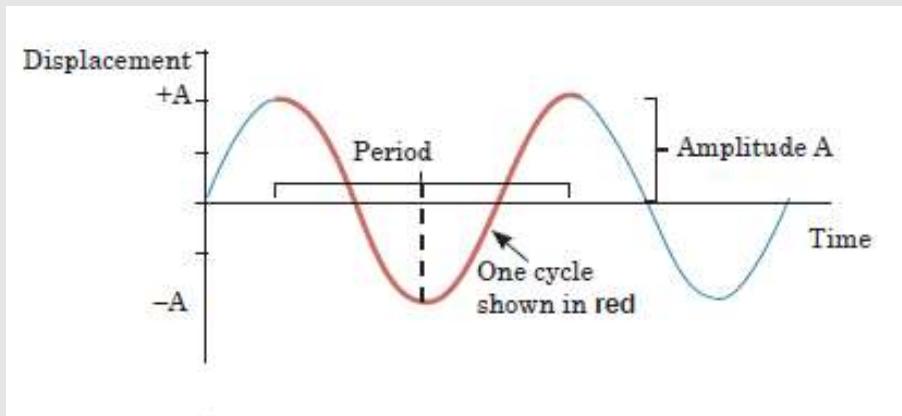
Where f is frequency in one oscillation and T is the time period

The frequency is measured in s^{-1} . This unit is known as the **hertz (Hz)** in honor of the physicist Heinrich Hertz.

- ❖ **Amplitude, A** is the maximum displacement of the particle from its resting position (central position) or mean position.

➤ **Graphical representation of S.H.M**

The figure below shows a displacement-time graph of a periodic motion of a particle:



➤ **Mathematical representation of S.H.M**

From the above graph, the displacement for a body in SHM can be represented as:

$$\text{Displacement, } x = A \sin(\omega t)$$

- ❖ **Angular velocity (ω)**: Angular velocity is the rate of change of angular displacement. It is measured in (rad/s).

This is related to periodic time according to equation $\omega = \frac{2\pi}{T}$

- ❖ **Linear velocity (v)**: Linear velocity is the rate of change of linear displacement. It is given by

$$v = \frac{dx}{dt} \Rightarrow v = A\omega \cos(\omega t) \text{ and measured in (m/s).}$$

But from trigonometric identities;

$$\sin^2 \omega t + \cos^2 \omega t = 1$$

Then

$$\cos \omega t = \pm \sqrt{1 - \sin^2 \omega t}$$

So

$$v = \pm \omega \sqrt{A^2 - A^2 \sin^2 \omega t}$$

Substituting equation, $x = A \sin(\omega t)$ into this equation gives;

$$v = \pm \omega \sqrt{A^2 - x^2}$$

❖ **Linear acceleration (a)** of a particle is the rate of change of linear velocity of that particle with time. It is measured in m/s^2 .

This is given by: $a = \frac{dv}{dt} \Rightarrow a = -A\omega^2 \sin(\omega t) \Leftrightarrow a = -\omega x$

Maximum acceleration is obtained when displacement x is maximum.

That is, when displacement is equal to amplitude A .

So, equation for acceleration becomes; $a = -\omega A$

EXAMPLE1

A particle moving with SHM has velocities 4 cm/s and 3 cm/s at distances 3 cm and 4 cm respectively from equilibrium position. Find

- (a) The amplitude of oscillation
- (b) The period
- (c) Velocity of the particle as it passes through the equilibrium position.

Solution:

Given $v_1 = 4 \text{ cm/s}$, $x_1 = 3 \text{ cm}$, $v_2 = 3 \text{ cm/s}$, $x_2 = 4 \text{ cm}$

From equation 2-9;

$$v_1 = \pm \omega \sqrt{A^2 - x_1^2}, v_2 = \pm \omega \sqrt{A^2 - x_2^2}$$

$$(a) \quad 4 = \pm \omega \sqrt{A^2 - 3^2}$$

$$3 = \pm \omega \sqrt{A^2 - 4^2}$$

Dividing these two equations gives

$$\frac{4}{3} = \frac{\sqrt{A^2 - 9}}{\sqrt{A^2 - 16}}$$

Squaring both sides will give;

$$\Rightarrow \frac{16}{9} = \frac{A^2 - 9}{A^2 - 16}$$

$$\Rightarrow A = 5 \text{ cm}$$

(b) Let us find the period at a velocity of 4 cm/s and displacement 3 cm. Both cases give the same value of angular velocity ω .

$$T = \frac{2\pi}{\omega} = \frac{2\pi\sqrt{5^2 - 9}}{4} = 2\pi \text{ or } 6.28 \text{ sec.}$$

$$\text{Hence, } \omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1 \text{ rad s}^{-1}$$

(c) As the particle passes the equilibrium position, it has the maximum velocity;

$$v_{\max} = +1 \times 5 = 5 \text{ cm/s}$$

EXAMPLE 2

A simple pendulum has a period of 2.0 s and amplitude of swing 5.0 cm. Calculate the maximum magnitude of

(a) velocity of the bob

(b) acceleration of the bob.

Solution:

$$(a) v_{\max} = \frac{2\pi A}{T} = \frac{4\pi^2 \times 5}{4} = 15.71 \text{ m/s}$$

$$(b) a_{\max} = -\frac{4\pi^2 A}{T^2} = \frac{4\pi^2 \times 5}{4} = 49.39 \text{ m/s}^2 \text{ (Negative sign will be neglected for minimum acceleration.)}$$

Other application exercises:

1. A particle oscillates with simple harmonic motion, so that its displacement varies according to the expression $x = (5 \text{ cm})\cos(2t + \pi/6)$ where x is in centimeters and t is in seconds. At $t = 0$ find the following:

- (a) the displacement of the particle,
- (b) its velocity, and
- (c) its acceleration.
- (d) Find the period and amplitude of the motion.

Solution:

- (a) The displacement as a function of time is $x(t) = A \cos(\omega t + \phi)$. Here $\omega = 2/\text{s}$, $\phi = \pi/6 \text{ rad}$, and $A = 5 \text{ cm}$. The displacement at $t = 0$ is $x(0) = (5 \text{ cm})\cos(\pi/6) = 4.33 \text{ cm}$.
- (b) The velocity at $t = 0$ is $v(0) = -\omega(5 \text{ cm})\sin(\pi/6) = -5 \text{ cm/s}$.
- (c) The acceleration at $t = 0$ is $a(0) = -\omega^2(5 \text{ cm})\cos(\pi/6) = -17.3 \text{ cm/s}^2$.
- (d) The period of the motion is $T = \pi s$, and the amplitude is $A = 5 \text{ cm}$.

2. A 20 g particle moves in simple harmonic motion with a frequency of 3 oscillations per second and an amplitude of 5 cm.

- (a) Through what total distance does the particle move during one cycle of its motion?
- (b) What is its maximum speed? Where does that occur?
- (c) Find the maximum acceleration of the particle. Where in the motion does the maximum acceleration occur?

Solution:

- (a) The total distance d the particle moves during one cycle is from $x = -A$ to $x = +A$ and back to $x = -A$, so $d = 4A = 20 \text{ cm}$.
- (b) The maximum speed of the particle is $V_{\max} = \omega A = 2\pi f A = 2\pi \times 15 \frac{\text{cm}}{\text{s}} = 0.94 \text{ m/s}$. The particle has maximum speed when it passes through the equilibrium position.
- (c) The maximum acceleration of the particle is $a_{\max} = \omega^2 A = (2\pi f)^2 A = 17.8 \text{ m/s}^2$. The particle has maximum acceleration at the turning points, where it has maximum displacement.

The displacement of an object undergoing simple harmonic motion is given by the equation $x(t) = 3.00 \sin \left(8\pi t + \frac{\pi}{4} \right)$. Where x is in meters, t is in seconds

and the argument of the sine function is in radians.

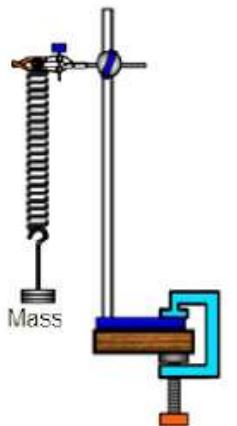
- (a) What is the amplitude of motion?
- (b) What is the frequency of oscillation?
- (c) What are the position, velocity and acceleration of the object at $t = 0$?



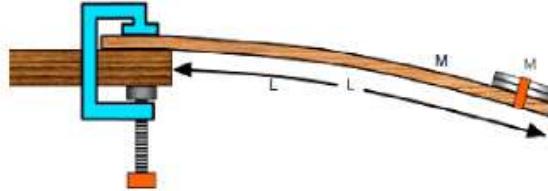
Practical learning Activity

Work in pairs to perform the following tasks:

Task1. Make a setup as shown in the figure below and explain what happens in each case when the mass is displaced. Use a stop watch to measure the time period in each case.



(a) Mass on the spring



(b) Mass on the meter rule

TASK2 :

Cantilever

Aim:

The aim of this activity is to determine the periodic time of a cantilever beam.

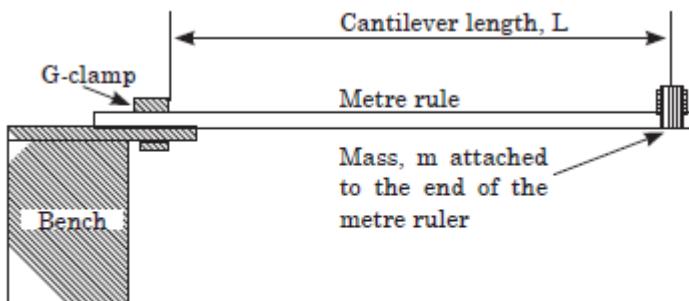
Required Materials:

Metre rule, G-clamp (or a wooden block), stop watch, set of masses ($4 \times 100\text{g}$), Cellotape and pair of scissors (can be shared).

Procedure:

(a) Use the apparatus, set up as shown in the diagram.

Start with a length L of 80.0 cm.



Determination of the periodic time of a cantilever

(b) Place a 200 g mass at 5 cm from the free end.

(c) Displace the mass slightly and release it.

(d) Use a stop-watch to measure the time taken t for 10 complete oscillations

(e) Calculate the time T for one oscillation.

(f) Repeat procedures from (b) to (e) for values of $L = 70.0, 60.0, 50.0, 40.0$ cm

(g) Record your observations in a table.

Also include the values of $\log T$ and $\log L$.

(h) Plot a graph of $\log T$ against $\log L$.

QUESTIONS:

(i) Measure the gradient, m of your graph.

(j) Calculate the intercept c on the vertical axis.

(k) Calculate the constant a of the rule from $c = \log a$.

(l) Calculate the period of a cantilever from $T = aL^m$

(m) Calculate the value of T from $\log T = m \log L + \log a$

for value of $L = 70.0$ cm.

(n) Compare and comment on the results in procedures (l) and (m).



Points to Remember

- ✓ **Time Period or Periodic Time T :** It is the time taken for the particle to complete one oscillation, that is, the time taken for the particle to move from its starting position and return to its original position.

This is generally denoted by the symbol T .

- ✓ **Frequency, f** means how many oscillations occur in one second.

Since the time period is the time taken for one oscillation, the frequency is expressed by; $f = \frac{1}{T}$

Where f is frequency in one oscillation and T is the time period

The frequency is measured in s^{-1} . This unit is known as the **hertz (Hz)** in honor of the physicist Heinrich Hertz.

- ✓ **Amplitude, A** is the maximum displacement of the particle from its resting position (central position) or mean position.

$$\text{Displacement, } x = A \sin(\omega t)$$

- ✓ **Angular velocity (ω):** Angular velocity is the rate of change of angular displacement. It is measured in (rad/s).

This is related to periodic time according to equation $\omega = \frac{2\pi}{T}$

- ✓ **Linear velocity (v):** Linear velocity is the rate of change of linear displacement. It is given by $v = \frac{dx}{dt} \Rightarrow v = A\omega \cos(\omega t)$ and measured in (m/s).
- ✓ **Linear acceleration (a)** of a particle is the rate of change of linear velocity of that particle with time. It is measured in m/s^2 .

This is given by: $a = \frac{dv}{dt} \Rightarrow a = -A\omega^2 \sin(\omega t) \Leftrightarrow a = -\omega x$



Content 3. Energies of the simple harmonic oscillator

➤ Kinetic energy of an oscillating system

Kinetic energy is the energy of a body in motion. Any change in velocity will also change the kinetic energy as shown in the equations below.

The velocity of an oscillating object at any point is given by equation:

$$v = \pm \omega \sqrt{A^2 - x^2}$$

From kinetic energy;

$$KE = \frac{1}{2}mv^2$$

$$KE = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$KE = \frac{1}{2}m\omega^2A^2 - \frac{1}{2}m\omega^2x^2$$

When the particle is in oscillatory motion, work is done against the force trying to restore it. The energy stored to perform this work is called **the potential energy**.

Force on the particle of mass m is given by; $F = ma$

But according to Hooke's law, the magnitude of the restoring force on the spring with spring constant k is given by: $F = kx$

The two forces are equal. Therefore, we have that;

$$F = ma = kx$$

but the magnitude of the acceleration is given by:

$$a = \omega^2x$$

Using this into the above equation gives us that;

$$ma = kx \Leftrightarrow m\omega^2x = kx$$

$$\Rightarrow k = m\omega^2$$

Thus, the kinetic energy of an oscillating system can also be expressed as:

$$K.E = \frac{1}{2} kA^2 - \frac{1}{2} kx^2$$

➤ **Elastic Potential energy of an oscillating system:**

The energy possessed by a stretched or a compressed spring is known as **the elastic potential energy**; as the spring's molecules are continually displaced or compressed relative to their normal distance apart. The potential energy (Pe) for an extension or compression of a spring can be obtained if we consider a force F acting on the spring and extends it through a small distance x, the initial value for the force (when the spring is at its equilibrium position) is Zero, $F_1=0$ and the final value for the force after stretching the spring is F.

The average value of the force is $F = \frac{F_1+F}{2} = \frac{0+F}{2} = \frac{F}{2}$, the elastic potential energy stored by the spring equals the average work done by the force in stretching the spring from its equilibrium position to the small distance x.

That is, $Pe = \text{average Work done}$,

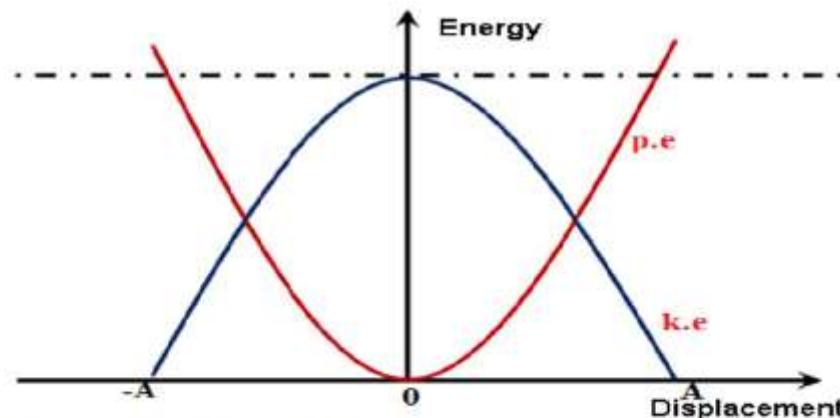
$$\text{work done} = \text{average spring force} \times \text{distance} = \frac{1}{2} Fx = \frac{1}{2} kx \cdot x = \frac{1}{2} kx^2$$

Therefore, the potential energy stored in a spring is given by the formula:

$$Pe = \frac{1}{2} kx^2 \text{ or } Pe = \frac{1}{2} m\omega^2 x^2$$

➤ **Total mechanical energy (T.E) of a simple harmonic oscillator**

In an oscillation, there is a constant interchange between the kinetic and potential forms and if the system does no work against resistive force its total energy is constant. The figure below illustrates the variation of potential energy and kinetic energy with displacement x.



Variation of potential and kinetic energy of an oscillating system

We know that the kinetic energy and potential energy are given by the expression:

$$K.E = \frac{1}{2}kA^2 - \frac{1}{2}kx^2 \quad \text{and} \quad P.E = \frac{1}{2}m\omega^2x^2$$

Using the equation $x = A\sin(\omega t)$ gives;

$$\begin{aligned} K.E &= \frac{1}{2}m\omega^2A^2 - \frac{1}{2}m\omega^2A^2\sin^2\omega t \\ K.E &= \frac{1}{2}m\omega^2A^2(1 - \sin^2\omega t) \quad \text{But } 1 - \sin^2\omega t = \cos^2\omega t \\ K.E &= \frac{1}{2}m\omega^2A^2\cos^2\omega t \\ P.E &= \frac{1}{2}m\omega^2A^2\sin^2\omega t \end{aligned}$$

The total energy of an oscillating system is given by:

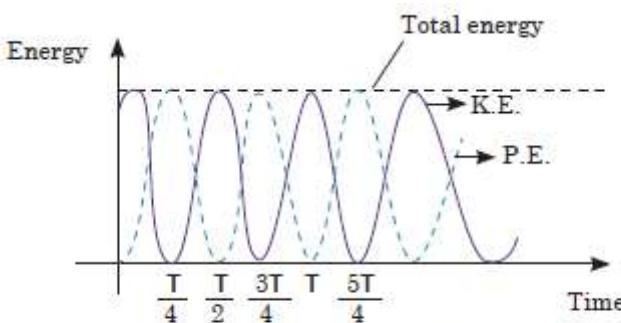
$$\text{Total energy} = K.E + P.E$$

$$E = \frac{1}{2}m\omega^2A^2 - \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2x^2$$

$$E = \frac{1}{2}m\omega^2A^2$$

➤ Representation of k.e, p.e and T.E on a graph

The figure below shows the variation of the kinetic energy, potential energy and the total mechanical energy with time for an oscillating system:



Variation of energy of an oscillating system with time

Example:

A 0.500-kg cart connected to a light spring for which the force constant is 20.0 N/m oscillates on a horizontal, frictionless air track.

- (a) Calculate the total energy of the system and the maximum speed of the cart if the amplitude of the motion is 3.00 cm.
- (b) What is the velocity of the cart when the position is 2.00 cm?
- (c) Compute the kinetic and potential energies of the system when the position is 2.00 cm.

Solution:

- (a) Using equation for the total energy:

$$E = \frac{1}{2}m\omega^2 A^2$$

$$E = \frac{1}{2} \times 2A^2 = \frac{1}{2} \times 20 \times (3 \times 10)^2 = 9 \times 10^{-3} J$$

Maximum K.E. energy is obtained when the cart is located at $x = 0$, and potential energy P.E = 0.

$$\therefore E_{\max} = \frac{1}{2}mv_{\max}^2$$

$$\begin{aligned}
 (b) \quad V_{\max} &= \sqrt{\frac{2E_{\max}}{m}} = \sqrt{\frac{2 \times 9 \times 10^{-3}}{0.5}} \\
 V_{\max} &= 0.19 m/s \\
 v &= \pm \sqrt{\frac{k}{m}(A^2 - x^2)} \\
 v &= \pm \sqrt{\frac{20}{0.5}(0.03^2 - 0.02^2)}
 \end{aligned}$$

$$v = \pm 0.141 \text{ m/s}$$

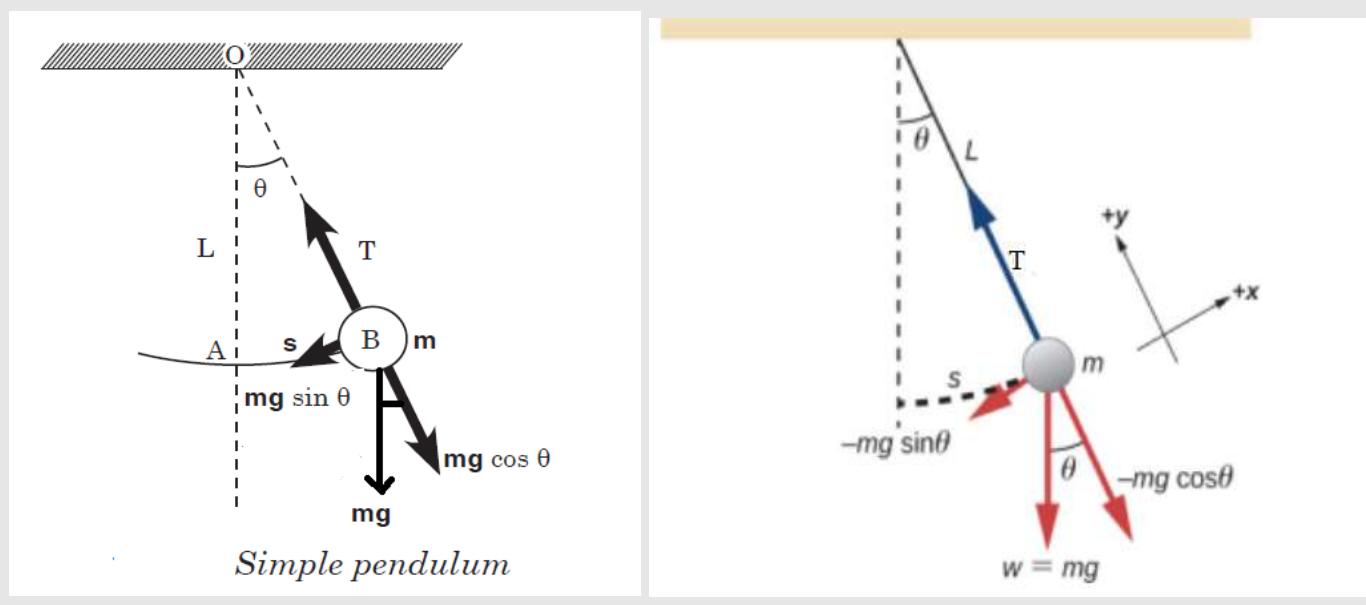
The positive and negative signs indicate that the cart could be moving to either the right or the left at this instant.

(c) $K.E = \frac{1}{2} m v^2 = \frac{1}{2} \times 0.5 \times 0.0141^2 = 5.0 \times 10^{-5} \text{ J}$

$$P.E = \frac{1}{2} kx^2 = \frac{1}{2} \times 20 \times 0.02^2 = 4.0 \times 10^{-3} \text{ J}$$

➤ Simple pendulum

A simple pendulum consists of a small bob of mass m suspended from a fixed support through a light, inextensible string of length L as shown on the figure below. This system can stay in equilibrium if the string is vertical. This position is called ***the mean position or the equilibrium position***. If the particle is pulled aside and released, it oscillates in a circular arc with the center at the point of suspension 'O'.



The driving force on the bob is always equal to the restoring force at any point during an oscillation but acts in opposite direction. This restoring force is a component of weight $mg \sin \theta$.

$$ma = -mg \sin \theta$$

$$a = -g \sin \theta$$

If the bob is slightly displaced and the angle θ is small, B is close to A and triangle AOB becomes a right angled triangle, then

$$\sin \theta = \frac{AB}{L} = \frac{s}{L}$$

Where s is the horizontal displacement of the bob, g is acceleration due to gravity and L is the length of the string;

$$\therefore a = -g \frac{s}{L}$$

$$\Rightarrow a = -\left(\frac{g}{L} \times s\right)$$

➤ **Factors affecting the periodic time of the simple pendulum**

From the above equation, the following are the factors affecting the periodic time of the simple pendulum;

- Length of string
- Angle from which pendulum is dropped
- Acceleration due to gravity
- Air resistance

Example1:

A small piece of lead of mass 40 g is attached to the end of a light string of length 50 cm and it is allowed to hang freely. The lead is displaced to 0.5 cm above its rest position, and released.

(a) Calculate the period of the resulting motion, assuming it is simple harmonic.

(b) Calculate the maximum speed of the lead piece. (Take $g = 9.81 \text{ ms}^{-2}$)

Solutions:

(a) To calculate the time period
equation 2-26 can be used

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$T = 2\pi \sqrt{\frac{0.5}{9.81}} = 1.42 \text{ s}$$

(b) The maximum speed is obtained by assuming that the total energy stored is equal to the total energy used.

$$\text{Total energy stored} = P.E$$

$$E_1 = mgh = 0.04 \times 9.81 \times 0.005 = 0.001962 \text{ J}$$

$$\text{Total energy used} = K.E$$

$$E_2 = \frac{1}{2}mv^2 = 0.5 \times 0.04 \times v^2 = 0.02 v^2$$

Since

$$E_1 = E_2,$$

$$0.02 v^2 = 0.001962$$

$$\Rightarrow v = \sqrt{\frac{0.001962}{0.02}} = 0.31 \text{ m/s}$$

Example2:

What happens to the period of a simple pendulum if the pendulum's length is doubled? What happens to the period if the mass of the suspended bob is doubled?

Solutions:

$$T_i = 2\pi \sqrt{\frac{L_i}{g}}$$

and

$$T_f = 2\pi \sqrt{\frac{L_f}{g}}$$

But

$$L_f = 2L_i$$

$$T_f = 2\pi \sqrt{2 \times \frac{L_i}{g}}$$

$$\Rightarrow T_f = \sqrt{2} \times T_i$$

Conclusion: The time period gets larger by $\sqrt{2}$ times. Changing the mass has no effect on the time period of a simple pendulum.

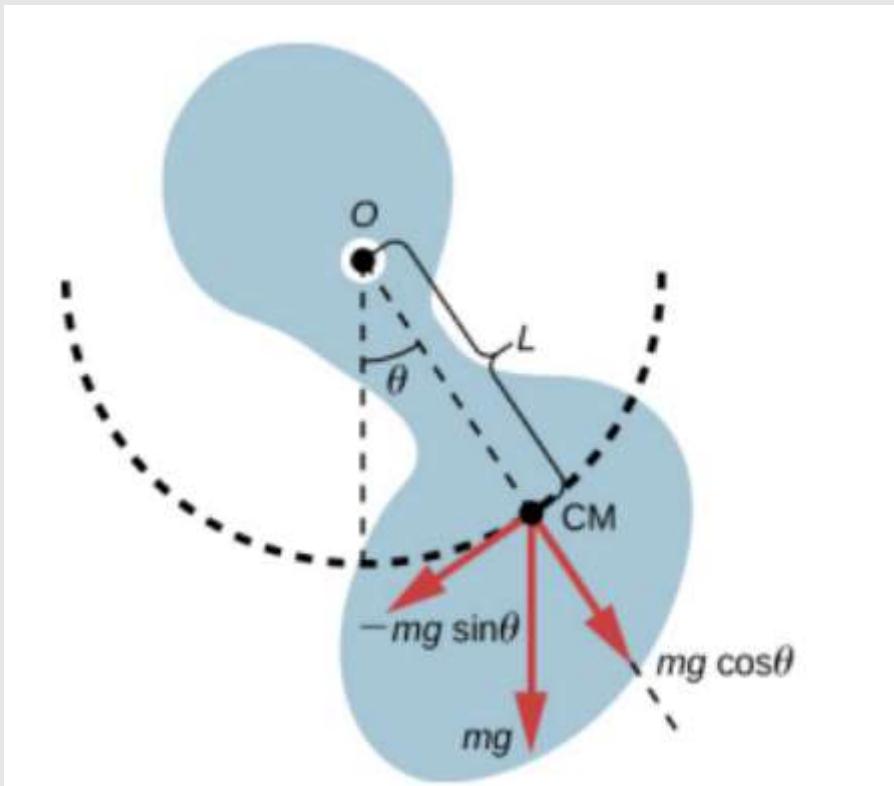
➤ Physical pendulum

A **physical pendulum** is any object that oscillates as a pendulum, but cannot be modeled as a point mass on a string.

As for the simple pendulum, the restoring force of the physical pendulum is the force of gravity.

With the simple pendulum, the force of gravity acts on the center of the pendulum bob. In the case of the physical pendulum, the force of gravity acts on the center of mass (CM) of an object. The object oscillates about a point O.

Consider an object of a generic shape as shown in the figure below:



The force of gravity acts on the center of mass (CM) and provides the restoring force that causes the object to oscillate. The minus sign on the component of the weight that provides the restoring force is present because the force acts in the opposite direction of the increasing angle θ .

When a physical pendulum is hanging from a point but is free to rotate, it rotates because of the torque applied at the CM, produced by the component of the object's weight that acts tangent to the motion of the CM. Taking the counterclockwise direction to be positive, the component of the gravitational force that acts tangent to the motion is $-mg \sin\theta$.

The magnitude of the torque is equal to the length of the radius arm times the tangential component of the force applied, $|\tau| = rF\sin\theta$.

Here, the length L of the radius arm is the distance between the point of rotation and the CM.

To analyze the motion, start with the **net torque**. Like the simple pendulum, consider only small angles so that $\sin\theta \approx \theta$.

The net torque is equal to the moment of inertia (I) times the angular acceleration (α)

$$I\alpha = \tau_{net} = L(-mg) \sin\theta.$$

Using the small angle approximation and rearranging gives:

$$\begin{aligned} I\alpha &= -L(mg)\theta; \\ I\frac{d^2\theta}{dt^2} &= -L(mg)\theta; \\ \frac{d^2\theta}{dt^2} &= -\left(\frac{mgL}{I}\right)\theta. \end{aligned}$$

Once again, the equation says that the second time derivative of the position (in this case, the angle) equals minus a constant ($-mgL/I$) times the position. The solution is:

$$\theta(t) = \Theta \cos(\omega t + \phi),$$

where Θ is the maximum angular displacement from the central (equilibrium) position.

The angular frequency is given by:

$$\omega = \sqrt{\frac{mgL}{I}}.$$

The time period is therefore given by:

$$T = 2\pi\sqrt{\frac{I}{mgL}}.$$

Note that for a simple pendulum, the moment of inertia is;

$$I = \int r^2 dm = mL^2 \text{ and the period reduces to } T = 2\pi\sqrt{\frac{L}{g}}.$$

➤ Torsional pendulum

A torsional pendulum consists of a disk (or some other object) suspended from a wire, which is then twisted and released, resulting in an oscillatory motion. The oscillatory motion is caused by a restoring torque which is proportional to the angular displacement.

The torsional pendulum is the disc suspended to the thin bar which creates twisting oscillations around the axis of the bar. The restoring force developed by twisting or torsional action creates oscillations in the disc.

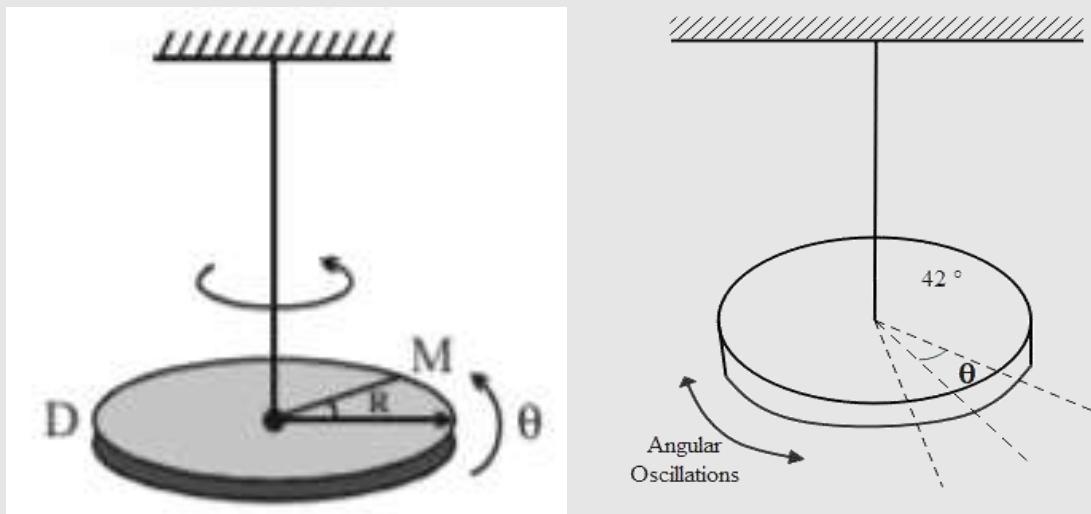


Figure 10: Torsional pendulum

If the initial angular displacement θ is given to the disc by applying a twisting torque, the thin rod generates the restoring torque, which causes the disc to revolve in the opposite direction.

The continuous twisting and releasing of the string/rod create oscillation in the torsional pendulum.

This mechanism creates simple harmonic motion in the torsional pendulum.

The required equations for the analysis of the torsional pendulum are listed below:

1) Restoring torque (T):

$$T = -C\theta$$

Here θ is angular twist and C is torque per unit twist of the pendulum that is given by,

$$C = \frac{\pi \times \eta \times r^4}{2l}$$

Where,

η = Modulus of rigidity

r = Wire radius

l = Length of wire

2) Torsional pendulum period equation:

The equation for the period of the original pendulum is given by,

$$T = 2\pi \sqrt{\frac{I}{C}}$$

3) Equation of modulus of rigidity for torsional pendulum:

The equation for the torsional rigidity of the torsional pendulum is given by,

$$\eta = \frac{8\pi I}{r^4} \left(\frac{L}{T^2} \right)$$

The restoring torque is directly proportional to the angle of twist in the wire that is given by,

$$T = -C\theta \quad (1)$$

Where C = Torsion constant

In angular motion, the equation for torque is,

$$T = I \times \alpha$$

$$\text{As, } \alpha = \frac{d^2\theta}{dt^2}$$

$$\text{Therefore, } T = I \frac{d^2\theta}{dt^2} \quad (2)$$

Where, I = Moment of inertia of the disc

Equating the equations 1 and 2 we get,

$$I \frac{d^2\theta}{dt^2} = -C\theta$$

$$\frac{d^2\theta}{dt^2} + \left(\frac{C}{I}\right)\theta = 0 \quad (3)$$

The equation of angular simple harmonic motion is given by,

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \quad (4)$$

Now by comparing equations 3 and 4 we get,

$$\omega^2 = \frac{C}{I}$$

$$\therefore \omega = \sqrt{\frac{C}{I}} \quad (5)$$

This is the equation for the angular speed of torsional pendulum.

Now the frequency of oscillation is given by,

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{C}{I}} \quad (6)$$

This is the equation for frequency of oscillation of the torsional pendulum.

Now the period of torsional pendulum is given by,

$$t = \frac{1}{f} = \frac{1}{\frac{1}{2\pi} \sqrt{\frac{C}{I}}}$$

$$\text{Period of torsional pendulum} = t = 2\pi \sqrt{\frac{I}{C}} \quad (7)$$

This is the required equation for the period of oscillation of the torsional pendulum.

From equation 7, the period of oscillation is,

$$t = 2\pi \sqrt{\frac{I}{C}}$$

Put value of $C = \frac{\pi \eta r^4}{2l}$

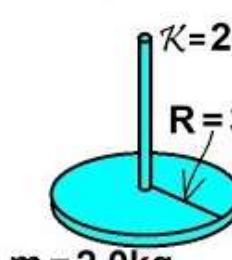
$$\therefore t = 2\pi \sqrt{\frac{I2l}{\pi \eta r^4}}$$

$$\therefore \eta = \frac{8\pi I}{r^4} \left(\frac{l}{t^2} \right)$$

This is the equation to determine **torsional rigidity of pendulum**

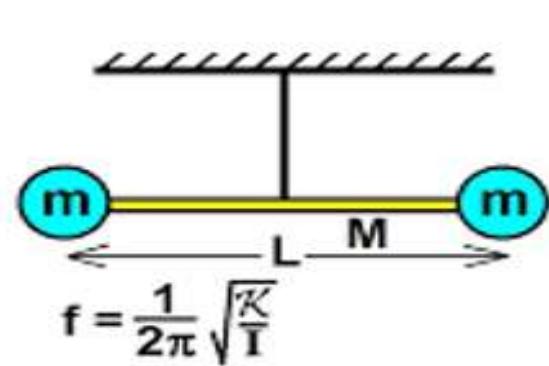
Exercises:

1. The figure below shows a torsional pendulum, find the frequency and the expression for the angular displacement as a function of time.



$\mathcal{K} = 20 \text{ Nm}$ $\theta_{\max} = 1.2 \text{ rad}$ $f = ?$
 $\theta(t) = ?$
 $R = 30 \text{ cm}$
 $m = 2.0 \text{ kg}$
 $I = \frac{1}{2} m R^2 = 0.09 \text{ kg-m}^2$
 $f = \frac{1}{2\pi} \omega = \frac{1}{2\pi} \sqrt{\frac{\mathcal{K}}{I}} = 2.4 \text{ Hz}$
 $\theta(t) = \theta_{\max} \sin \omega t$
 $= \theta_{\max} \sin(2\pi f t) = \dots$

2. For the torsional pendulum below, determine the value of the angular frequency and the time period of oscillation:

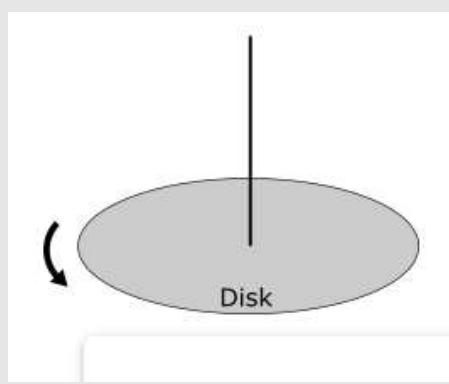


$$\begin{aligned}
 K &= 0.0010 \text{ Nm/rad} & f &= ? \\
 L &= 1.2 \text{ m} & T &= ? \\
 m &= 0.40 \text{ kg} \\
 M &= 2.6 \text{ kg} \\
 I &= 2(m)\left(\frac{L}{2}\right)^2 + \frac{1}{12}ML^2 \\
 I &= 0.6 \text{ kg} \cdot \text{m}^2
 \end{aligned}$$

Other exercises:

A torsion pendulum is made from a disk of mass $m = 6.7 \text{ kg}$ and radius $R = 0.7 \text{ m}$. A force of $F = 46.1 \text{ N}$ exerted on the edge of the disk rotates the disk $1/4$ of a revolution from equilibrium.

- What is the torsion constant of this pendulum?
- What is the minimum torque needed to rotate the pendulum a full revolution from equilibrium?
- What is the angular frequency of oscillation of this torsion pendulum?





Theoretical learning Activity:

In groups of four, brainstorm about the following problems:

- ✓ Explain the terms: simple pendulum, physical pendulum and torsional pendulum.
- ✓ Derive the time period for the above pendulums.



Practical learning Activity

In pair perform the following practical task:

Aim:

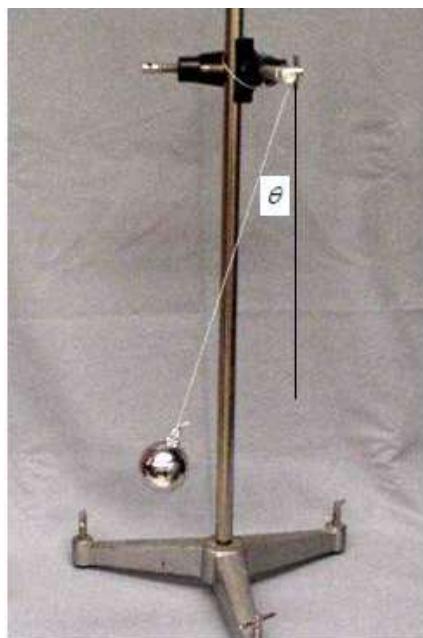
The aim of this activity is to determine the acceleration due to gravity using oscillation of a simple pendulum

Apparatus:

Cotton thread, small pendulum bob, metre rule, stopwatch, retort stand/clamp stand.

Procedure:

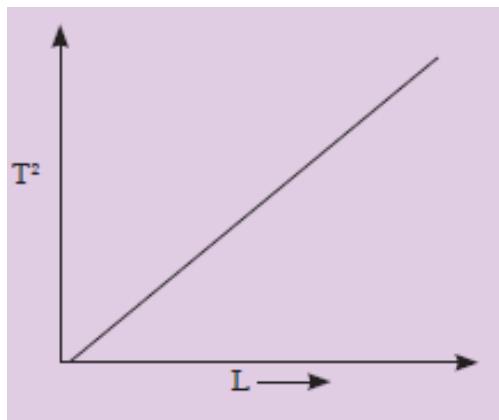
- (a) Set up a small simple pendulum, as shown in the diagram below:



(b) Keeping the angle of swing, θ less than 10° (approximately) for the values of $L = 20, 30, 40, 50, 60, 70, 80, 90$ cm respectively, measure the time period t for 10 oscillations and calculate time period T for one oscillation.

(c) Record your results in a suitable table including the values of T^2 .

(d) Plot a graph of T^2 against L as shown in the figure below:



(e) Calculate the slope S of the graph

(f) Find the value of acceleration due to gravity g from $S = \frac{4\pi^2}{g}$



Points to remember

- ❖ Kinetic energy is the energy of a body in motion.
- ❖ The kinetic energy of an oscillating system can be expressed as:
$$K.e = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$
- ❖ The energy possessed by a stretched or a compressed spring is known as **the elastic potential energy**.
- ❖ The potential energy stored in a spring is given by the formula:
$$Pe = \frac{1}{2}kx^2$$
- ❖ A simple pendulum consists of a small bob of mass m suspended from a fixed support through a light, inextensible string of length.
- ❖ A physical pendulum is any object that oscillates as a pendulum, but cannot be modeled as a point mass on a string.



Learning outcome 2.1 : Formative Assessment

1. For a pendulum with period of oscillation, T , if the mass of the bob is reduced by half what will be the new period of oscillation?

a. $2T$

b. $T/2$

c. T

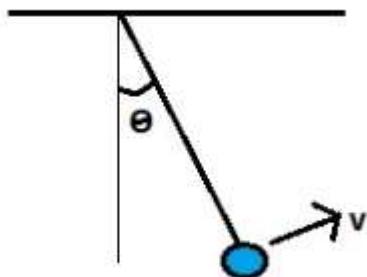
d. $\sqrt{2} T$

2. A pendulum makes 43 vibrations in exactly 50 s.

(a) What is its period?

(b) What is its frequency?

3. One end of a cord is fixed and a small 0.580 kg object is attached to the other end, where it swings in a section of a vertical circle of radius 2.50 m. When $\theta=28^\circ$, the speed of the object is 6.90 m/s.



At this instant find:

(a) Tension in the string.

(b) Tangential and radial component of acceleration.

(c) Total acceleration.

1. Sketch a graph that shows the variation of the kinetic, potential and total mechanical energy with time for an oscillating system:
2. What is the magnitude of the force required to stretch two springs of constants $k_1 = 100 \text{ N/m}$ and $k_2 = 200 \text{ N/m}$ by 6 cm if they are stretched together in parallel?
3. What is the magnitude of the force required to stretch two springs of constants 100 N/m and 200 N/m by 6 cm if they are connected in series?

4. A baby in a 'baby bouncer' is a real-life example of a mass-on-spring oscillator. The baby sits in a sling suspended from a stout rubber cord, and can bounce himself up and down if his feet are just in contact with the ground. Suppose a baby of mass 5.0 kg is suspended from a cord with spring constant 500 N m^{-1} . Assume $g = 10 \text{ N kg}^{-1}$.

(a) Calculate the initial (equilibrium) extension of the cord.
(b) What is the value of angular velocity?
(c) The baby is pulled down a further distance, 0.10 m, and released. How long after his release does he pass through equilibrium position?
(d) What is the maximum speed of the baby?

5. A pendulum can only be modelled as a simple harmonic oscillator if the angle over which it oscillates is small. Why is this so?
6. What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s? State the assumptions made.
7. A geologist uses a simple pendulum that has a length of 37.10 cm and a frequency of 0.8190 Hz at a particular location on the Earth. What is the acceleration due to gravity at this location?
8. Find the time taken for a particle moving in S.H.M. from $\frac{1}{2}A$ to $-\frac{1}{2}A$. Given that the period of oscillation is 12s.
9. A spring is hanging from a support without any object attached to it and its length is 500 mm. An object of mass 250 g is attached to the end of the spring. The length of the spring is now 850 mm.

(a) What is the spring constant?

The spring is pulled down 120 mm and then released from rest.

(b) Describe the motion of the object attached to the end of the spring.

(c) What is the displacement amplitude?

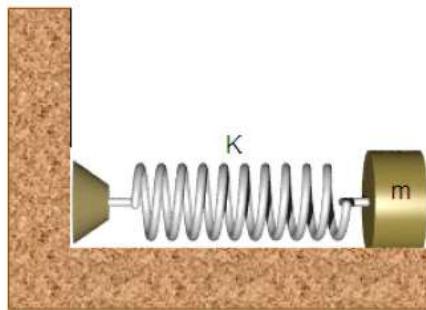
(d) What is the natural frequency of oscillation and period of motion?

Another object of mass 250 g is attached to the end of the spring.

(e) Assuming the spring is in its new equilibrium position, what is the length of the spring?

(f) If the object is set vibrating, what is the ratio of the periods of oscillation for the two situations?

10. The diagram below shows a spring of stiffness K, attached to a mass m.



The mass is pulled by a distance a to the left and released. Show that the velocity of the mass can be modeled by;

$$v = \pm \omega \sqrt{a^2 - x^2}$$

Where x is the extension in the spring. What important assumption has to be made about the system?



Practical Assessment:

In groups of four, perform the following task:

Aim:

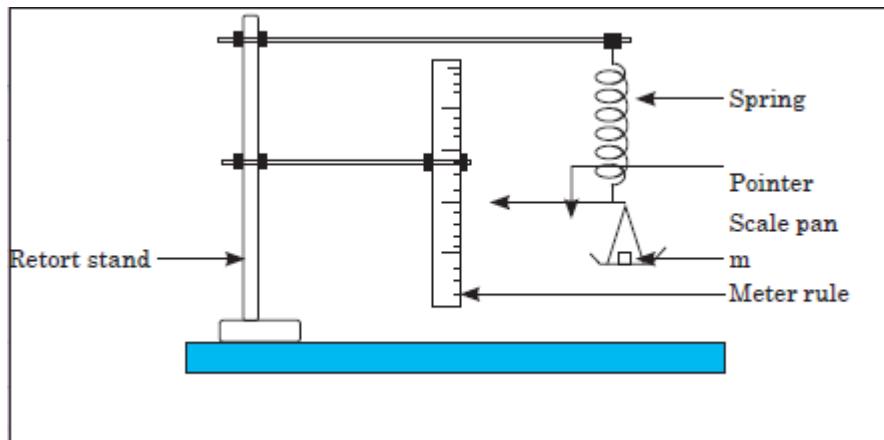
The aim of this activity is to determine the acceleration due to gravity, g , using mass on spring.

Required materials:

1 retort stand, one spiral spring, slotted masses ($5 \times 100\text{g}$), 1 meter rule

Procedure:

- Clamp the given spring and a meter rule as shown in the figure below:



- Read and record the position of the pointer on the meter rule.

(c) Place mass m equal to 0.100 kg on the scale pan and record the new position of the pointer on the meter rule.

(d) Find the extension of the spring x in meters.

(e) Remove the meter rule.

(f) Pull the scale pan downwards through a small distance and release it.

(g) Measure and record the time for 20 oscillations. Find the time T for one oscillation.

(h) Repeat the procedures (f) and (g) for values of m equal to 0.20, 0.30, 0.40 and 0.50kg.

(i) Record your results in a suitable table including values of T^2 .

(j) Plot a graph of T^2 (along the vertical axis) against m (along the horizontal axis).

(k) Find the slope, s , of the graph.

(l) Calculate g from $g = \frac{4\pi^2 x}{s}$

Learning Outcome 2.2: : Describe effects of oscillations on systems



Duration: 5hours



Learning outcome 2.2 Objectives:

By the end of the learning outcome, the trainees will be able to:

1. Describe clearly damped oscillations as used in Physics.
2. Describe clearly forced oscillations.
3. explain correctly the Effects of resonance on physical systems.



Resources

Equipment	Tools	Materials
- Projector	- PhET simulations	- Whiteboard and markers
- Computer	of damped and forced oscillators. - Simulation software	- Chalkboard and chalks - Textbooks - Stop watch



Advance preparation: Avail materials that can be used to make a simple endulum and demonstrate damped and forced oscillations.



Content1. Damped oscillations

➤ **Definition of damped oscillations**

Damped motion is a motion in which the mechanical energy of the system diminishes with time.

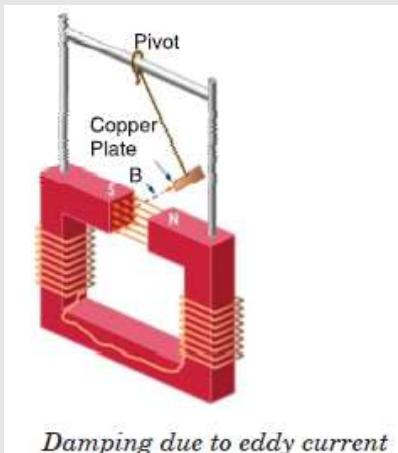
Unless maintained by some source of energy, the amplitude of vibration of any oscillatory motion becomes progressively smaller and the motion is said to be **damped**. The majority of the oscillatory systems that we encounter in everyday life suffer this sort of irreversible energy loss while they are in motion due to frictional or viscous heat generation generally. We therefore expect oscillations in such systems to eventually be damped.

Damping is the gradual decrease of amplitude of an oscillating system due to friction force (air resistance) and losses of energy. As work is being done against the dissipating force, energy is lost. Since energy is proportional to the amplitude, the amplitude decreases exponentially with time.

➤ **Example of a damped oscillator**

In everyday life we experience some damped oscillations like:

(i) Damping due to the eddy current produced in the copper plate.



(ii) Damping due to the viscosity of the liquid

➤ **EQUATION OF DAMPED OSCILLATIONS**

Consider a body of mass **m** attached to one end of a horizontal spring, the other end of which is attached to a fixed point. The body slides back and forth along a straight line, which we take as x-axis of a system of Cartesian coordinates and is subjected to forces all acting in x-direction (they may be positive or negative).

The motion equations for constant mass are based on Newton's second law which can be expressed in terms of derivatives. In all derivations assume that m is the mass of an oscillating object, b is the damping constant and k is the spring constant.

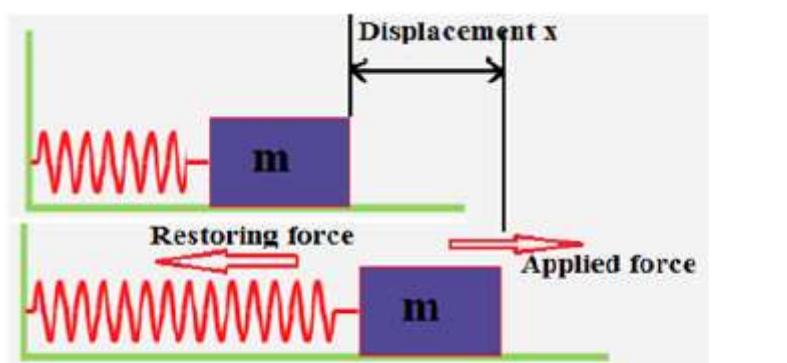


Figure: Mass attached with the spring

$$F_{\text{net external}} = ma \quad \dots \dots \dots \text{Equation 3-1}$$

$$\Rightarrow F_{\text{net external}} = m \frac{d^2x}{dt^2} \quad \dots \dots \dots \text{Equation 3-2}$$

Where x is displacement. The force that causes damping is directly proportional to the speed of oscillation. i.e.

$$\begin{aligned} F_{\text{damping}} &\propto v \\ F_{\text{damping}} &\propto \frac{dx}{dt} \\ F_{\text{damping}} &= -b \frac{dx}{dt} \quad \dots \dots \dots \text{Equation 3-3} \end{aligned}$$

Where b is the damping constant and the negative sign means that damping force always opposes the direction of motion of the mass.

The spring itself stores the energy that is used to restore the position of the mass once released after being slightly displaced. The restoring force of the spring is directly proportional to the displacement.

$$F_{\text{restoring}} \propto x$$

$$F_{\text{restoring}} = -kx \quad \dots \dots \dots \text{Equation 3-4}$$

Where k is the spring constant and the negative sign means that the restoring force opposes the direction of motion of the mass. With this restoring force and the resisting force of the spring, the resultant force on the mass is :

$$\begin{aligned}
 F_{\text{net external}} &= F_{\text{damping}} + F_{\text{restoring}} \\
 m \frac{d^2x}{dt^2} &= -b \frac{dx}{dt} - kx \\
 m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx &= 0 \quad \dots \dots \dots \text{Equation 3-5}
 \end{aligned}$$

Equation 3-5 is the differential equation of damping.

➤ THE SOLUTION OF EQUATION OF DAMPING

To solve the differential equation 3-5 (of damping), we try a solution of the form :

$$\begin{aligned}
 x &= e^{yt} \\
 \text{So,} \quad \frac{dx}{dt} &= ye^{yt} \quad \text{and} \quad \frac{d^2x}{dt^2} = y^2 e^{yt}
 \end{aligned}$$

Substituting in equation 3-5,

$$\begin{aligned}
 my^2 e^{yt} + b ye^{yt} + ke^{yt} &= 0 \\
 e^{yt}(my^2 + by + k) &= 0 \\
 my^2 + by + k &= 0 \quad \dots \dots \dots \text{Equation 3-6}
 \end{aligned}$$

Equation 3-6 is called the auxiliary quadratic equation of the differential equation. Solving this equation for y gives;

$$y = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} \quad \dots \dots \dots \text{Equation 3-7}$$

Assume that

$$\omega_d = \frac{\sqrt{b^2 - 4mk}}{2m} \quad \dots \dots \dots \text{Equation 3-8}$$

The value ω_d is called the damped angular frequency or damped circular frequency.

Equation 3-7 has three cases to consider based on whether the quantity under the square root is positive or negative. These three cases define the types of damping oscillations.



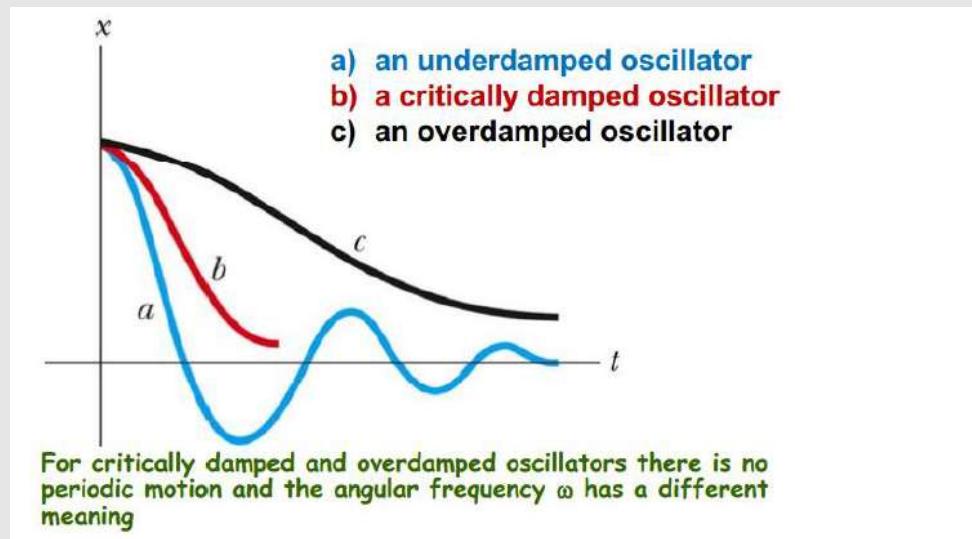
Content2. Types of damping and their representation on a graph

➤ Types of damping and their representation on a graph

There exist three types of damping namely ;

- ✚ Underdamped oscillator
- ✚ Critically damped oscillator
- ✚ Overdamped oscillator

➤ Graphical representation of the three types of damping



✚ Under damping oscillation

This is also called a lightly damped oscillation. For this oscillation, the displacement keeps varying with time and oscillations keep dying away slowly and slowly. The vibrating system keeps passing its original position and more time is taken by it to come to rest.

This is the case where the value of equation 3-8 under the square root is negative :

$$b^2 < 4mk$$

This means that damping constant b is small relative to mass m and spring constant k . So the solution of equation 3-7 becomes;

$$y = \frac{-b}{2m} \pm j \sqrt{\frac{b^2 - 4mk}{4m}}$$

$$y = \frac{-b}{2m} \pm j\omega_d \quad \dots \dots \dots \text{Equation 3-9}$$

Equation 3-9 has two values for y :

$$y_1 = -\frac{b}{2m} + j\omega_d \quad \text{and} \quad y_2 = -\frac{b}{2m} - j\omega_d$$

So the general solution of equation 3-5 becomes;

$$x(t) = C_1 e^{y_1 t} + C_2 e^{y_2 t}$$

$$(x)_t = C_1 e^{\left(-\frac{b}{2m} + j\omega_d\right)t} + C_2 e^{\left(-\frac{b}{2m} - j\omega_d\right)t}$$

$$(x)_t = C_1 \left[e^{-\frac{b}{2m}t} \times e^{j\omega_d t} \right] + C_2 \left[e^{-\frac{b}{2m}t} \times e^{-j\omega_d t} \right]$$

$$x(t) = C_1 e^{-\frac{b}{2m}t} (\cos \omega_d t + j \sin \omega_d t) + C_2 e^{-\frac{b}{2m}t} (\cos \omega_d t - j \sin \omega_d t) \quad \dots \dots \dots \text{Equation 3-10}$$

C_1 and C_2 are constants. Simplifying the expression of equation 3-10 give;

$$x(t) = C_1 e^{-\frac{b}{2m}t} \cos \omega_d t + C_2 e^{-\frac{b}{2m}t} \sin \omega_d t$$

$$x(t) = e^{-\frac{b}{2m}t} (C_1 \cos \omega_d t + C_2 \sin \omega_d t) \quad \dots \dots \dots \text{Equation 3-11}$$

Equation 3-11 represents the displacement of under damped oscillation. From this equation it can be interpreted that the value of x decreases as time increase, but due to the trigonometric function of sine and cosine in bracket makes its graph on Fig.3-3 cross the horizontal axis so many times.

Examples of slightly damped oscillations include :

Acoustics

- (i) A percussion musical instrument (e.g. a drum) gives out a note whose intensity decreases with time. (slightly damped oscillations due to air resistance)
- (ii) The paper cone of a loud speaker vibrates, but is heavily damped so as to lose energy (sound energy) to the surrounding air.

Over damped oscillation

Over damping is also called **excessive or heavy damping**. In this oscillation, displacement decreases with increase in time but the vibrating system takes a longer time to come to rest. This is the case where the value of equation 3-8 under the square root is positive.

$$b^2 > 4mk$$

One extremely important thing to notice is that in this case the roots are both negative. You can see this by looking at equation 3-7 where the square root is less than b . The term under the square root is positive by assumption, so the roots are real.

$$y_1 = -\frac{b}{2m} + \omega_d \quad \text{and} \quad y_2 = -\frac{b}{2m} - \omega_d$$

The solution for expression 3-5 is;

$$x(t) = C_1 e^{y_1 t} + C_2 e^{y_2 t}$$

$$x(t) = C_1 e^{\left(-\frac{b}{2m} + \omega_d\right)t} + C_2 e^{\left(-\frac{b}{2m} - \omega_d\right)t}$$

$$(x)t = C_1 e^{-\frac{b}{2m}t} \times e^{\omega_d t} + C_2 e^{-\frac{b}{2m}t} \times e^{-\omega_d t}$$

$$(x)t = e^{-\frac{b}{2m}t} \left(C_1 e^{\omega_d t} + C_2 e^{-\omega_d t} \right)$$

..... Equation 3-12

Equation 3-12 represents the displacement of over damped. This equation is fully exponential and keeps the value of x decreasing towards zero in a quite long time.

✚ Critically damped oscillation

This is also called natural damping and is when there is an intermediate dissipating force and the system reaches equilibrium position as fast as possible without oscillating. This rapid return to the equilibrium position ($x = 0$) reduces the motion to rest in a shortest possible time. This is the case where the term (of equation 3-8) under the square root is 0 and the characteristic polynomial has repeated roots, i.e.

$$b^2 = 4mk$$

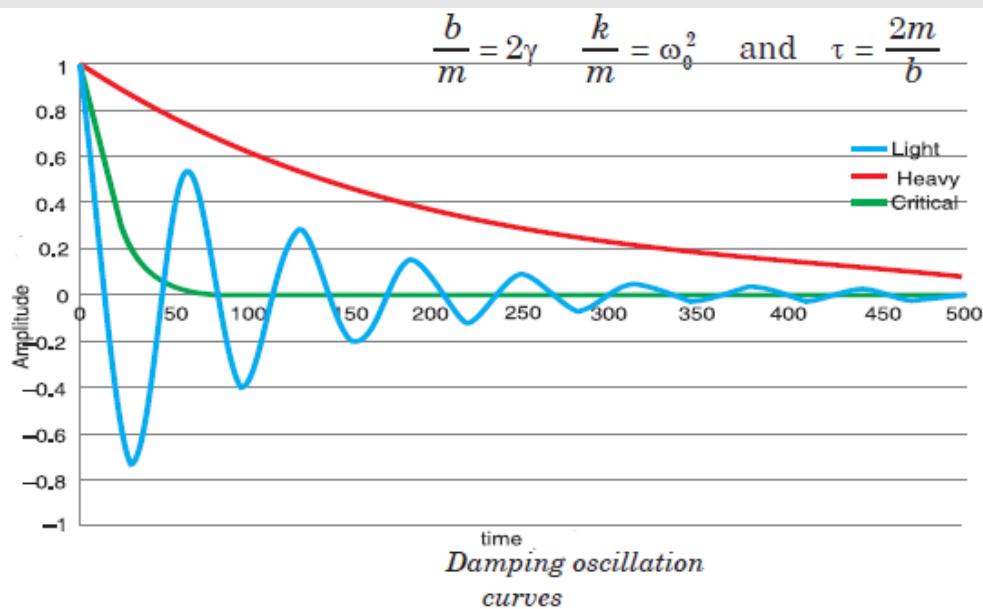
$$b = \pm 2\sqrt{mk} \quad \dots \dots \dots \text{Equation 3-13}$$

Repeated roots are; $-\frac{b}{2m}, -\frac{b}{2m}$

Now we use the roots to solve equation 3-5 in this case. We have only one exponential solution, so we need to multiply it by t to get the second solution.

$$x(t) = e^{-\frac{bt}{2m}} (C_1 + C_2 t) \quad \dots \dots \dots \text{Equation 3-14}$$

Equation 3-14 represents the displacement of critically damped oscillation and shows that the displacement critically dies to zero in a short period of time. It is possible to use the following values:



Where γ is the damping coefficient, ω_0 is the natural frequency and τ is the decay constant or the damping constant. Plotting equations 11, 12 and 14 on the same amplitude-time axes gives the general curve for damping oscillation as shown on the figure above.

➤ Examples of Critical damping

✚ Shock Absorber

It critically damps the suspension of the vehicle and so resists the setting up of vibrations which could make control difficult or cause damage. The viscous force exerted by the liquid contributes to this resistive force.

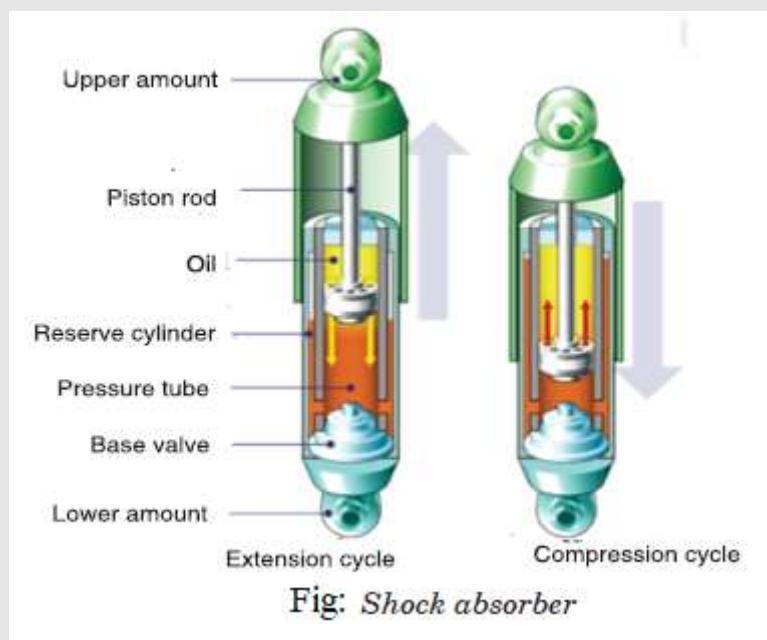


Fig: *Shock absorber*

✚ **Electrical Meters** They are critically damped (i.e. dead-beat) oscillators so that the pointer moves quickly to the correct position without oscillation.



Content3. Forced oscillations

➤ Definition of forced oscillation

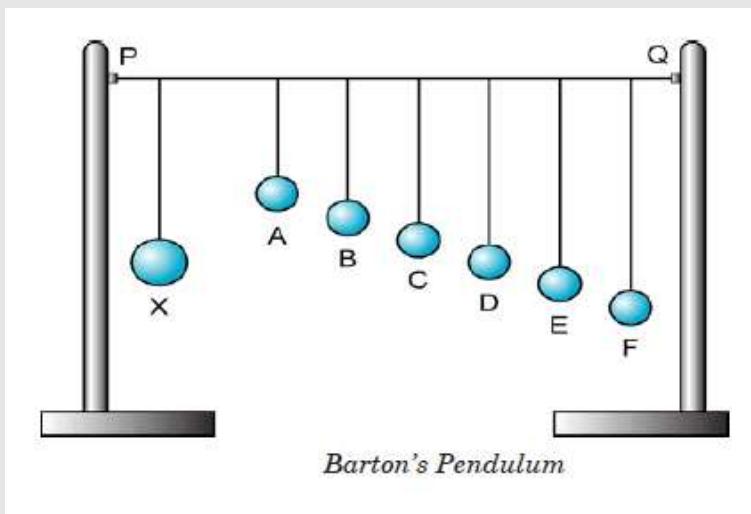
In the conventional classification of oscillations by their mode of excitation, oscillations are called **forced** if an oscillator is subjected to an external periodic influence whose effect on the system can be expressed by a separate term, a periodic function of the time, in the differential equation of motion. We are interested in the response of the system to the periodic external force. The behaviour of oscillatory systems under periodic external forces is one of the most important topics in the theory of oscillations.

➤ Examples of forced oscillating systems

Examples on forced oscillation include:

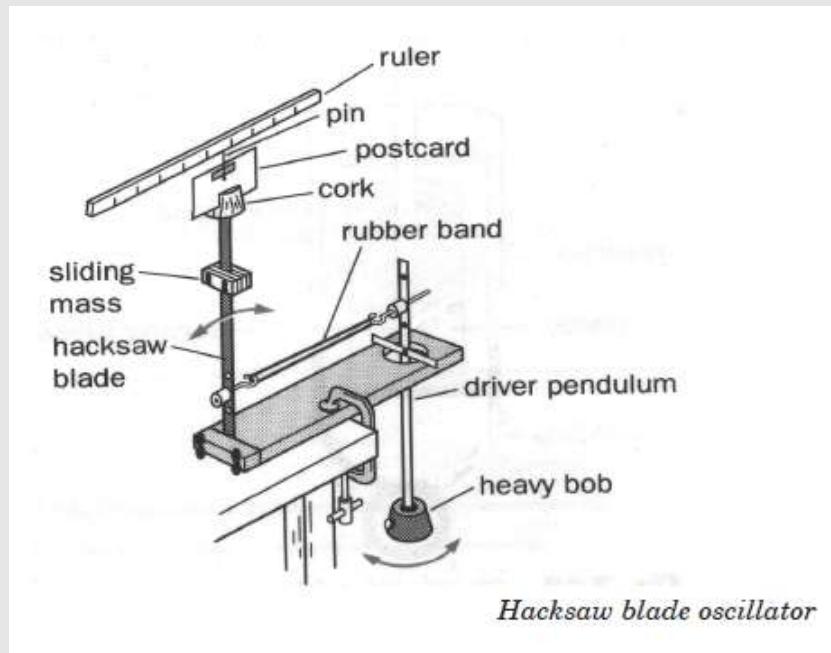
✚ Barton's Pendulum

The oscillation of one pendulum by application of external periodic force causes the other pendulums to oscillate as well due to the transfer of energy through the suspension string. The pendulum having the same pendulum length and pendulum bob mass will have the same natural frequency as the original oscillating pendulum and will oscillate at maximum amplitude due to being driven to oscillate at its natural frequency causing resonance to occur.



► Hacksaw blade oscillator

This is another example of resonance in a driven system. If the period of oscillation of the driver is changed by increasing the length of thread supporting the moving mass, the hacksaw blade will vibrate at a different rate. If we get the driving frequency right the slave will reach the resonant frequency and vibrate widely. Moving the masses on the blade will have a similar effect.



➤ Difference between damped and forced oscillation:

In a Damped oscillation, an object or a system is oscillating in its own natural frequency without the interference of an external periodic force or initial motion.

Damped oscillation is similar to forced oscillation except that it has continuous and repeated force.



Content4. Resonance

- **Definition of Resonance**

Resonance occurs when an object capable of oscillating has a force applied to it with a frequency equal to its **natural frequency** of oscillation.

- **Description of resonance and natural frequency**

Each time the force is applied it transfers energy to the oscillation and increases its amplitude. A very large amplitude occurs after a short time. e.g. pushing a child on a swing. You record your pushes to have the same frequency as **the swing**. The windows rattle when a truck goes by if the frequency of the sound made by the truck's engine is the same as the natural frequency of the glass when it is tapped.

The oscillator resonates when the amplitude in equation 3-22 is maximum, and this occurs if:

$$\frac{dA}{d\omega} = 0$$

This condition of deriving the amplitude A with respect to frequency ω gives;

$$\omega = \sqrt{\frac{4km - 2b^2}{2m}} \quad \dots \dots \dots \text{Equation 3-25}$$

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$\omega = \sqrt{\frac{k}{m} \left(\frac{b}{2m} \right)^2} \quad \text{but } \frac{b}{m} = 2\gamma \text{ and } \frac{k}{m} =$$

$$\omega = \sqrt{\omega_0^2 - \gamma^2} \quad \dots \dots \dots \text{Equation 3-26}$$

In these equations, γ is the damping coefficient and ω_0 is the natural frequency.

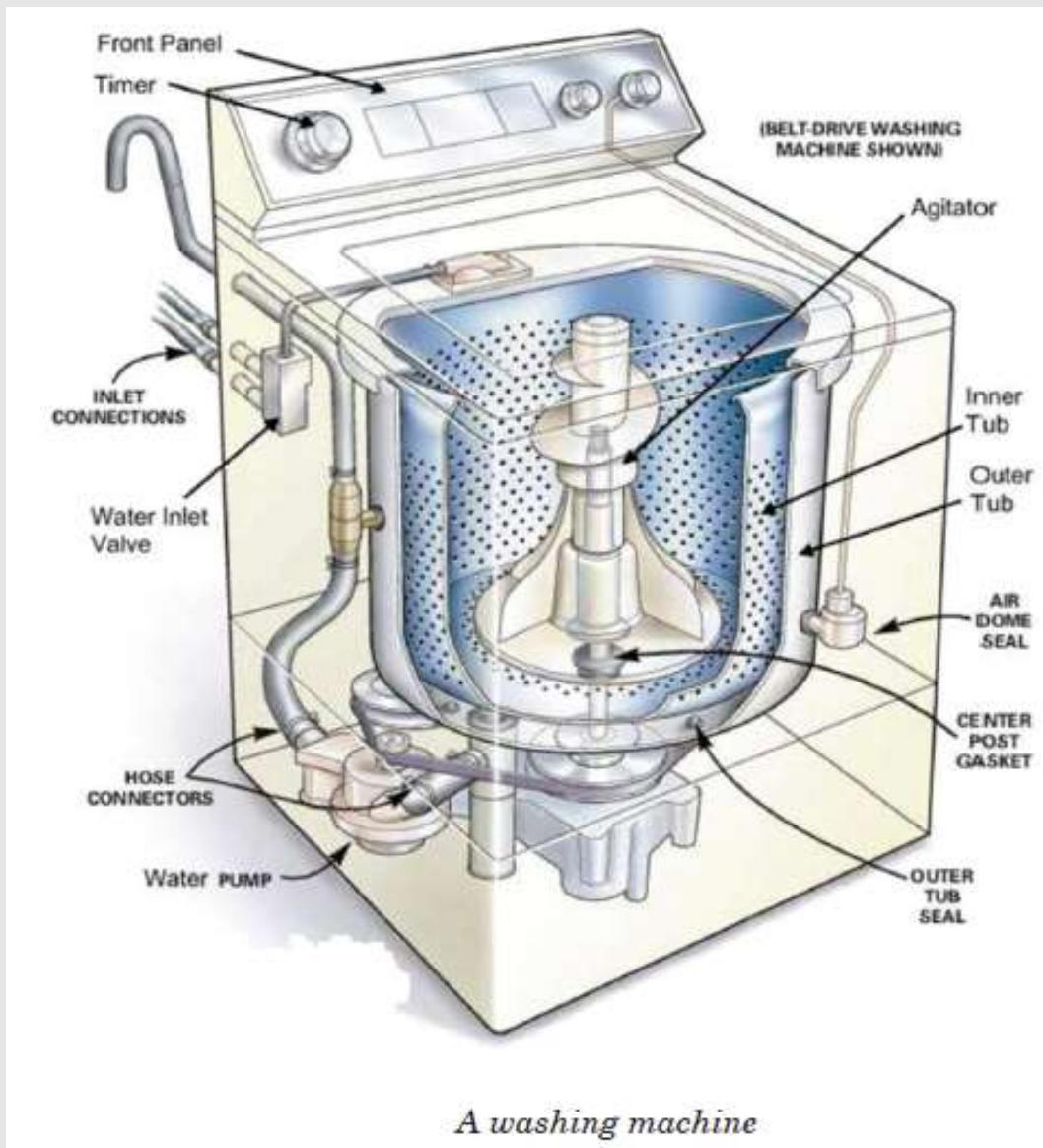
- **Applications and examples of resonance in everyday life**

The phenomenon of resonance depends upon the whole functional form of the driving force and occurs over an extended interval of time rather than at some particular instant.

Below are examples of resonance in different applications:

A washing machine

A washing machine may vibrate quite violently at particular speeds. In each case, resonance occurs when the frequency of a rotating part (motor, wheel, drum etc.) is equal to a natural frequency of vibration of the body of the machine. Resonance can build up vibrations of large amplitude.



Breaking the glass using voice

You must have heard the story of an opera singer who could shatter a glass by singing a note at its natural frequency. The singer sends out a signal of varying frequencies and amplitudes that makes the glass vibrate. At a certain frequency, the amplitude of these vibrations becomes maximum and the glass fails to support it and breaks it.

This scenario is shown on figure below:



Opera singer breaking the glass

Breaking the bridge

The wind, blowing in gusts, once caused a suspension bridge to sway with increasing amplitude until it reached a point where the structure was over-stressed and the bridge collapsed. This is caused by the oscillations of the bridge that keep varying depending on the strength of the wind. At a certain level, the amplitude of oscillation becomes maximum and develops cracks on it and suddenly breaks.



Vibrations breaking the bridge

✚ Musical instruments

Wind instruments such as flute, clarinet, trumpet etc. depend on the idea of resonance. Longitudinal pressure waves can be set up in the air inside the instrument. The column of air has its own **natural frequencies** at which it can vibrate. When we blow, we use the mouthpiece to start some vibrations. Those which happen to match exactly the natural frequencies of the instrument are picked out and magnified.



Figure 11: Howard Johnson's musical trailblazing

✚ Tuning circuit

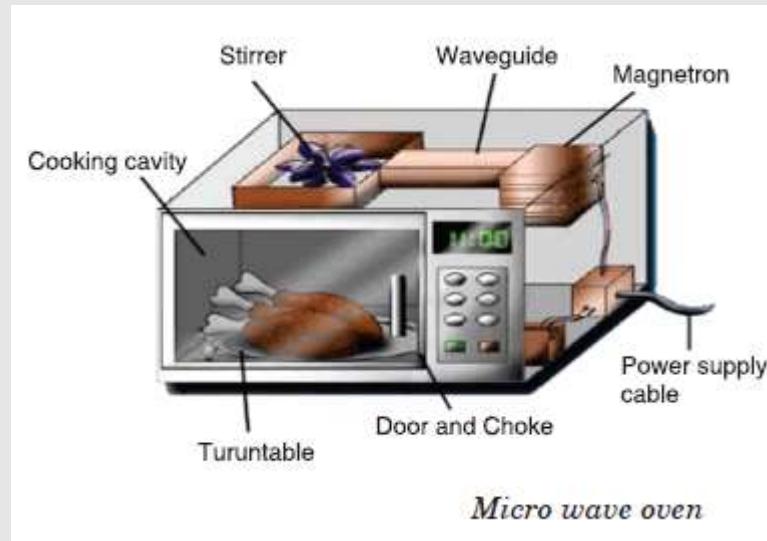
The another example of useful resonance is the tuning circuit on a radio set. Radio waves of all frequencies strike the aerial and only the one which is required must be picked out. This is done by having a capacitance-inductance combination which resonates to the frequency of the required wave. The capacitance is variable; by altering its value other frequencies can be obtained.



Radio receiver tuning circuit

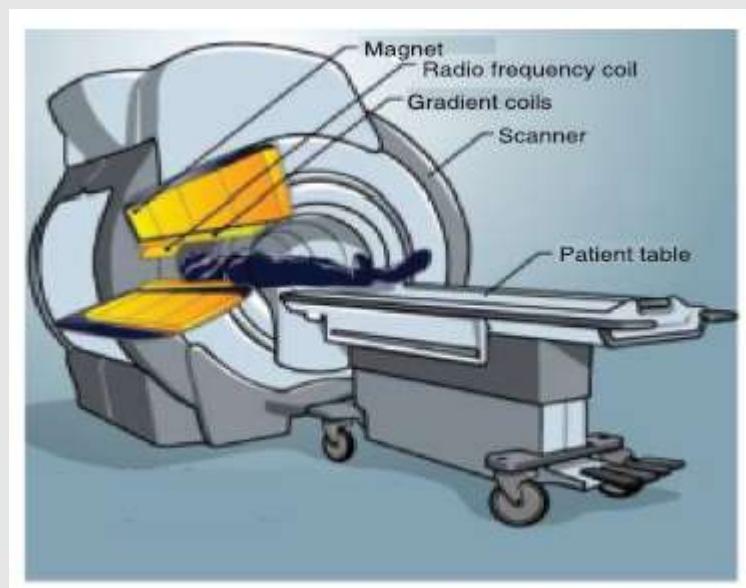
Microwave Ovens

Microwave ovens use resonance. The frequency of microwaves almost equals the natural frequency of vibration of a water molecule. This makes the water molecules in food to resonate. This means they take in energy from the microwaves and so they get hotter. This heat conducts and cooks the food.



Magnetic Resonance Imaging (MRI)

The picture showing the insides of the body was produced using magnetic resonance imaging (MRI). Our bodies contain a lot of hydrogen, mostly in water. The proton in a hydrogen spins. A spinning charged particle has a magnetic field, so the protons act like small magnets. These are normally aligned in random directions. Placing a patient in a strong magnetic field keeps these mini magnets align almost in line. Their field axis just rotates like a spinning top. This is called **processing**.





Content5. Effects of resonance on a system

The following are some effects of resonance on a system:

- Vibrations at resonance can cause bursting of the blood vessel.
- In a car crash a passenger may be injured because their chest is $\frac{1}{2}$ thrown against the seat belt.
- The vibration of kinetic energy from the wave resonates through the rock face and causes cracks.
- It is also used in a guitar and other musical instruments to give loud notes.
- Microphones and diaphragm in the telephone resonate due to radio waves hitting them.
- Hearing occurs when eardrum resonates to sound waves hitting it.
- Soldiers do not march in time across bridges to avoid resonance and large amplitude vibrations. Failure to do so caused the loss of over two hundred French infantry men in 1850.
- If the keys on a piano are pushed down gently enough it is possible to avoid playing any notes. With the keys held down, if any loud noise happens in the room (e.g. Somebody shouting), then some of the notes held down will start to sound.
- An opera singer claims to be able to break a wine glass by loudly singing a note of a particular frequency.



Theoretical learning Activity:

In groups of four, brainstorm about the following problems:

- ✓ Discuss the effects of resonance on physical systems.
- ✓ Represent the three types of damped oscillations on a graph and describe the graph.
- ✓ Derive the equation for a damped oscillation



Practical learning Activity

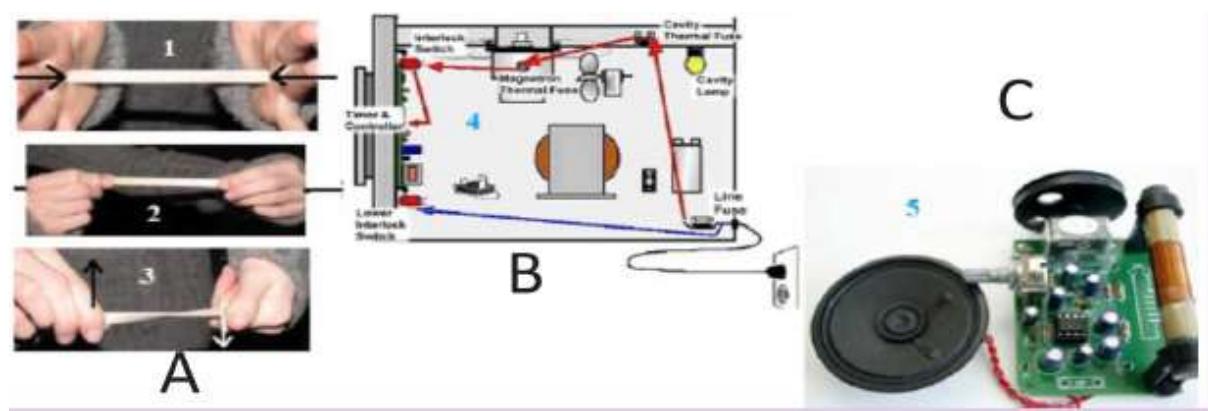
In pair perform the following practical task:

Aim:

The activity aims at understanding the causes and conversion trends of energies and forces for applications of resonance in everyday life.

PROCEDURE:

Clearly analyse the pictures shown below and answer the questions that follow:



- a) Parts 1, 2 and 3 of the figure are the kinds of forces. Name these types of forces.
- b) Explain the concept of these forces and how they can result into breaking of the bridge.
- c) Part 4 of the figure is the side view of an interior structure of the applications explained in this section.
- d) Explain the use of the instrument in (c) above.

e) Part 5 of the figure is the inner parts of one of the applications discussed in this section. What do you think it is?

f) Label each part of the instrument in (e) above and explain how they are used in everyday life.



Points to Remember!

❖ **Damping** is a dissipating force that is always in the opposite direction to the direction of motion of the oscillating particle and is represented by equation;

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

❖ The **natural frequency** of an object is the frequency of oscillation when released. e.g. a pendulum.

❖ A **forced oscillation** is where an object is subjected to a force that causes it to oscillate at a different frequency than natural frequency. It is represented by differential equation;

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F_0 \cos \omega t$$

❖ **Resonance** occurs when an object capable of oscillating, has a force applied to it with a frequency equal to its natural frequency of oscillation. Resonance occurs when angular frequency of oscillation is related to natural angular frequency according to equation;

$$\omega = \sqrt{\omega_0^2 - \gamma^2}$$

❖ **In real life, resonance is applied in;**

1. A washing machine
2. Breaking the glass using the voice
3. Breaking the bridge
4. Musical instruments
5. Tuning circuit
6. Microwave ovens
7. Magnetic Resonance Imaging (MRI)



Learning outcome 2.2 : Formative Assessment

I. Answer by **true** or **false** the following questions :

- Damped motion is a motion in which the mechanical energy of the system increases with time.
- For a critically damped oscillation, the system returns to equilibrium as quickly as possible without oscillating.
- For underdamped oscillation, the system oscillates (at a reduced frequency compared to the undamped case) with the amplitude gradually decreasing to zero
- For an overdamped oscillation, the system returns (exponentially decays) to equilibrium without oscillating.

II. Choose the best answer from the following options :

- When we immerse an oscillating block of mass in a liquid, the magnitude of damping will.....
 - Decrease
 - Increase
 - Remain the same
 - None of the above
- Justify the answer you have given in (i) above.

III. (A) Solve the following initial value problem. For each problem, determine whether the system is under, over, or critically damped:

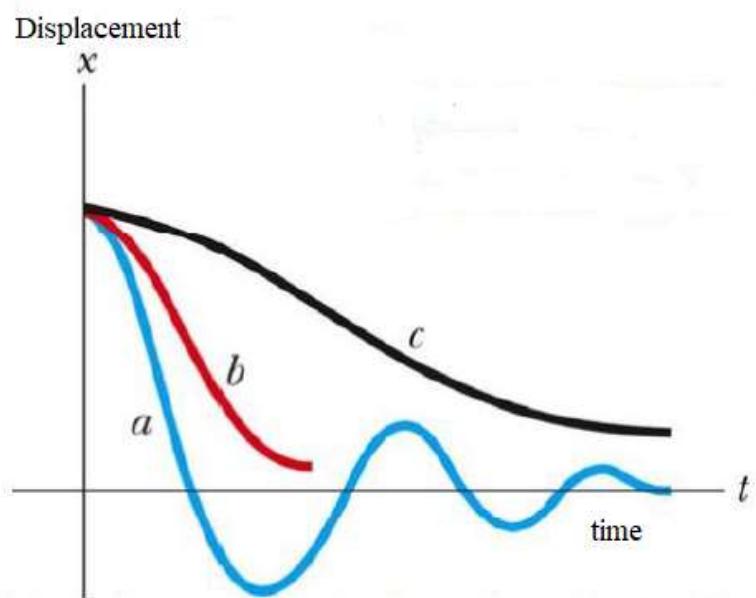
$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 0, y(0) = 1, \frac{dy}{dt} = 1$$

(B) Solve the following initial value problem. For each problem, determine whether the system is under, over, or critically damped.

$$3\frac{d^2y}{dt^2} + 24\frac{dy}{dt} + 48y = 0, y(0) = -5, \frac{dy}{dt} = 6$$

(C) For a critically damped oscillator (not driven by any external force) find the time T^* after which the displacement drops to half its maximum value in terms of **b** and **m**.

IV. Identify the types of damping shown by the letters **a**, **b** and **c** in the figure below:



Learning Outcome 2.3 Apply oscillatory motion in mechanical waves



Duration: 5hours



Learning outcome 2.3 Objectives:

By the end of the learning outcome, the trainees will be able to:

1. Describe clearly wave motion as used in Physics.
2. Describe clearly Mechanical waves as a source of oscillation.
3. Identify properly the sources of mechanical waves.



Resources

Equipment	Tools	Materials
- Projector - Computer	- PhET simulations of wave propagation - Simulation software	- Whiteboard and markers - Chalkboard and chalks - Textbooks - Stop watch



Advance preparation: Prepare simulation softwares of wave propagation and present them to trainees.



Content1. Wave motion

➤ Definition of wave motion

In physics, a wave is a disturbance that occurs in a material medium and in such process, energy is transferred from one place to another.

Waves can be defined as a disturbance in a medium that transfers energy from one place to another, although the medium itself does not travel.

➤ Propagation of a disturbance

The term wave is often intuitively understood as referring to a transport of spatial disturbances that are generally not accompanied by a motion of the medium occupying this space as a whole. In a wave, the energy of a vibration is moving away from the source in the form of a disturbance within the surrounding medium.

Other properties, however, although usually described in terms of origin, may be generalized to all waves. For such reasons, wave theory represents a particular branch of physics that is concerned with the properties of wave processes independently of their physical origin.

➤ Terms used and characteristics of waves

All waves are characterized by the following terms;

- **The Time period (T)** of the wave is the time it takes for one wavelength of the wave to pass a point in space or the time for one cycle to occur. It is also defined as the time taken between two successive wave crests or trough. It is measured in seconds (s).
- **The frequency (f)** is the number of wavelengths that pass a point in space per second. In another words, it can be defined as the number of complete oscillations or vibrations per second. Its SI unit is hertz (Hz). Mathematically;

$$f = \frac{1}{T}$$

- **The wavelength (λ)** is the horizontal distance in space between two nearest points that are oscillating in phase (in step) or the spatial distance over which the wave makes one complete oscillation.
Its SI unit is metre (m).
- **The wave speed (v)** is the speed at which the wave advances.
Its SI unit is m/s.

$$\text{Speed} = \frac{\text{distance travelled}}{\text{time period}} = \frac{\lambda}{T}$$

$$v = \lambda f$$

That is, wave speed = wavelength \times frequency.

This is the relationship between wavelength, frequency and velocity.

⊕ **Amplitude** is defined as the maximum distance measured from equilibrium position (mean position).

The amplitude is always taken as positive and is measured in metres.

⊕ **Phase difference (phase angle)** is the angular difference between two points on the wave or between two waves.

The wave number, also called the propagation number k , is the spatial frequency of a wave, either in cycles per unit distance or radians per unit distance.

It can be envisaged as the number of waves that exist over a specified distance (analogous to frequency being the number of cycles or radians per unit time). Its unit is per metre (m^{-1}).

Mathematically;

$$k = \frac{\Phi}{x} = \frac{2\pi}{\lambda}$$

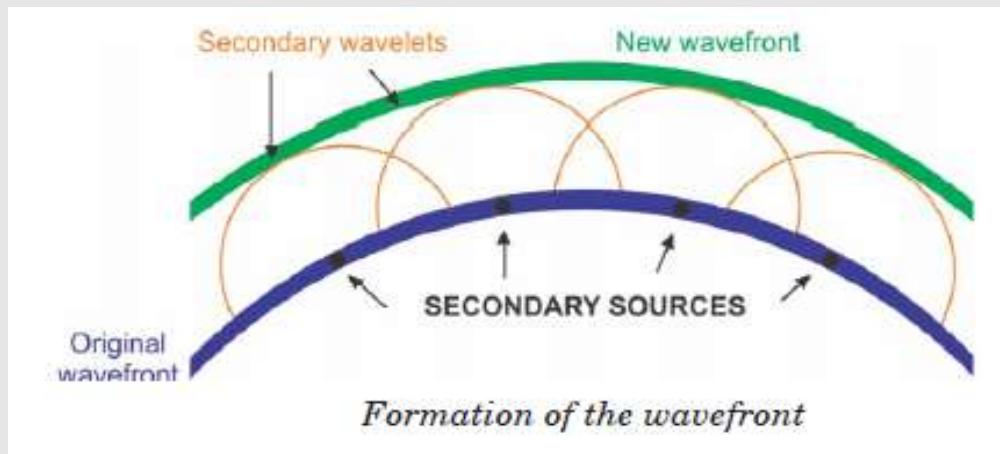
⊕ **The Intensity (I)** of a wave or the power radiated by a source are proportional to the square of the amplitude (x).

$$I \propto x^2$$

- Wavefront is a line or surface in the path of the wave motion on which the disturbance at every point have the same phase.

This can also be defined as the surface which touches all the wavelets from the secondary sources of waves.

Consider the Huygens construction principle for the new wavefront.





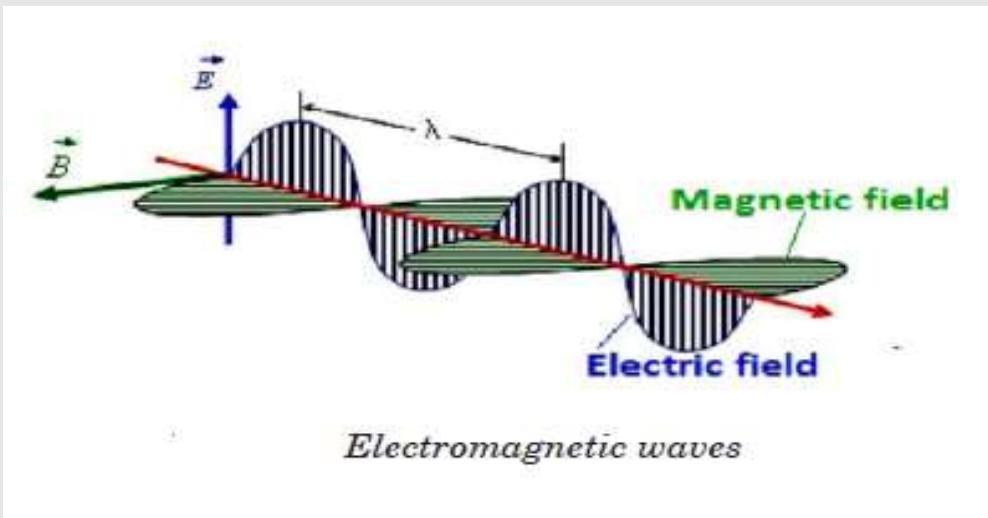
Content2. Types of Waves

➤ Types of waves

According to medium of propagation, there are two types of waves: ***mechanical waves*** and ***electromagnetic waves***. These waves are classified based on conditions necessary for the wave to propagate.

✚ Electromagnetic waves

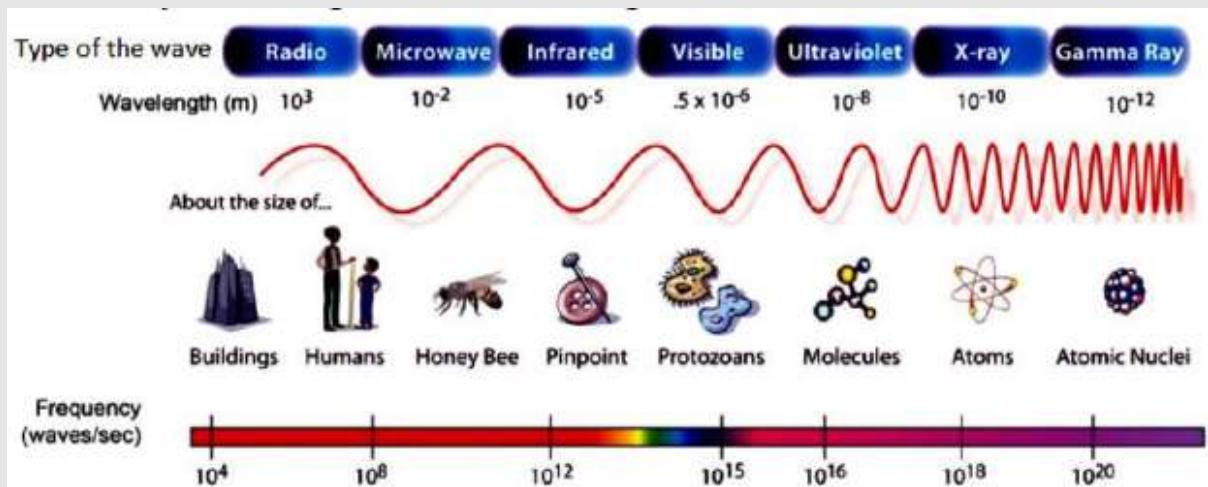
These waves consist of disturbances in the form of varying electric and magnetic fields. No material medium is necessary for their movement and they travel more easily in vacuum than in matter.



➤ Examples of electromagnetic waves are:

Radio waves, Microwaves, Infrared radiation, Visible light, Ultraviolet light, X-rays and Gamma rays.

These waves vary according to their wavelengths.



Examples of electromagnetic waves

✚ Mechanical waves

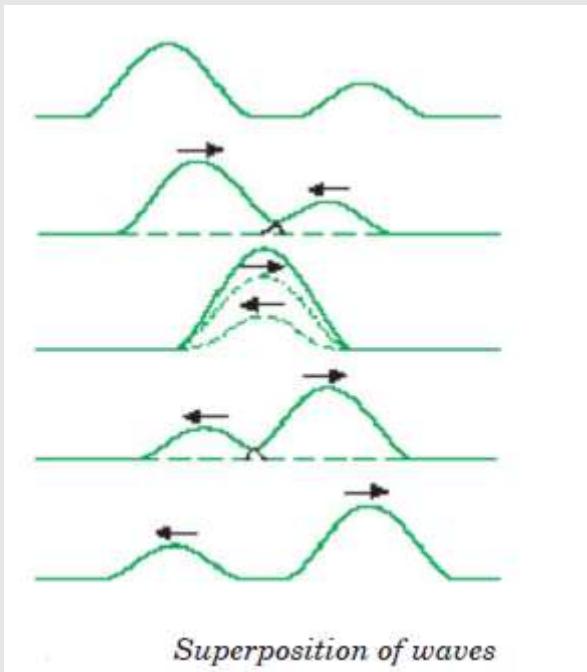
These waves are produced by the disturbance in a material medium and they are transferred by particles of the medium.

These waves include waves in strings, water waves and sound waves. Mechanical waves are classified as **progressive** or **standing waves**.

• Stationary waves

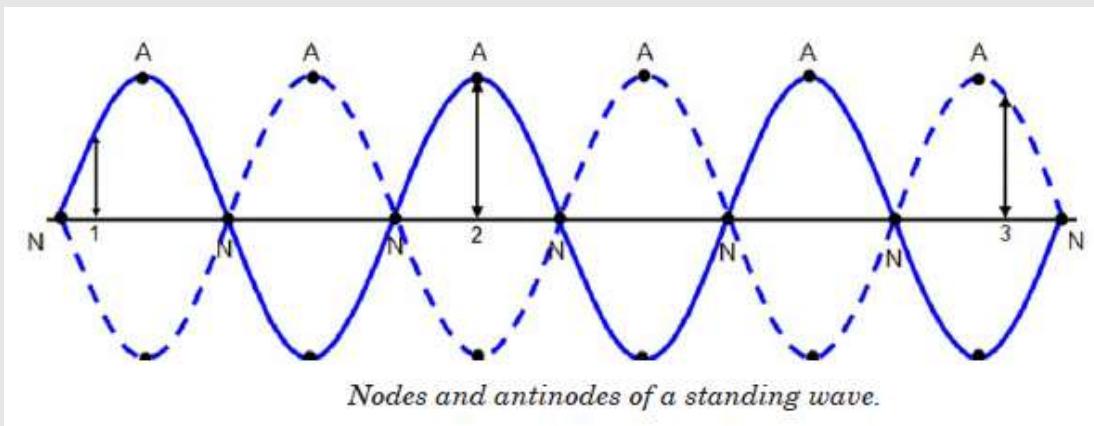
A stationary wave (or a standing wave) is a wave which results when two waves travelling in opposite directions and having the same speed, frequency and approximately equal amplitudes are superposed. When these two waves meet, they create regions of **nodes** and **antinodes**.

A standing wave is shown in the figure below:



Superposition of waves

The creation of nodes and antinodes is shown in the figure below:



Here, **A** denotes the antinodes and **N** denotes the nodes.

- **Progressive waves**

A progressive wave is also called a **travelling wave** which consists of a disturbance moving from one point to another. As a result, energy is transferred between points.

Progressive mechanical waves can be categorised according to the direction of the effect of the disturbance relative to the direction of travel.

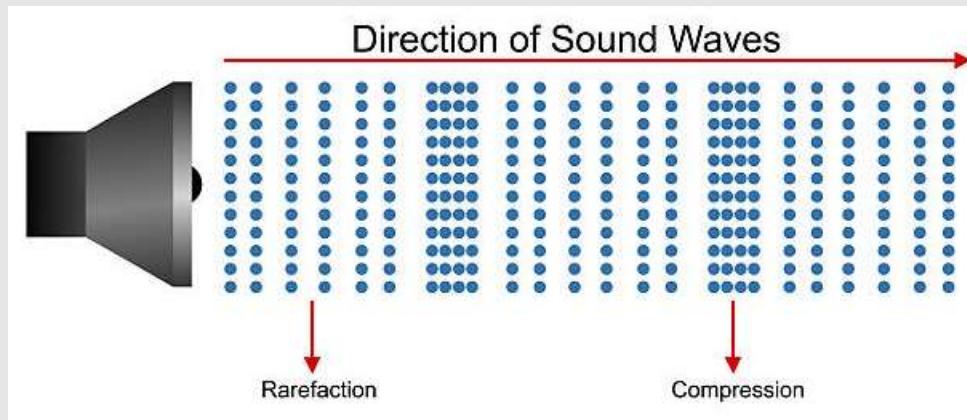
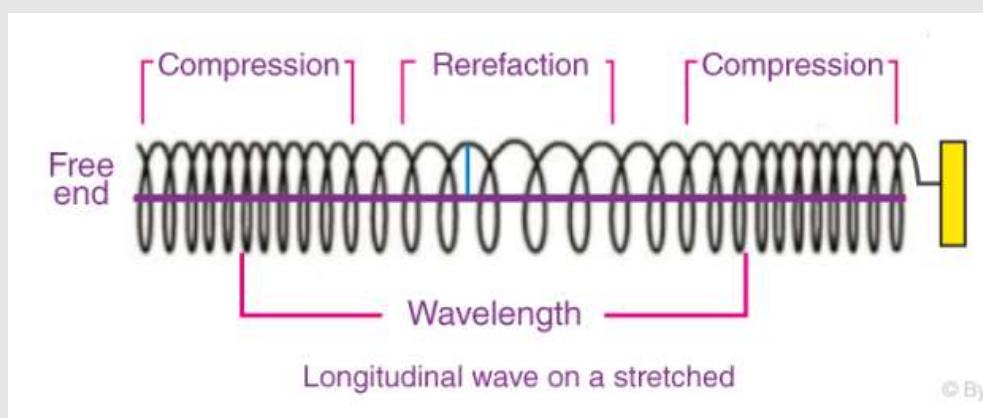
According to direction of propagation, Progressive waves are classified as **longitudinal** and **transverse waves**.

✚ **Longitudinal waves :**

The particles of the medium vibrate about their equilibrium position in a direction parallel to the direction of propagation of the wave is called a longitudinal waves.

Longitudinal waves require a medium with only elasticity of volume (or Bulk modulus) for its propagation. In this type of wave motion, the waves travel through a medium in the form of compression and rarefaction.

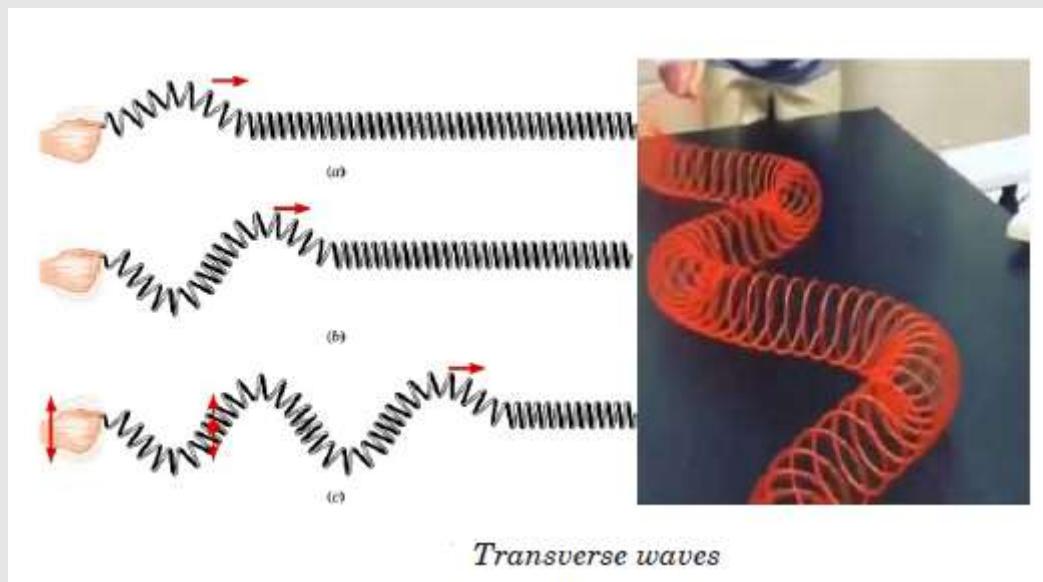
An example of a longitudinal wave is the pulse that can be sent along a stretched slinky by shaking one end of the slinky along its length. The pulse moves along the line of the slinky and ultimately makes the other end move. Notice that in this case, the individual coils of the slinky vibrate back and forth about some equilibrium position, but there is no net movement of the slinky itself.



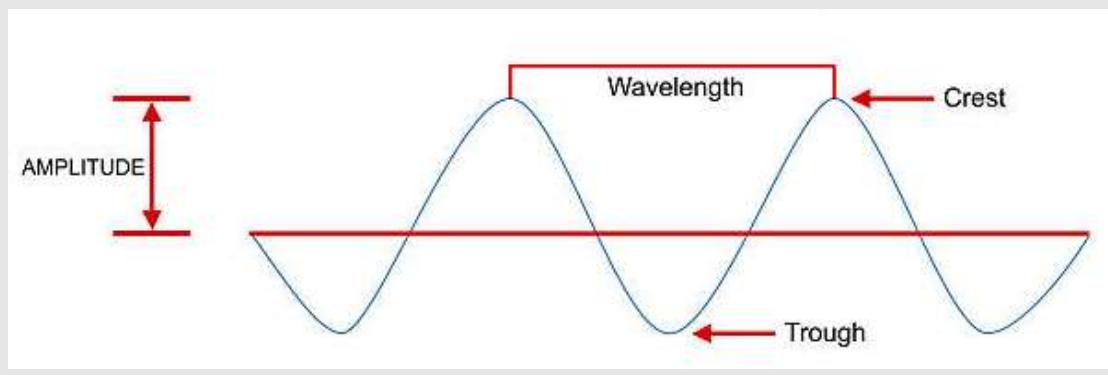
The region of high pressure is called **compression** and the region of low pressure is called **rarefaction**. For example, Sound waves in the tube.

Transverse waves:

These are waves in which the direction of disturbance is perpendicular to the direction of travel of the wave. The particles do not move along with the wave; they simply oscillate up and down about their individual equilibrium positions as the wave passes by.

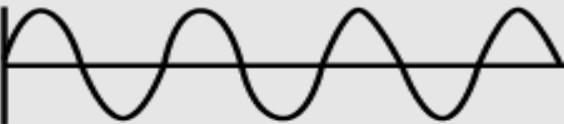
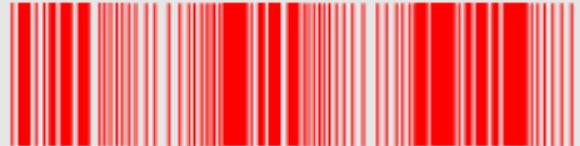


The main parts of a transverse wave are illustrated in the figure below :



➤ **Difference between transverse and longitudinal waves**

The table below summarizes the difference between transverse and longitudinal waves :

Transverse waves	Longitudinal waves
<p>1.</p>  <p>2. Particles vibrate in a direction perpendicular to the direction of propagation of the wave.</p> <p>3. Crests and troughs are formed</p> <p>4. Formed on the surface of solids and liquids</p> <p>5. May be elastic waves or non elastic waves eg.light wave,radio wave</p> <p>6. Do not create pressure difference in the medium</p>	<p>1.</p>  <p>2. Particles vibrate in a direction parallel to the direction of propagation of the wave.</p> <p>3. Compression and rarefactions are formed.</p> <p>4. Formed in solids,liquids and gas</p> <p>5. Only elastic waves (mechanical)eg.sound wave,seismic wave.</p> <p>6. Creates pressure difference in the medium</p>

➤ **Mechanical waves as a source of oscillation**

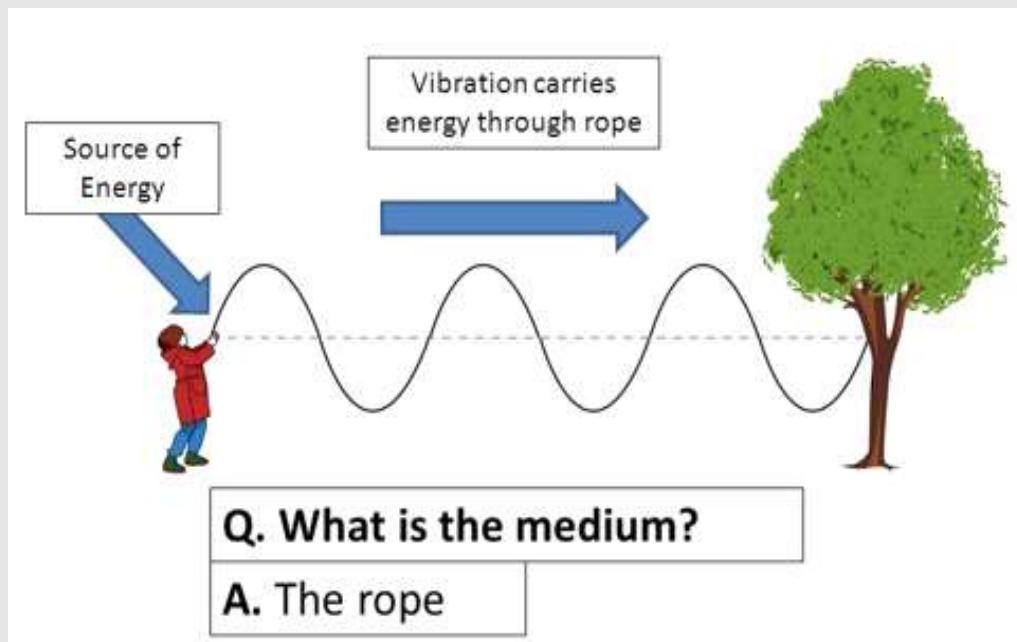
Definition of mechanical wave

Mechanical waves are the waves that require a medium in order to transfer energy away from their source.

Some common examples of mechanical waves are they earthquakes waves that travel through layers within the earth. Soundwaves are also mechanical waves that travel through the air, water and even solid matter.

Source of mechanical waves

Mechanical waves are created by the interaction between neighbouring particles in the medium. Energy and momentum are transferred from one particle to the next by this interaction and the net effect is to pass these quantities along from the source to the receiver.



Some examples of mechanical waves

Mechanical waves, being progressive and stationary, are seen in different forms as described in this section.

Sound waves

Sound waves are longitudinal waves. Sound waves travel fastest in solids, slower in liquids and slowest in gases. This means the air particles (or particles of the medium) move back and forth on paths that are parallel to the direction of wave propagation and thus take the form of compressions and rarefactions of the molecules in the air itself.

Water waves

Water waves are a combination of both transverse and longitudinal waves. These waves are periodic disturbances that move away from the source and carry energy as they go.



Water waves

Ocean waves

These waves are longitudinal waves that are observed moving through the bulk of liquids, such as our oceans. Ocean waves are powerful forces that erode and shape of the world's coastlines. Most of them are created by the wind. Winds that blow over the top of the ocean, create friction between the air and water molecules, resulting in a frictional drag as waves on the surface of the ocean.

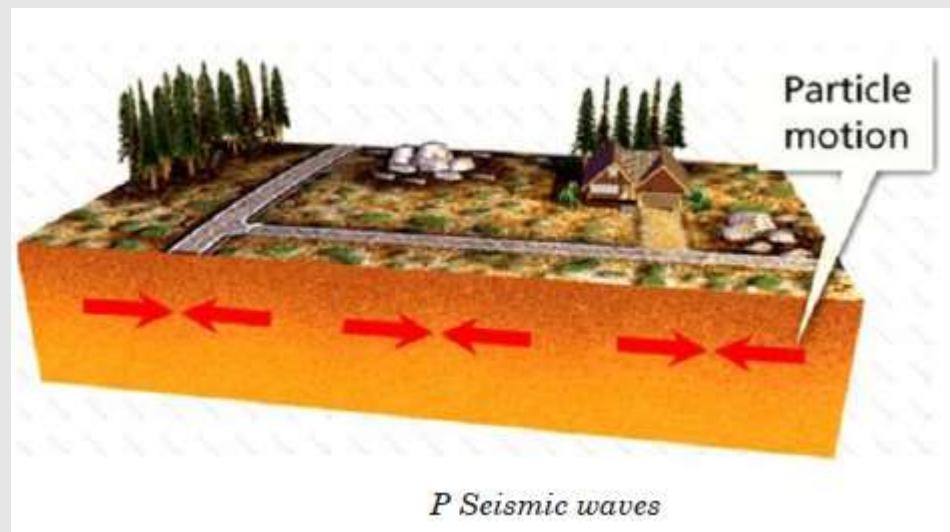


Ocean waves

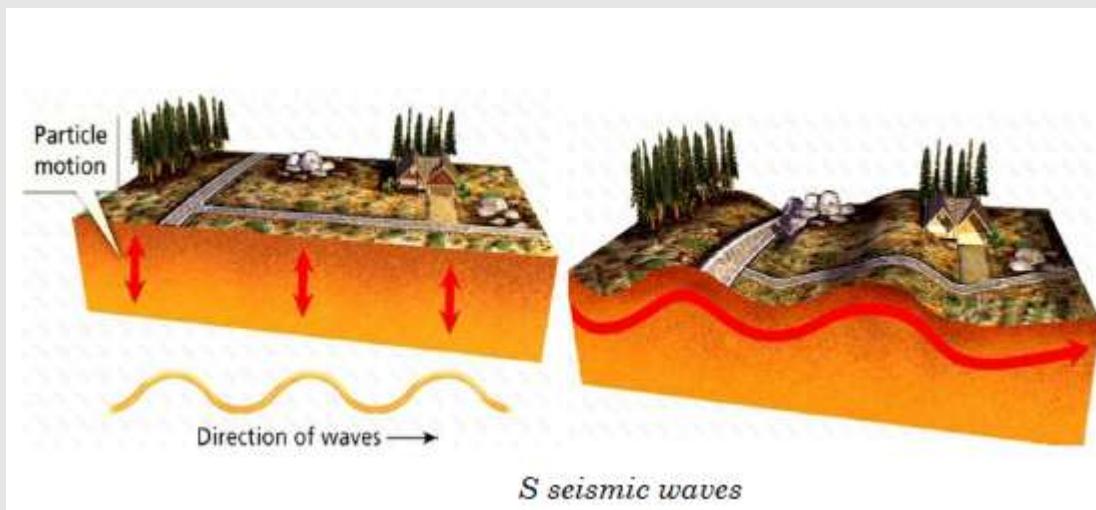
✚ **Earthquake waves**

Earthquakes occur when elastic energy is accumulated slowly within the Earth's crust (as a result of plate motions) and then released suddenly along fractures in the crust called faults. Earthquake waves are also called seismic waves and actually travel as both transverse and longitudinal waves.

The P waves (Primary waves or compressional waves) in an earthquake are examples of longitudinal waves. The P waves travel with the fastest velocity and are the first to arrive.



The S waves (Secondary waves or shear waves) in an earthquake are examples of transverse waves. S waves propagate with a velocity slower than P waves, arriving several seconds later.



Surface Waves

When waves occur at or near the boundary between two media, a transverse wave and a longitudinal wave can combine to form a surface wave. Examples of surface waves are a type of seismic wave formed as a result of an earthquake and water waves.



Surface water waves.

➤ **APPLICATIONS OF WAVES**

1. They are used in radar, broadcasting and radio communication.
2. They are used in MRI in hospitals.
3. They are also used in radio communication which forms an integral part of wireless communication.



Theoretical learning Activity:

In groups of four, brainstorm about the following problems:

Give the appropriate word as used in wave propagation:

- 1) The part of a longitudinal wave where the particles of the medium are close together.
- 2) A wave which needs to travel through a medium.
- 3) A repeated back-and-forth or up-and-down motion.
- 4) A wave which moves the medium in a direction across the direction the energy is traveling.
- 5) A disturbance that transfers energy from place to place.
- 6) The highest point of a wave.
- 7) The part of a longitudinal wave where the particles of the medium are far apart.
- 8) The lowest part of a transverse wave.
- 9) The material through which a wave travels.
- 10) The vertical distance between the line of origin and the crest of a wave.
- 11) Waves that do not require a medium.

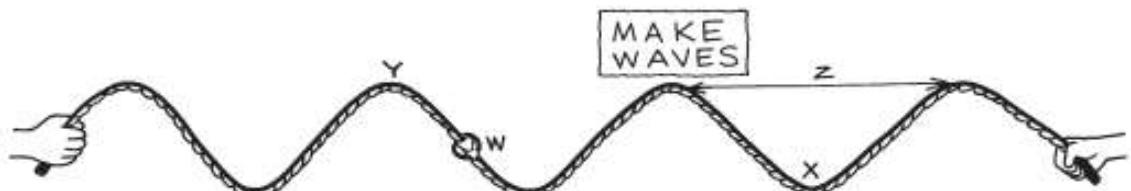


Practical learning Activity

In pair perform the following practical task:

Requirements:

A manila paper with the drawing of the wave shown below:



- a) How do you call the distance represented by arrow z?
- b) What letter is labelling the wave's trough?
- c) What letter is labelling a wave's crest?
- d) The number of waves that pass the poster per second is called the of the waves.
- e) If the knot (w) travels 2 meters in 1 second, we say that it has of 2 m/s.
- f) If the wavelengths were shortened, would the frequency be higher or lower?
- g) The greatest distance the knot (w) travels from its resting position is called of the wave.
- h) What kind of wave are these in the rope?



Points to Remember

- ⊕ **Waves** can be defined as a disturbance in a material medium that transfers energy from one place to another.
- ⊕ The time **period (T)** of the wave is the time it takes for one complete vibration of the wave.
- ⊕ The **frequency, f** is the number of wavelengths that pass a point in space in one second.
- ⊕ The **wavelength, λ** is the horizontal distance in space between two nearest points that are oscillating in phase.
- ⊕ The **wave speed, v** is the speed at which the wave advances.
- ⊕ **Phase difference** (phase angle) is the angular difference between two points on the wave or between two waves.
- ⊕ The **wave number** also called the propagation number k is the spatial frequency of a wave.
- ⊕ **The Intensity** of a wave or the power radiated by a source are proportional to the square of the amplitude.
- ⊕ **Wavefront is** a line or surface in the path of the wave motion on which the disturbance at every point have the same phase.
- ⊕ **Mechanical waves** are waves produced by the disturbance in a material medium.
- ⊕ **A progressive wave** consists of a disturbance moving from one point to another.
- ⊕ **Longitudinal wave** propagates through some medium with vibrations in the direction of propagation of the disturbance.
- ⊕ **In Transverse waves**, the direction of vibrations is perpendicular to the direction of propagation of the wave.
- ⊕ **Electromagnetic waves** are disturbances in form of varying electric and magnetic fields.



Learning outcome 2.3 : Formative Assessment

1. State whether the statement is TRUE or FALSE:

- (i) Mechanical waves form when a source of energy causes a medium to compress.
- (ii) A transverse wave moves a medium at an obtuse angle to the wave.
- (iii) Mechanical waves can travel with or without a medium.

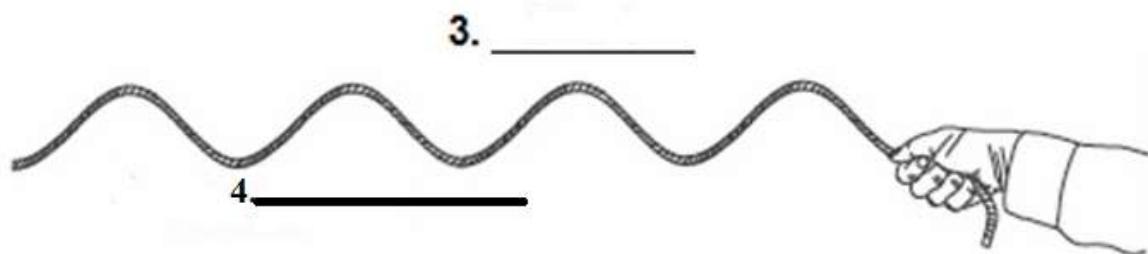
2. Complete the following statements:

- a. A wave that can travel only through matter is a.....
- b. The empty space is called.....
- c. A material in which mechanical waves travel is known as.....
- d. Mechanical waves can not move through a.....
- e. Sound waves, water waves and seismic waves are.....waves
- f. Anis a wave that can travel through vacuum and through matter.

3. (a) What happens when a source of energy causes a medium to vibrate?

(b) what are the types of mechanical waves?

4. Label the parts of the wave shown in the illustration below:



For the figure above, answer the following related questions:

- i. What medium is the wave traveling through?
- ii. What is the source of energy causing the wave?
- iii. How do you know that the wave is mechanical?
- iv. What type of mechanical wave is this?

5. Complete the sentence with the appropriate word

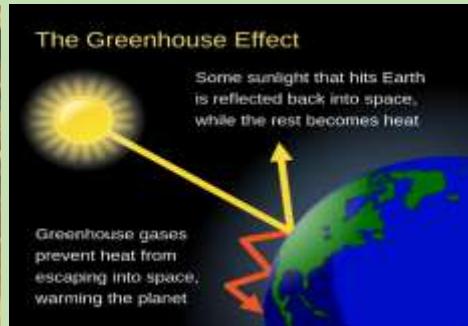
- i) In a _____ wave the particles of the medium vibrate parallel to the direction the wave is traveling.
- ii) In a _____ wave the particles of the medium vibrate perpendicular to the direction the wave is traveling.
- iii) Sound is an example of _____ wave.

iv) Light is an example of _____ wave.

6. (a) Draw a transverse wave and label; Wavelength ,Amplitude, Crest and Trough.
(b) Draw a longitudinal wave and label; Wavelength, Compression and rarefaction.
(c) identify the type of wave in the figure below and mention the parts labelled **1**, **2** and **3**.



Learning Unit 3: Describe Climate change and Greenhouse effect



Learning outcomes:

- 3.1 Describe intensity of the sun's radiation reaching planets.
- 3.2 Explain Greenhouse effects.
- 3.3 Explain climate change and mitigation measures.

Learning Outcome 3.1: Describe intensity of the sun's radiation reaching planets.



Duration: 4hours



Learning outcome 3.1 Objectives:

By the end of the learning outcome, the trainees will be able to:

1. Describe intensity of the Sun's radiation reaching planets on climate change.
2. Explain Greenhouse effect according to climate change.
3. Explain climate change and relate facts based on the concepts of Physics.



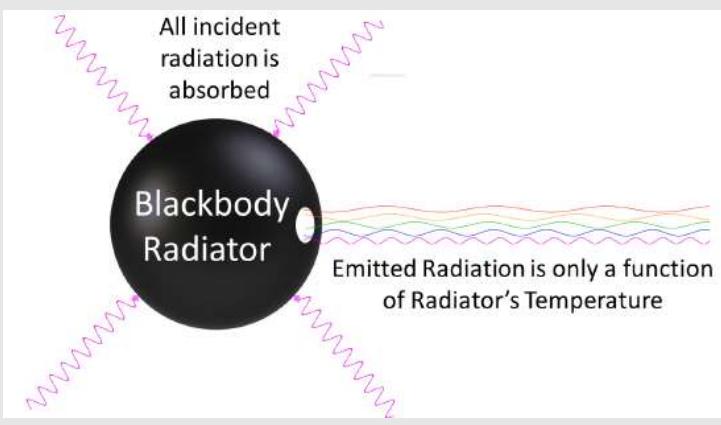
Resources

Equipment	Tools	Materials
<ul style="list-style-type: none">- Computer- Projector	<ul style="list-style-type: none">- Search engine- Scenarios of environmental journals- PhET simulations of Greenhouse effect	<ul style="list-style-type: none">- Whiteboard and markers- Chalkboard and chalks- Textbooks



Content1. Definition of black body radiation

- A **black body** is an object that absorbs all radiation falling on it and reflects none.
- A **black body** is an object that has no reflective power and completely absorbs all radiations falling on it.
- A perfect blackbody is one that absorbs all incoming light and does not reflect any.
- At room temperature, such an object (black body) would appear to be perfectly black (hence the term blackbody). However, if heated to a high temperature, a blackbody will begin to glow with thermal radiations.





➤ Content2. Description of Intensity of the sun's radiation and Albedo

➤ Definition of intensity of the Sun's radiation

Intensity of the sun's radiation is the power of solar energy transferred per unit area on earth's surface.

The Sun's intensity refers to the amount of incoming solar energy or radiation that reaches the Earth's surface.

The Sun produces heat of very high intensity that is spread and then received by all surrounding objects.

These objects include all planets and other objects around the Sun.

The intensity of the Sun on the surface of the earth is approximately **1400 W/m²**. This value is known as the solar constant. The value of the solar constant can still change by a certain percentage due to different factors.

➤ The main factors responsible for the variation in intensity of the sun

⊕ ***The shape of the earth:*** The earth has a spherical shape and therefore the sunlight is more spread out near the poles than other parts of the Earth.

There are also fewer atmospheres at the equator, allowing more sunlight to reach the earth. Therefore, the intensity varies depending on the geographical latitude of the earth's location.

⊕ ***The angle at which the sun's radiations strike the earth's surface:*** When the Sun is directly overhead, its rays strike Earth perpendicular to the ground and so deliver the maximum amount of energy.

⊕ ***Seasons:*** The Sun's radiation is maximum in the **summer** and it is minimum in **winter**.

⊕ ***The earth's rotation:*** during rotation of the earth some parts of the earth receive more solar energy than others.

➤ **Definition of Albedo**

Albedo is the fraction of radiation returning from a given surface compared to the amount of radiation initially striking that surface.

It describes the reflectivity of surface.

Scientists use a quantity called "albedo" to describe the degree to which a surface reflects light that strikes it.

Albedo can be calculated by the ratio of reflected radiation from the surface to the incident radiation upon it.

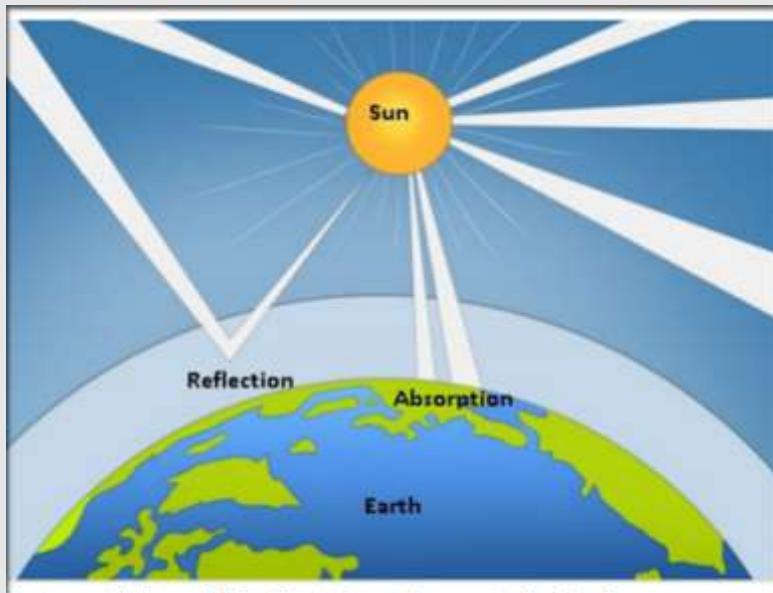


Figure: The sun's radiations on the Earth

Of the power radiated by the sun on earth, one part is **absorbed** while another part is reflected back.

➤ **Range of values for albedo:**

Being a dimensionless fraction, albedo can also be expressed as a percentage

It is measured on a scale from 0 (0%), corresponding to a black body that absorbs all incident radiation (no reflective power), to 1 (100%), corresponding to a body that reflects all incident radiations such as a white body (or perfect reflectors).

➤ ***The formula used to calculate Albedo:***

$$\text{albedo} = \frac{\text{total scattered power}}{\text{total incident power}}$$

Or $\text{albedo} = \frac{\text{reflected radiation}}{\text{total incident radiation}}$

Note: The albedo has **no units** since it is a ratio of the similar quantities but it can be expressed as a percentage (%)

➤ **How does the albedo value affect the reflectivity of a surface?**

A high albedo value means that the surface reflects the majority of the radiation that hits it and absorbs the rest.

A low albedo value means that a surface reflects a small amount of the incoming radiation and absorbs the rest.

Solved exercises:

1. If a surface absorbs 90% of all incident solar energy what is the albedo of the surface?
2. Of 1000watts light falling on a sand desert, 300watts are reflected. Calculate the albedo.
3. If a surface absorbs 25% of all incident solar energy, what is albedo of the surface?

Solutions:

$$1. \text{ Albedo} = \frac{\text{reflected radiation}}{\text{total incident radiation}}$$

$$= \frac{10}{100} = 0.1 \text{ or } A = 10\%$$

$$2. \text{ Albedo} = \frac{300w}{1000w} = 0.3 \text{ or } A = 30\%$$

$$3. \text{ Albedo} = \frac{75}{100} = 0.75 \text{ or } A = 75\%$$

The table below gives some values of estimated albedo for various surfaces expressed as percentages:

Surface	Albedo (%)
Fresh snow	80-95
Water bodies	10-60
Grass	25-30
Crops, grasslands	10-25
Forests	10-20
Light roof	35-50
Dark roof	8-18
Brick, stone	20-40
Concrete, dry	17-27

Surface	Albedo%
Tropical forests	13
Woodland	14
Sandy desert	37
Sea ice	25-60
Grassland	20
Snowy vegetation	20-80
Stony desert	24
Snowy Ice	80
Water	8-10

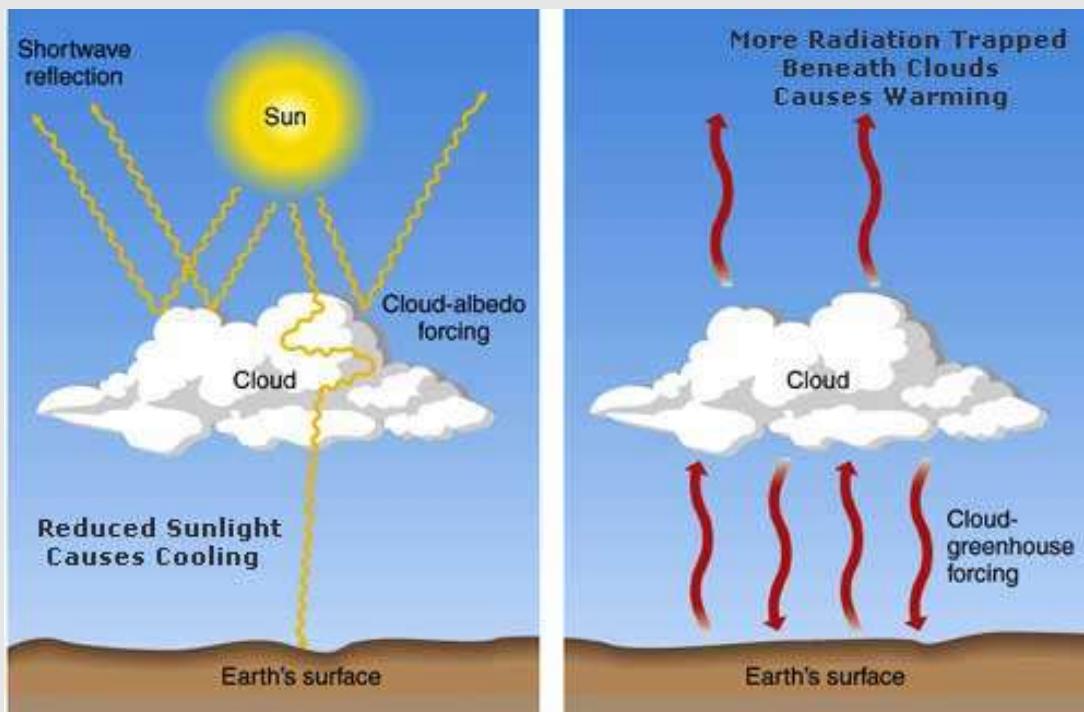


Content3. Factors affecting the planet's Albedo

Some factors such as Clouds, Oceans, Thick vegetation covers or forested areas and Surface albedo affect the earth's albedo.

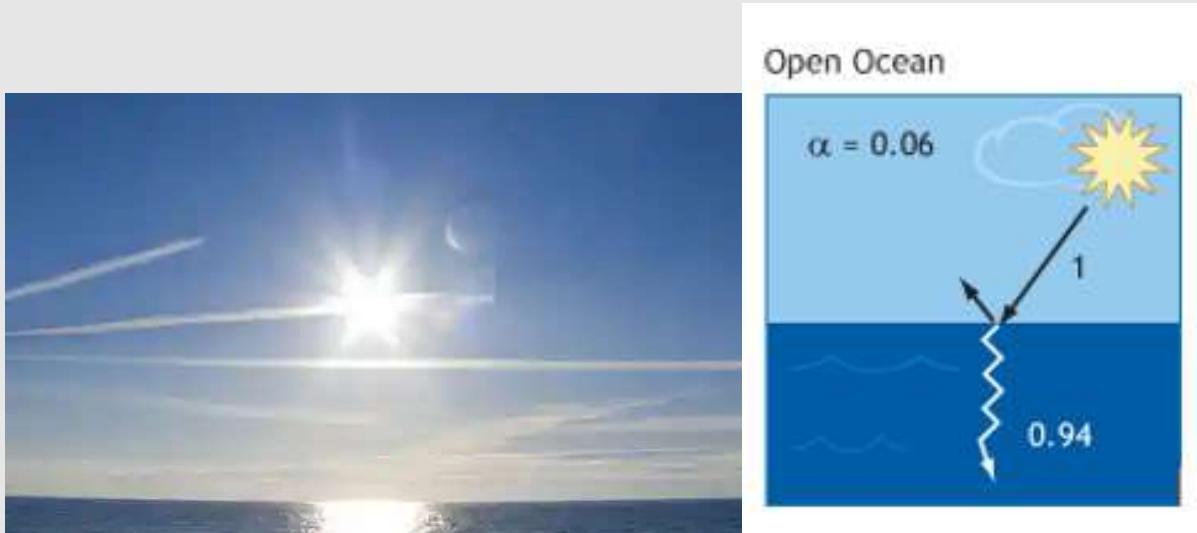
 **Clouds:** The atmosphere is usually covered with clouds that usually pass over the earth's surface.

These clouds may absorb or reflects back the sun's light to the free space depending on the distance from which the clouds are from earth's surface.



- ⊕ **Oceans:** While observing from the space, you will find out that water bodies appear differently from land surfaces.

They appear darker and therefore absorb more sun's radiations than land.

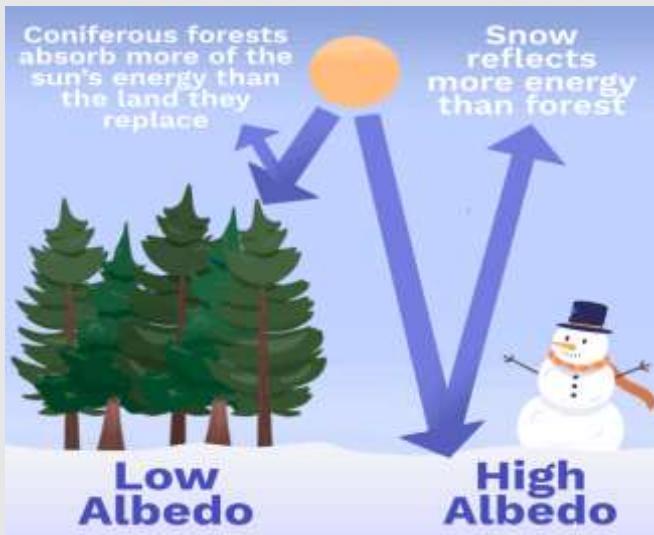


However, some of the radiations heating the water surface (ocean) may be carried away by the currents while others may form water vapor.

- ⊕ **Thick vegetation covers or forested areas:**

Places covered with vegetation absorb a lot of sun's radiation.

This is because the vegetation cover provides a dark surface which absorbs more radiations than the bare land.

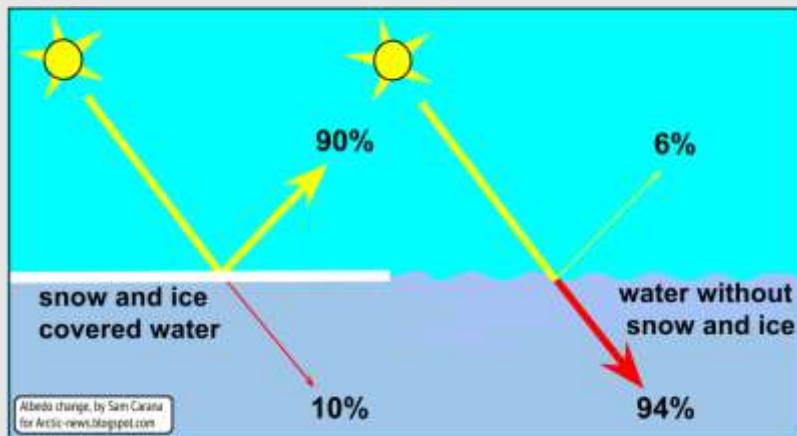


✚ **Surface albedo:**

Different surfaces appear differently. Bright colored surfaces absorb different amounts of radiations than dark colored surfaces.

Snow covered areas are highly reflective.

They thus absorb less amounts of energy (Sun's radiation). The snow cover reduces the heating effect of the earth's surface.



However, if temperatures reduce, the snow cover reduces leading to the absorption of radiation by the exposed ground surface.

➤ **Applications of albedo**

Albedo is a fraction of light that is reflected by a body or surface.

It is commonly used in astronomy to describe the reflective properties of :

- ✚ Planets,
- ✚ Satellites, and
- ✚ Asteroids.



Theoretical learning Activity:

In groups of four, brainstorm about the following problems:

1. What is a blackbody? Give any two examples of black bodies.
2. Explain the term intensity of the sun's radiation.
3. What is albedo?
4. Explain the factors affecting the earth's albedo.



Point to remember

- ❖ **Intensity of the sun's radiation** is the power of solar energy transferred per unit area on earth's surface.
- ❖ **Albedo is** the fraction of radiation returning from a given surface compared to the amount of radiation initially striking that surface. It describes the reflectivity of surface.

$$\text{albedo} = \frac{\text{total scattered power}}{\text{total incident power}}$$

- ❖ **Some of the factors that affect the albedo of the earth:**

- ✓ Clouds.
- ✓ Oceans
- ✓ Thick vegetation covers or forested areas.
- ✓ Surface albedo



Learning outcome 3.1: Formative Assessment

- I. Answer by true or false:

- a. Scientists use a quantity called "**albedo**" to describe the degree to which a surface reflects light that strikes it.
- b. A high **albedo** value means that the **surface** reflects a small amount of the radiation that hits it and absorbs the rest.

II. We have two planets A&B. The Planet A is covered by an ocean and it has an overall average albedo of **20%**. The Planet B is covered by clouds, and has an overall average albedo of **70%**. Which planet reflects more Sunlight back into space?

Choose among the following options:

- a. Planet A
- b. Planet B
- c. The two planets reflect the same amount of light.
- d. More information is required to answer this question.

III. (a) Calculate the albedo of a surface that receives a solar intensity of **10,000 Wm⁻²** and reflects **1500W.m⁻²** of the received amount.

Express your answer as a percentage.

(b) Explain any three of the scientific factors that affect planet's Albedo.

IV. (a) If a surface receives 90% of all incident solar energy what is the albedo?

(b) If a surface receives 25% of all incident solar energy, what is albedo of surface?

Learning Outcome 3.2 : Explain Greenhouse effects.



Duration: 4hours



Learning outcome 3.2 Objectives:

By the end of the learning outcome, the trainees will be able to:

1. Define correctly the term global warming as used in Physics.
2. Explain clearly the human activities causing global warming.
3. Explain clearly the natural activities causing global warming.



Resources

Equipment	Tools	Materials
<ul style="list-style-type: none">- Projector- Computer-Search engine	<ul style="list-style-type: none">PhET simulations of Greenhouse effect.- Simulation software-Scenarios of environmental journals- Videos	<ul style="list-style-type: none">- Whiteboard and markers- Chalkboard and chalks- Textbooks



Advance preparation: prepare PhET simulations of Greenhouse effect and other related videos then present them to trainees.



Content1. *Definition of greenhouse effects, global warming and greenhouse gases*

- **Greenhouse effect** is the process by which thermal radiation from the sun is prevented from leaving the atmosphere and then re-radiated in different directions.
- **Greenhouse gases** are gases that contribute to the greenhouse effect by absorbing infrared radiations from the Sun.
- **Examples of greenhouse gases:** Carbon dioxide (CO₂), Methane , Nitrous oxide and Fluorinated gases.

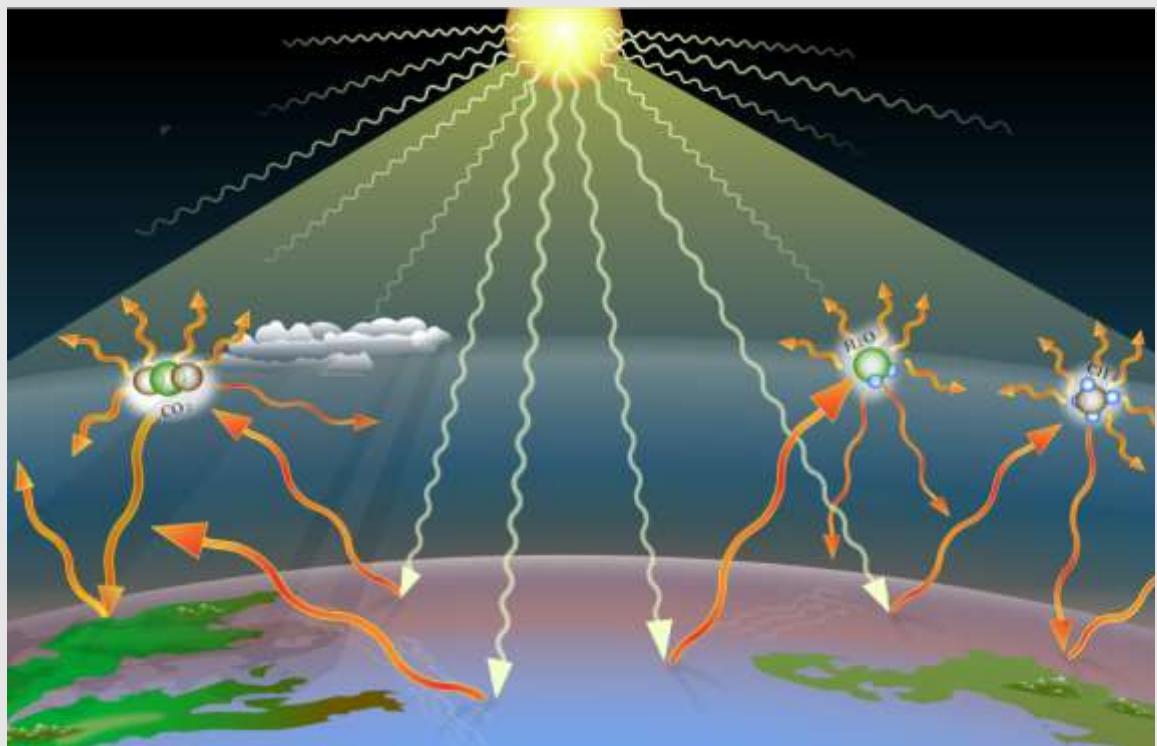


Figure: Greenhouse gases

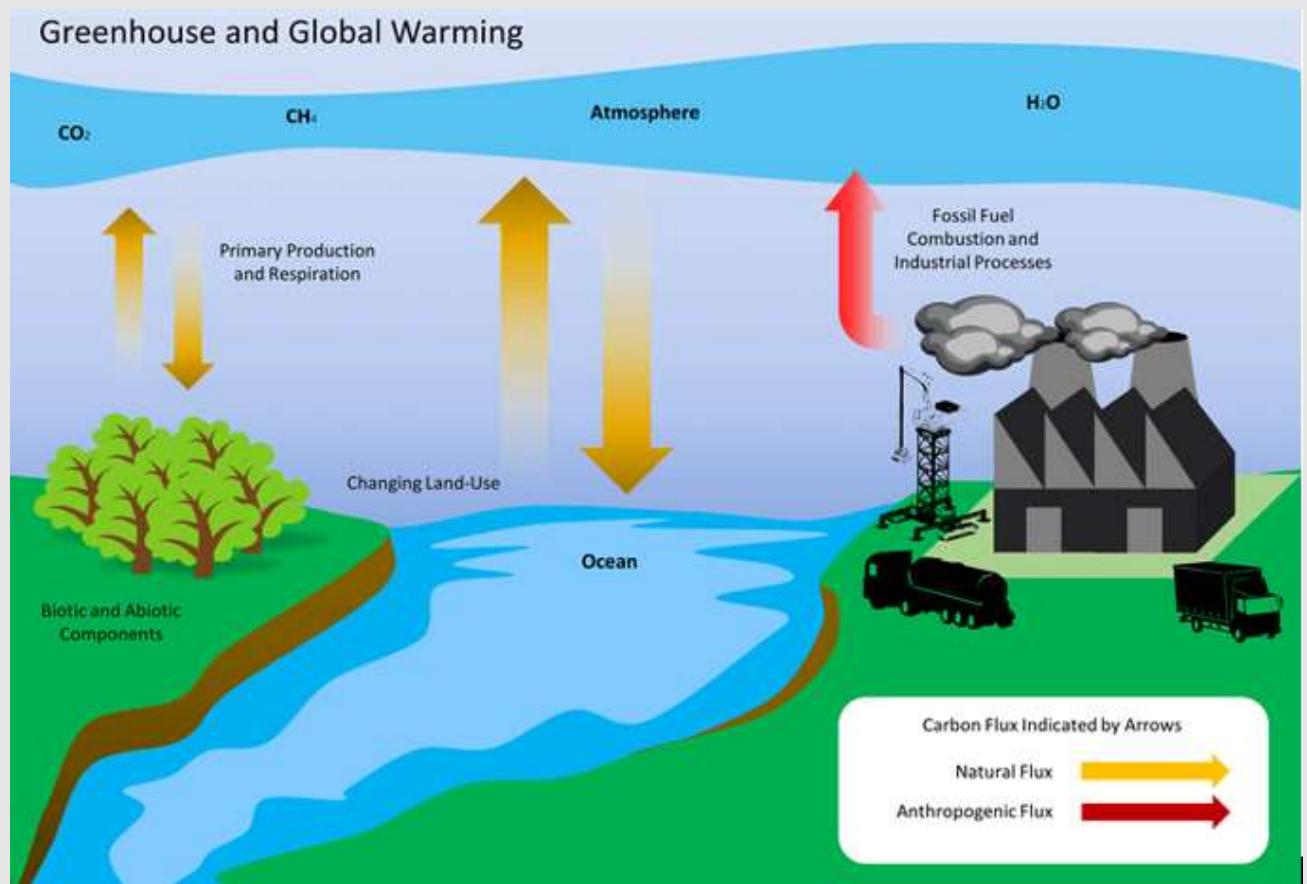
Note: If these green gases were completely not there, the Earth would be too cold for humans, plants and other creatures to live.

✚ Global warming:

Global warming is the increase in the temperature of the earth's surface due to the solar radiations bounced off by greenhouse gases in the atmosphere. or

Global warming is the persistent increase in temperature of the earth's surface (both land and water) as well as its atmosphere.

Global warming damages both the earth's climate and its physical environment.



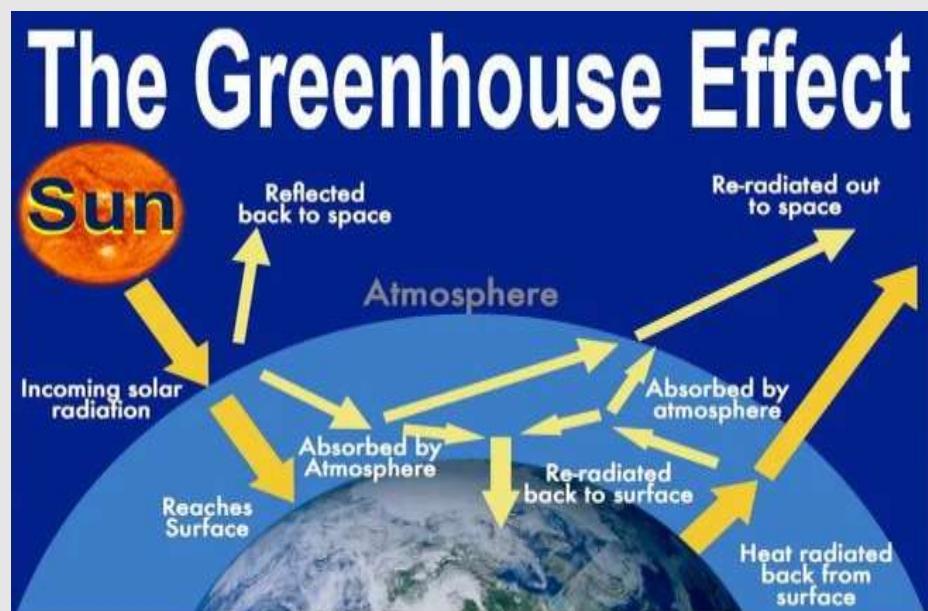


Content2. Explanation of Impact of greenhouse effect on climate change

With the greenhouse effect, the earth is unable to emit the excess heat to space and this leads to increase in atmosphere's temperature and global warming.

Scientists have recorded about 0.75°C increase in the planet's overall temperature during the course of the last 100 years.

The increased greenhouse effect leads to other effects on our climate.



➤ According to REMA, greenhouse effect has already caused the following impacts on climate change:

- ✓ **Greater strength of extreme weather events** like heat waves, tropical cyclones, floods, and other major storms.
- ✓ **Increasing number and size of forest fires.**
- ✓ **Melting of glaciers and polar ice.**
- ✓ **Rising sea levels** (predicted to be as high as about 5.8 cm at the end of the next century).
- ✓ **Increasing acidity in the ocean**, resulting in bleaching of coral reefs and damage to oceanic wildlife.



Figure: Effects of greenhouse effect

➤ **CAUSES OF GLOBAL WARMING:**

The causes of global warming are classified into two categories: **human activities** and **natural activities**.

➤ *Explanation of Human activities causing global warming*

These activities include the following:

Burning of fossil fuels, Deforestation, Agriculture , Dumping waste in landfill and infrastructure development.

✓ **Burning of fossil fuels:**

These emit carbon dioxide gases when heated that accumulate in the atmosphere.

- These gases absorb and may not allow sun's radiation to pass through.
- This in turn leads to an increase in the temperature of the earth's atmosphere, hence global warming.
- These fuels are burnt in vehicles and in industries.

✓ **Agriculture:**

Agriculture is a good practice. Though if not handled with care, it may lead to destruction of nature which is a big problem.

Agricultural practices such as applying fertilizers to crops lead to the release of nitrogen oxides into the atmosphere.

✓ **Infrastructure development:**

These include, urbanization, road construction and places for settlement. These have contributed to the destruction of nature hence global warming.

✓ **Deforestation:**

Plants use carbon dioxide in manufacturing their food in a process known as photosynthesis.

This implies that trees reduce amount of carbon dioxide in the atmosphere.

Therefore, Cutting down trees leads to part of carbon dioxide that remains unused and there will be higher amounts of carbon dioxide in the atmosphere causing global warming.

✓ **Dumping waste in landfill:**

When waste materials decompose, they produce methane gas and this has serious impact in causing global warming as it is among greenhouse gases.

➤ *Explanation of Natural activities causing Global warming*

These activities are the following:

✓ *Volcanicity:*

When volcanoes erupt, they throw out large volumes of Sulphur dioxide (SO₂), water vapor, dust, ash and CO₂ into the atmosphere.

These gases trap heat from solar radiations and prevent them from leaving our planet hence increasing the space temperature.

✓ *Ocean currents:*

Oceans plays a major role in the change of climate. Oceans cover about 71% of the Earth and absorb about twice as much of the sun's radiation as the atmosphere or the land surface.

✓ *Solar output:*

Sometimes there are some fluctuations in the amount of radiation from the Sun. Therefore, if there is a high amount of radiation emitted there will be an increase in the Earth's temperature.

✓ *Orbital changes:*

The earth has natural warming and cooling periods caused by the variations in the orbit of the earth around the Sun.

➤ *Solution to reduce the impact of greenhouse gases:*

To reduce the impact of greenhouse gases, the following measures should be taken:

- Apply greater efficiency of power production.
- Planting more trees.
- Replacing coal and oil with natural gas.
- Use Combined heating and power systems (CHP). (CHP is an energy efficient technology that generates electricity and captures the heat that would otherwise be wasted to provide useful thermal energy).
- Use Renewable energy sources and nuclear power.
- Make carbon dioxide capture and storage.
- Use of hybrid vehicles. (A hybrid vehicle is one that uses two or more distinct types of power)



Theoretical learning Activity:

In groups of four, brainstorm about the following problems:

1. Complete the following by writing either 'INCREASING' or 'DECREASING'.

a. The surface temperature of the Earth has been _____ over the past 100 years.

b. The thickness of the ice at the North Pole is _____.

c. In some parts of the world, summers are getting hotter and drier. The effects of this are that:

i. Water levels in lakes and ponds are _____

ii. The risk of forest fires is _____

iii. The numbers of wild animals and farm animals dying from lack of food is _____

2. Use the words in the box below to complete the sentences. Some words may be used more than once, or not at all.

The amount of _____ in the atmosphere is _____, and may be leading to _____ warming. In many countries, _____ are being cut down and _____, which adds _____ to the air. It also removes _____ that could have used up carbon dioxide to produce _____.

atmosphere	burned	carbon dioxide	decreasing	Earth	forests
global	greenhouse	increasing	oxygen	sea	trees

3. Write TRUE or FALSE for the following statements.

a. Greenhouses are made of clear glass that allow light in to warm inside. _____

b. Without the greenhouse effect, temperatures on Earth would be too cold for life. _____

c. Two greenhouse gases are oxygen and nitrogen. _____

d. Deforestation leads to global warming because there are no trees to remove carbon dioxide. _____

e. Global warming does not change the weather and climate of an area. _____



Points to remember

- ⊕ **Greenhouse effect** is the process by which thermal radiation from the sun is prevented from leaving the atmosphere and then re-radiated in different directions.
- ⊕ **Greenhouse gases** are gases that contribute to the greenhouse effect by absorbing infrared radiations from the Sun.
- ⊕ **Examples of greenhouse gases:** Carbon dioxide (CO₂), Methane , Nitrous oxide and Fluorinated gases.
- ⊕ **Global warming** is the increase in the temperature of the earth's surface due to the solar radiations bounced off by greenhouse gases in the atmosphere
- ⊕ **Impact of greenhouse effect on climate change:**

Greater strength of extreme weather events, Rising sea levels, Melting of glaciers and polar ice and Increasing acidity in the ocean

✓ **Human activities causing global warming include the following:**

Burning of fossil fuels, Deforestation, Agriculture , Dumping waste in landfill and infrastructure development.

⊕ **Natural phenomena causing Global warming include:**

- ⊕ Volcanic eruption
- ⊕ Ocean currents



Learning outcome 3.2: Formative Assessment

1. Match each item of column A with its corresponding definition from column B:

Write your here	Column A	Column B
1→....	1. Global warming.	X. A substance that contributes to the global warming by absorbing infrared radiations from the Sun.
2→....	2. Greenhouse gas.	Y. The persistent increase in temperature of the earth's surface (both land and water) as well as its atmosphere.

2. The following are natural activities causing global warming except:

Select one or more items:

- a. Ocean currents
- b. Plants
- c. Volcanic activity
- d. Deforestation
- e. Solar output

3. Answer by True or False:

- a. Oceans cover about **71%** of the Earth and absorb about twice as much of the Sun's radiations as the atmosphere or the land surface....
- b. Scientists have recorded about **0.75°C** increase in the planet's overall temperature during the course of the last **100** years.....
- c. Global warming only damages the earth's climate....

4. Explain any three human activities causing global warming.

Learning Outcome 3.3: Explain climate change and mitigation measures.



Duration: 2hours



Learning outcome 3.3 Objectives:

By the end of the learning outcome, the trainees will be able to:

1. Define correctly climate and climate change as used in Physics.
2. Explain clearly the mitigation measures.



Resources

Equipment	Tools	Materials
<ul style="list-style-type: none">- Projector- Computer-Search engine	<p>PhET simulations of Greenhouse effect.</p> <ul style="list-style-type: none">- Simulation software-Scenarios of environmental journals- Related videos	<ul style="list-style-type: none">- Whiteboard and markers- Chalkboard and chalks- Textbooks



Advance preparation: Prepare PhET simulations of Greenhouse effect and climate change to be presented to trainees.



Content1. Explanation of climate change related facts

- ⊕ **Climate** is usually defined as the “average weather”. However, climate change has been observed over a longer period of time.
- ⊕ **Climate change** refers to any significant change in the climate parameters such as temperature, precipitation, or wind patterns that occur over several decades or longer.

Natural and human systems have adapted to the prevailing amount of sunshine, wind, and rain.

⊕ *Climate change related facts:*

- **Climate feedback:** This refers to the process that acts to amplify or reduce direct warming or cooling effects.
- **Climate lag:** This is the change in radiation.
- **Climate model:** This is a quantitative way of representing the interactions of the atmosphere, oceans, land surface and ice.



Content2. Explanation of the causes of climate change and mitigation measures.

Most of activities done by human being leads to high concentration of greenhouse gases. Such activities include the following: bush burning, burning of fossil fuels, deforestation and agriculture.

- **Burning coal, oil and gas** produces carbon dioxide and nitrous oxide responsible for global warming.
- **Cutting down forests (deforestation)**. Trees help to regulate the climate by absorbing CO₂ from the atmosphere. So when they are cut down, that beneficial effect is lost and the carbon stored in the trees is released into the atmosphere, adding to the greenhouse effect.
- **Increasing livestock farming**. Cows and sheep produce large amounts of methane when they digest their food.
- **Fertilizers containing nitrogen** produce nitrous oxide emissions.
- **Fluorinated gases** produce a very strong warming effect, up to 23 000 times greater than CO₂.

➤ Explanations of mitigation measures.

What is climate change mitigation?

Climate change mitigation refers to efforts made to reduce or prevent emission of greenhouse gases.

Mitigation can mean using new technologies and renewable energies, making older equipment more energy efficient, or changing management practices or consumer behavior.

Climate change mitigation is much needed as Climate change is one of the most complex issues facing us today.



Theoretical learning Activity:

In groups of four, brainstorm about the following problems:

Discuss the following terms as used in Physics:

- a. Climate change
- b. Climate lag
- c. Climate model
- d. Climate feedback



Points to remember

- ✚ **Climate** is usually defined as the “average weather”.
- ✚ **Climate change** refers to any significant change in the climate parameters such as temperature, precipitation, or wind patterns that occur over several decades or longer.
- ✚ **Climate feedback:** This refers to the process that acts to amplify or reduce direct warming or cooling effects.
- ✚ **Climate lag:** This is the change in radiation.
- ✚ **Climate model:** This is a quantitative way of representing the interactions of the atmosphere, oceans, land surface and ice.
- ✚ **Climate change mitigation** refers to efforts made to reduce or prevent emission of greenhouse gases.



Learning outcome 3.3: Formative Assessment

I. Answer by **true or false**:

- a. Global warming only damages the earth's climate.....
- b. Scientists use a quantity called "**albedo**" to describe the degree to which a surface reflects light that strikes it.....
- c. A high **albedo** value means that the **surface** reflects a small amount of the radiation that hits it and absorbs the rest.....
- d. **Greenhouse effect** is a process by which thermal radiation from the Sun is prevented from leaving the atmosphere and then re-radiated in different directions.....
- e. **Climate change mitigation** refers to the efforts made to increase the emission of greenhouse gases.....

II. The following are natural activities causing global warming except:

Select one or more items:

- a. Ocean currents
- b. Exposing trash
- c. Volcanic activity
- d. Deforestation
- e. Solar output

III. Discuss any five measures that should be taken to reduce global warming.

IV. Fill in the blanks using the words: **Climate change, Climate feedback, climate model, Climate lag, greenhouse effect, global warming**

(a) _____ is a term used to a process that acts to amplify or reduce direct warming or cooling effects.

(b) _____ is the change in radiation.

(c) _____ is a quantitative way of representing the interactions of the atmosphere, oceans, land surface, and ice.

(d) _____ refers to any significant change in the climate parameters such as temperature, precipitation, or wind patterns that occur over several decades or longer.

(e) _____ is the increase in the temperature of the earth's surface due to the solar radiations bounced off by greenhouse gases in the atmosphere.

V. The following are examples of greenhouse gases except:

A. Carbon dioxide	C. Methane
B. Nitrous oxide	D. Oxygen

VI. Answer the following questions:

Fill in the Blank: Use the words to fill in the blanks.

risen
warm up
temperature

gases
sunlight
Earth

radiated
absorbed
surface

global warming
greenhouse effect

Why does a closed car get so hot on a sunny day? (1) _____ shining through the car's windows is (2) _____ by the objects and materials in the car. Then some of this energy is (3) _____ back from the objects and materials in the form of heat. But this heat cannot pass back out through the glass windows. The (4) _____ inside the car rises. This process of warming takes place in glass greenhouses, and it also happens to (5) _____. Much of the heat radiated from Earth's (6) _____ does not go out into space; it is reflected back down to Earth by (7) _____ in the atmosphere. This reflected heat causes the atmosphere to (8) _____. The process by which heat is trapped and reflected by gases in Earth's atmosphere is known as the (9) _____. As a result, average temperatures on Earth have (10) _____ 0.5°C in the last hundred years. This temperature increase is known as (11) _____.

Write each term after its definition.

global warming greenhouse effect deforestation

12. The process by which heat is trapped by gases in Earth's atmosphere. _____

13. The removal of forests. _____

14. A rise in Earth's average temperatures caused by increased amounts of gases in Earth's Atmosphere. _____

Fill in the Blank: Using your reading, find the words or phrases to complete each sentence or statement.

15. _____ describes the weather over a long period of time.

16. We now face the prospect of change. Many scientists believe this change is being brought on primarily by _____.

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Online resources:

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2. <https://www.physicsforums.com/threads/gravitational-constant-measurement.644074/>
3. <https://www.qrg.northwestern.edu/projects/vss/docs/space-environment/1-what-causes-an-orbit.html>
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5. https://www.lehman.edu/faculty/anchordoqui/101-P4_s.pdf
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