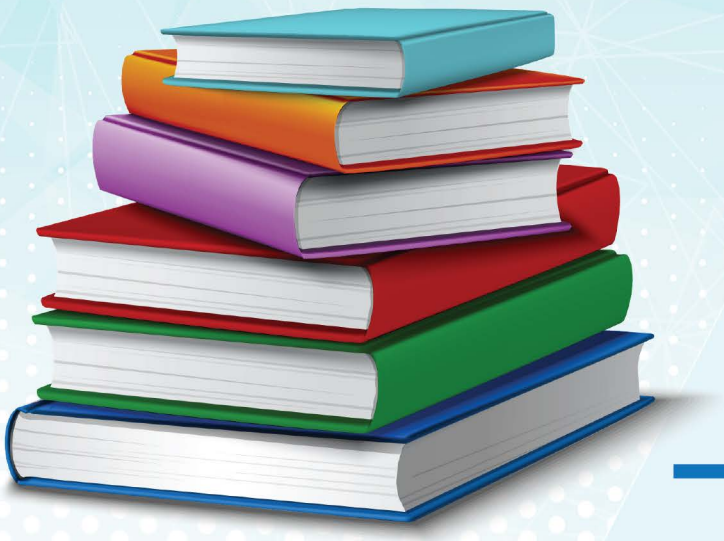
RWANDA POLYTECHNIC

TVET LEVEL II



BASIC EDUCATION

Math

TRAINER MANUAL

RWANDA POLYTECHNIC

Approved by: Workforce Development Authority

USAID FROM THE AMERICAN PEOPLE

EDC Education Development Center

2020

Acknowledgements

Rwanda Polytechnic (RP) would like to officially recognize all parties who contributed actively to the preparation of the Trainer and Trainee manuals of this module. We wish to extend our thanks to various organizations such as Workforce Development Authority (WDA), EDC through its USAID Huguka Dukore Akazi Kanoze (USAID - HDAK), TVET schools, Private Industries, GIZ Hanga Ahazaza Project and other individuals who greatly contributed from the initial concept towards publication of this training manual.

Under Rwanda Polytechnic (RP) supervision and involvement

Under Workforce Development Authority (WDA) guiding policies and directives

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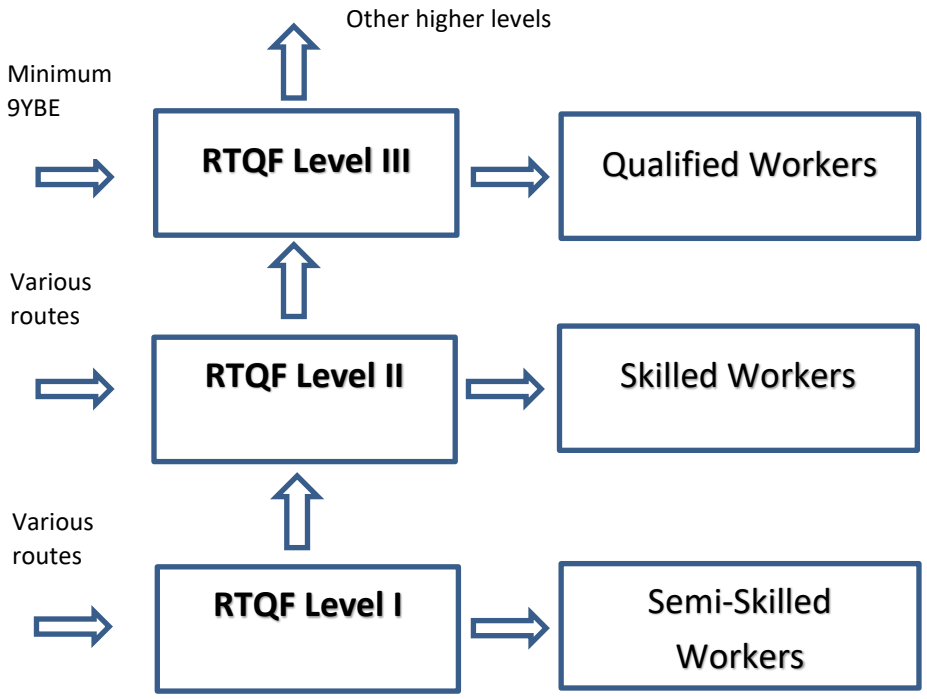
Introduction to RTQF Level II Training Modules

Background

Rwanda Polytechnic, with support of and in collaboration with USAID Huguka Dukore Akazi Kanoze, has developed RTQF TVET Level II programs that combine basic education, soft skills and vocational skills modules. Bridging the gap between Level I and Level III programmes, Level II aims to prepare learners who have a minimum education level of Primary 6 or equivalent to continue with their education or become skilled workers in the labour force.



Following the Workforce Development Authority (WDA) curriculum development process that involved experts from Rwanda Polytechnic, Rwanda Education Board, Ministry of Agriculture, technical vocational institutions, Education Development Center, Akazi Kanoze Access and other technical experts, training modules were developed in basic education, soft skills (work readiness) and, initially, agriculture. Additional vocational areas will be added over time. Trainees will be trained in all Basic Education and Soft Skills modules listed below, as well as in 6 - 8 modules that make up their chosen technical vocational programme.

Module Requirements:

Basic Education	Soft Skills	Vocational Skills
• English • Kinyarwanda • Mathematics • Integrated Science (Physics, Chemistry, Biology)	• Basic Entrepreneurship Skills • ICT Essentials • Communication Skills • Safety, Health and Sustainable Environment	• Vocational programmes will have a set of 6 – 8 required technical modules. E.g. Food Crop Production and Processing includes the following modules: 1. Food Crop Production

	• Personal Development and Career Guidance	2. Small Scale Post-Harvest Operations 3. Growing Medium 4. Food Safety and Sanitation 5. Food Preservation and Storage 6. Flour Processing

Organization of the Training Manuals

For each module there is a Trainer Manual and a Trainee Manual. These manuals, based on the curricula for each subject, are divided into Learning Units, and each Learning Unit includes 3 – 5 Learning Outcomes. The learning outcomes make up the essential skills, knowledge and attitudes to be acquired by trainees. To make the Trainee Manual more user friendly, Unit and Topic are used respectively for Learning Unit and Learning Outcome. The number of hours per training module varies, ranging between 30 and 120 hours.

Teaching & Learning Methodology of RTQF Level II 2 TVET Materials

The teaching and learning methodology used in the materials is based in experiential and adult learning. Activities are designed to engage trainees, build upon what they know and learn and provide them with opportunities to build their skills in the classroom and in the workplace. More specifically, guiding principles in the development of the manuals include:

- Building on participants' knowledge, skills and experiences
- Facilitating a learning process through active engagement of participants rather than through lecturing
- Providing opportunities to practice – inquiry based and hands on practice, both in the classroom and workplace
- Using simple and clear language
- Connecting to the real world: use local resources and the environment for learning
- Promoting critical thinking through properly debriefing activities and asking questions that get learners to think, analyze, relate issues and topics to their own lives and come up with solutions
- Applying social inclusion principles: Finding ways to include all types of youth (and trainers) – males and females; different cultural/ethnic/religious backgrounds, people with disabilities (PWD); people with different types of health status …
- Encouraging risk taking – promote questioning and being free to explore

- Promoting habits of mind that support life-long learning: curiosity and wonder, open mindedness, creativity

These principles are reflected in the layout and flow of activities in the manuals:

1. **Key Competencies:** Table found at the beginning of each Learning Outcome that describes the main knowledge, skills and attitudes to be gained by the end of the activities.

2. **Self-Assessment:** Conducted at the beginning and end of each Learning Unit to get a sense of trainees' knowledge and skills going into it and what they have gained by the end of the Learning Unit (and steps they need to take to further their understanding and skills).

3.  **Getting Started Activity:** Typically, a quick activity or questions to 1) give the trainer a sense of trainees' existing knowledge and skills; 2) spark the interest of trainees in the topic; 3) introduce the objectives and key competencies of the topic.

4.  **Problem Solving Activity:** A challenging activity to get trainees engaged and to learn through discovery instead of memorization of facts. A variety of teaching and learning methodologies are used, including individual and group work such as reading real life work-based scenarios and answering accompanying questions to activities such as identifying proper tools and equipment from the school workshop to conduct a certain activity. Following the sharing of responses, the trainer guides trainees through the content and processes being introduced.

5.  **Guided Practice Activity:** Building on the concepts and skills gained in the Problem Solving Activity, the trainer guides trainees through practical examples.

6.  **Application Activity:** Consolidates trainees' knowledge and skills through a real-life application of the topic in the classroom, community or workplace. Trainees are given more independence in applying what they have learned.

7. **Key Facts boxes:** Throughout the Trainee Manual, one will find Key Facts boxes. These contain the main information or content for a given Learning Outcome. They are there for the trainees' reference and are used throughout the different types of activities.

8.  **Points to Remember:** List of the top key learning points or "take-aways" from the topic.

9.  **Formative Assessment:** Questions and activities to assess trainees' level of understanding of the concepts introduced.

10.  **Summative Assessment:** Based on the integrated, real life situation approach used in other TVET levels, this is done at the end of every module for agricultural modules and, with some variations, at the end of each Learning Unit for Basic Education and Soft Skills modules.

11.  **Self-Reflection:** Trainees re-take the Self-Assessment given at the beginning of the Learning Unit and identify their strengths, challenges and actions to improve their level of competence.

The Trainer and Trainee Manuals are meant to be used in conjunction with each other and are well coordinated through the headings and labelling of activities. The trainer will always be able to refer trainees to specific activities by the coordinated numbering system. For instance, a specific exercise might be labelled Topic 1.2 Task 2. The Topic is the number of the Learning Outcome and the task is the specific exercise to be done. The Key Facts are also numbered for easy reference. These nor the Self-Assessment tables are in the Trainer's Manual so the trainer should have a copy of both manuals.

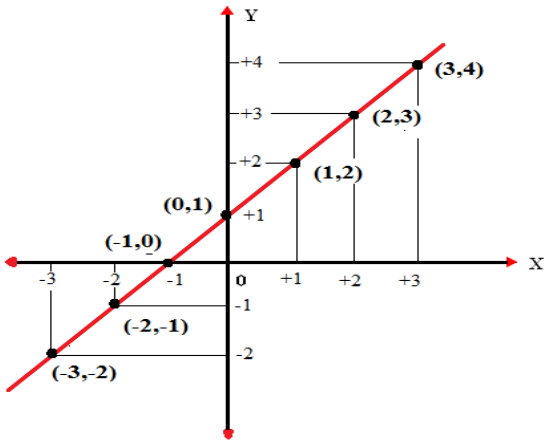
The Trainer's Manual includes answers (or guidelines to the trainer as appropriate) to Formative and Summative Assessments as well as to problems given throughout the activities. Summative Assessments are not included in the Trainee's Manual. These are meant to be used as a guide for those who will be developing a context-appropriative Summative Assessment at the end of the Module or Learning Unit. Basic Education and Soft Skills modules include Summative Assessments at the end of every Learning Unit while the technical modules include it only at the end of the module.

Lastly, there is a section in the Trainer's Manual for additional information to the trainer that includes either specific information or references to information that can help them deepen their understanding of the particular content.

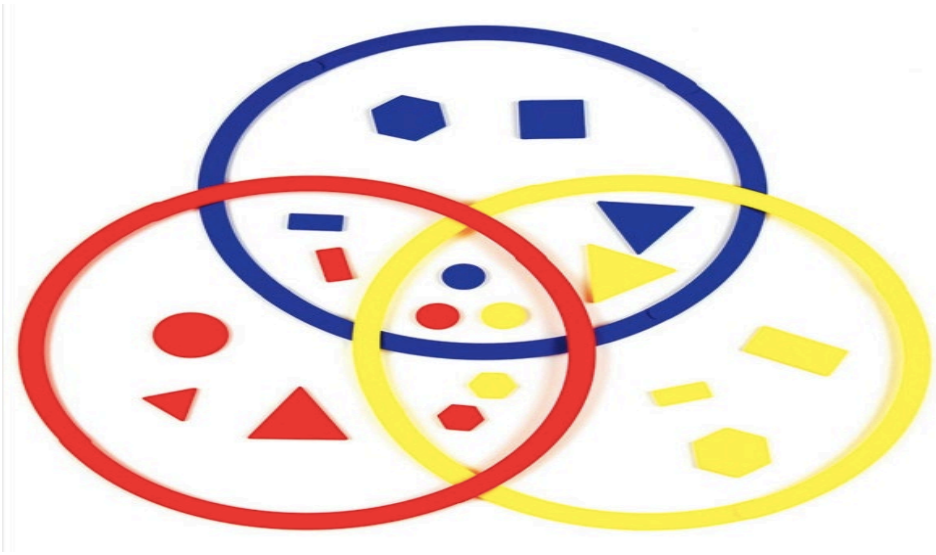
BASIC MATHEMATICS AT THE WORKPLACE

Learning Units	Learning Hours	Learning Outcomes
Learning Unit 1: Apply algebraic and set concepts	10	**1.1** Use Venn diagrams correctly to represent set operations and properties
		1.2 Calculate proportions, ratios, percentages, and mixtures in workplace and real-life situations
		1.3 Use percentages to calculate discount, commission, profit, loss, interest, and taxes in real life situations
		1.4 Manipulate and solve simple algebraic equations and inequalities for daily life and workplace
		1.5 Plot and interpret graphs of linear and quadratic functions based on real life and work situations
Learning Unit 2: Apply geometric properties to shapes and angles	25	**2.1** Sketch shapes and angles to illustrate relevant relationships and properties
		2.2 Draw regular and irregular shapes as well as angles to represent properties of the given figures
		2.3 Measure lengths and calculate areas of shapes using formulas, and Thales theorem
		2.4 Construct mathematical arguments using the angle properties of parallel lines
		2.5 Calculate, using given formulas, the surface area and volume of common geometrical solids
Learning Unit 3: Apply statistical concepts, interpret data, and statistical calculations	15	**3.1** Record and organize statistical data for analysis
		3.2 Compute the measures of central tendency (Mean, Mode, and Median) using data sets from life situations
		3.3 Present statistical data and analysis using mathematical representations (graphs, charts, and tables)

Learning Unit 1: Apply Algebraic and Set Concepts



$$y = x + 1$$



Learning Outcomes

By the end of the Learning Unit, trainees will be able to:

1.1 Use Venn diagrams to represent operations and properties

1.2 Calculate proportions, ratios, percentages, and mixtures in workplace and real-life situations

1.3 Use percentages to calculate discount, commission, profit, loss, interest, and taxes

1.4 Manipulate and solve simple algebraic equations and inequalities

1.5 Plot and interpret graphs of linear and quadratic functions

Learning Unit 1 Self-Assessment

1. Ask trainees to look at the illustration above (in their Trainee Manuals) and discuss what they see. What topics do they think this unit will include based on the illustrations? After some brainstorming, share the main topics for the unit to come.

2. Explain that this Learning Unit is going to focus on **sets**—groups of objects that share common traits or characteristics. We will be learning about sets of numbers, organizing sets of objects, and using sets to solve problems in addition to reviewing algebraic concepts and properties.

3. Ask trainees to fill out the self-assessment at the beginning of the unit in their Trainee Manuals. Explain that the purpose of the self-assessment is to become familiar with the topics in the unit and for them to see what they know (or do not know) at the beginning. At the end of the unit, they will do a self-reflection, which includes re-taking the same self-assessment and identifying their strengths, areas where they have improved, and areas that still need improvement as well as listing out actions to take so each trainee can feel confident with the details of this learning unit. The self-assessment is not a test!

Learning Outcome 1.1: Use Venn diagram representations to carry out set operations (union, intersection, complement) and solve problems

Objectives: By the end of the learning outcome, trainees will be able to:

a. Represent and determine union of 2 or 3 sets
b. Represent and determine intersection of 2 or 3 sets
c. Solve problems using sets represented by Venn diagrams

Time Required: 2 hours

Learning Methodologies: group discussions; pair, group, and individual work

Materials Needed:

- Standard materials: student workbook/notebook

Preparation:

- ❑ Prepare a Venn Diagrams Representing Mr. Alloys work on a flip chart

Cross Cutting Issues:

- ✔ **Gender:** trainer will provide equal opportunities to both boys and girls during the lesson as well as balance gender when forming groups
- ✔ **Peace/value education:** trainer promotes team work and respect for others' views and work
- ✔ **Inclusive education:** trainer is sensitive to differences in learning abilities and uses a variety of activities and materials to cater to different learning styles

Prerequisites: Trainees should already be comfortable with and able to:

- Use addition, subtraction, multiplication, and division
- Understand and use set notation and vocabulary (set, element, sub-set, union, intersection, complement, and null (or empty set))

Key Competencies:

1. Describe the purpose of representing of 2 or 3 sets in a Venn diagram	**1.** Create a Venn diagram to represent 2 or 3 finite sets then use the diagram to solve related problems	**1.** Creative
2. Explain Union and Intersections of sets	**2.** Represent and determine union of 2 or 3 sets	**2.** Innovative
3. Describe the complement of a set.	**3.** Represent and determine intersection of 2 or 3 sets	**3.** Accurate

Steps:

Getting Started: What do we know and where are we going?

1. Write the words following words on the blackboard:

Set, Element, Sub-set, Intersection, Union, and Complement

Ask trainees to turn to **Topic 1.1 Task 1** and write the words in their notebooks and to try to remember what they can about each vocabulary word. They should then write a good definition beside each word and give an example.

2. After 3 minutes ask a trainee to share their definitions. Let other trainees add to or question the given definition to come to a final class definition with a good example. Write the definition on the board. Trainees should copy definition and examples into their notebooks.

Possible Answers:

Set: a group of objects that share a common characteristic. Or a collection of objects…

Element: an object inside a set, is called an element or a "member of the set."

We use the notation $\in$ to mean "is an element of the set" so we could say $2 \in A$ to mean that 2 is an element of the set A.

If x is not an element of set A, then we write $x \notin A$. And we say "x does not belong to A."

Note:

a. We use capital letters to name sets.

b. We can list out the elements of a set: A= {2, 4, 6, 8, 10}, thus 4 is an element of A.

c. OR we can describe a set: B is the set of vowels in the alphabet, thus $u \in B$.

Sub-Set: a set that is completely contained inside another set. We use the notation $\subset$ to mean "is a subset of" so if X = {1,2,3,4,5} and Y = {2,4} then $Y \subset X$ means "Y is a subset of X" BUT X is NOT a subset of Y because X has elements that are not included in Y, we write $X \not\subset Y$.

Note: Set X above is a subset of the set C= {1,2,3,4,5,6,7,8,9,10} –and so is Y. $X \subset C$ and $Y \subset C$.

Another example is that Set B (the vowels) is a subset of the set of all letters in the alphabet.

Intersection: The intersection of two sets is the set containing all the elements that are in BOTH of the given sets. This happens when sets overlap or share elements. We write ∩ which means 'intersection.' So, X ∩ Y + {2, 4}

Example: In the illustration at the beginning of the chapter there are two triangles and three circles contained in the intersection of the Yellow and the Blue sets.
(Ask trainees what is contained in the INTERSECTION of the Red and Blue sets)

Union: The union of two sets is ALL the elements in BOTH sets (but do not double up the elements—make sure elements in the intersection are listed only once.)
We write this using the symbol ∪ which means 'union.'

Example: If P= {1, 2, 3, 4} and Q= {3, 4, 5} then:
The **intersection** of P and Q (written P∩ Q) will be P∩ Q= {3, 4}
The **union** of P and Q (written P∪ Q) will be P∪ Q = {1, 2, 3, 4, 5}

Complement of a Set: is all the elements that are in the population but NOT inside the set. Complement is written as **A'** which means the Complement of A—all elements **not** in A.
Empty Set or the **Null Set** is a set with NO elements is called the**,** use the symbol ∅.

3. Assign each trainee a number starting at 1 and going up (whole numbers only) until each trainee has their own number. Explain that trainees will now stand and form sets according to properties of the numbers they have been assigned.

4. Read the following group descriptions aloud. Give time for trainees to move into correct groups. Discuss the groupings so all understand who belongs inside and who is NOT in the set.

Topic 1.1 Task 2
Read descriptions aloud. Trainees move into given groups. Discuss.

- **a.** All odd numbers go to the front of the class and all even numbers go to the back of the class
- **b.** All numbers less than 10 go to the back of the room and all even number go to the front of the room.
- **c.** All the numbers that are multiples of 4 go to the back of the room and all the numbers that are 2 go to the front of the room.
- **d.** Trainer will provide other sets – or think of one to share with the class!

Answer: (and notes on activity)

- **a.** The students (representing numbers) form two separate groups…there is no intersection because no elements are shared, a number is EITHER odd or even—not both.
- **b.** Some trainees need to be in BOTH groups-other trainees are not in either group. Explain that trainees belonging to BOTH groups are the elements of the INTERSECTION of the two sets.
  Ask the trainees to describe the UNION of the two sets. All grouped trainees are in the union.
- **c.** This shows a subset because all multiples of 4 are ALSO inside the set multiples of 2
- **d.** Trainer can continue to describe two different sets of numbers and trainees can move to join the given set—or sit down if their number is NOT in the set.
  For example: Set of numbers that are multiples of 3---and---set of odd numbers
  Or: Set of numbers Greater than 10—and—set of prime numbers
  Or: Trainees may have ideas of sets that they could use to divide up the class, or??

5. Continue until trainees have understood the concept of sets, sub-sets, intersection, and union.

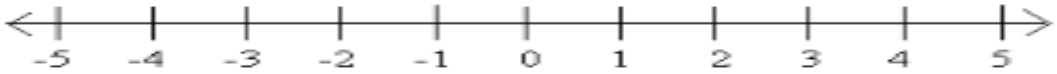
6. Introduce the learning outcome and objectives and describe the type of knowledge, skills, and attitudes trainees will gain during this learning outcome.

Problem Solving Activity

1. Trainees organize into groups of 4 to complete **Task 3**. Groups will work together to discuss and find answers, but trainees each write in their own notebook.

Topic 1.1 Task 3
Draw a number line along the top of the notebook page.
Label the number 0 in the centre of their number line, then label also the numbers -3, -2, -1, 0, 1, 2, 3, 4, and 5 on their number line.

Answer: Trainer will draw on the blackboard. The number line will look like the following:



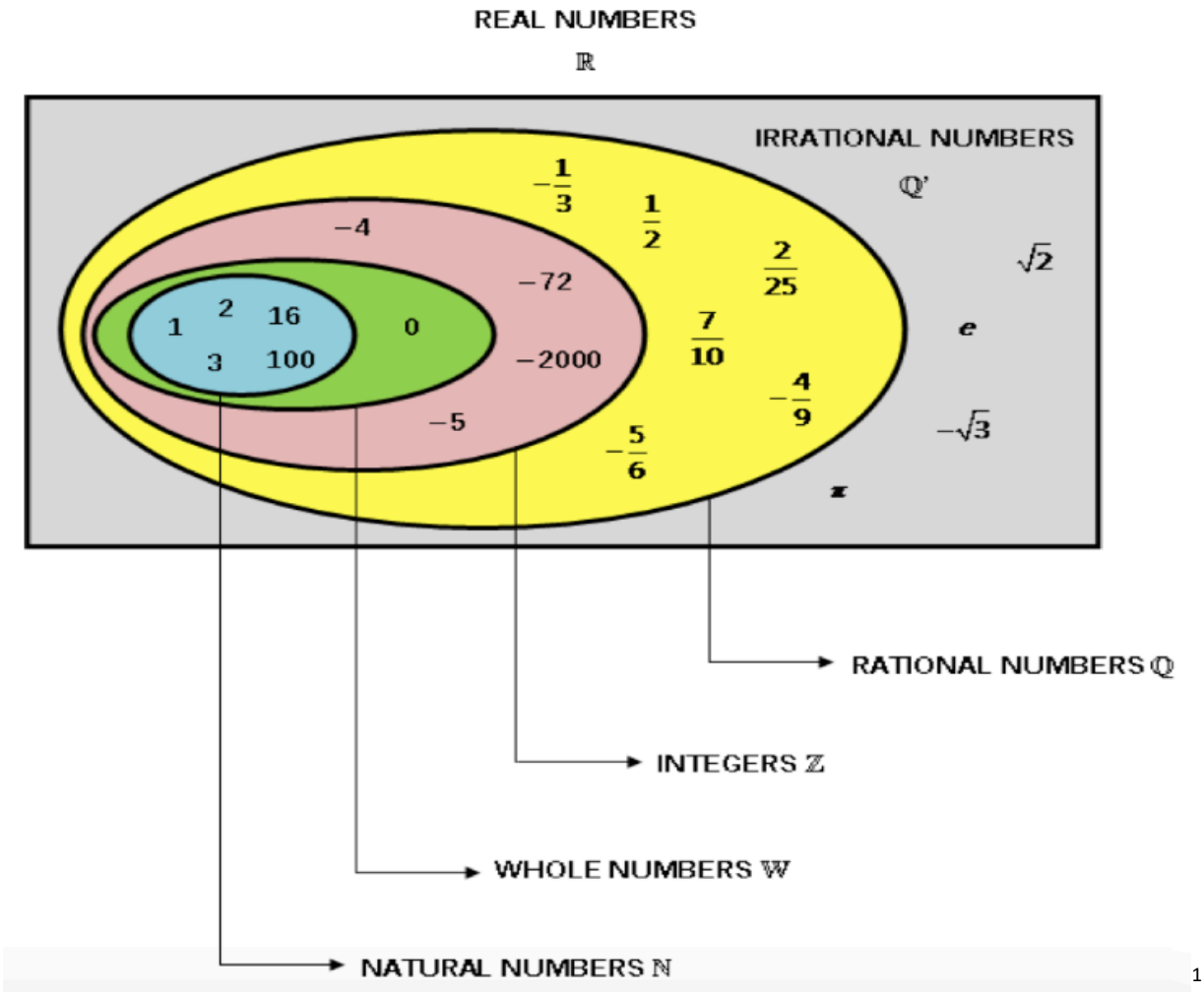
2. Ask all trainees to put their pencil tip on the number 2 and their finger on the number 3. Ask the students to discuss how many numbers there are between the numbers they are touching.

Answer: There are many, for example 2.1, and 2.225, and 2 ½, and 2.75 and so forth. Ask for examples of numbers we can find on the drawn number line—as trainees give examples, touch the space where that number is seen on the number line on the black board.

3. Remind trainees that every number on a number line is called a REAL NUMBER. Inside Real Numbers there are sub-sets that are important to know.

4. Ask trainee to review the Venn diagram of Real Numbers in their manual. Using the diagram, you can see that the number line (the set of Real Numbers) has subsets inside subsets that make up all the numbers. The clear sets of numbers are identified below the diagram.

Topic 1.1 Task 4: Describe the different sets shown in the diagram above and identify which are sub-sets of another set.



[1]

Answer:
Irrational numbers (cannot be written as fractions) are a subset of the Real numbers
Rational numbers (can be written as fractions or decimals) are a subset of the Real numbers
Integers (positive and negative counting numbers, including zero) are a subset of Real numbers
Note: Integers are also a subset of Rational numbers (because a whole number can be written as a fraction over 1, so 9 = 9/1
Whole numbers (positive counting numbers and zero) are a sub-set of Integers and of Rational Numbers and of Real Numbers
Natural numbers (positive counting numbers—not zero) are a subset of Whole, Integer, Rational and of Real numbers
Subsets are sets completely contained inside another set as shown in this illustration.

5. In groups of four, each trainee will work on 2 of the following questions. Their job is to solve 2 questions individually and then explain their answers to their group. Review the answers with the whole class by asking for volunteers from each group to share responses. Correct as needed.

Topic 1.1 Task 5 (possible answers follow each question in the parentheses)
Review the illustration from Task 4 to assist in responding to the questions below.

a. Give an element of the Integers that is NOT an element of the Whole numbers.

[1] Ndyabasa, E. et al (2016). Mathematics for Rwandan Schools Senior 1, Student's Book. Kigali, Rwanda: Longhorn Publishers Limited.

(-5, -1, …)

b. Give an element of the Rational numbers that is also an element of the Natural numbers.
(1, 5, 24, …)

c. Give an element of the Whole numbers that is NOT an element of the Natural numbers.
(the only answer is 0)

d. Give an element of the rational numbers that is also an irrational number.
(No elements are in both sets—there is no intersection. The two sets are complements)

e. Which sets does the number 0.75 belong to? (rational, real)

f. Which sets does the number -31 belong to? (integer, rational, real)

g. Give an example of a number that is an integer but not a whole number.
(negative numbers)

h. Give an element of the Rational numbers that is also an integer
(3/1, 19/1, …)

6. Look again at the Venn diagram illustration to help trainees understand these sets and the relationships between them. Note: ovals are sets. Sets are inside other sets; elements may be in one set, many sets, or no set.

Activity 3: Guided Practice

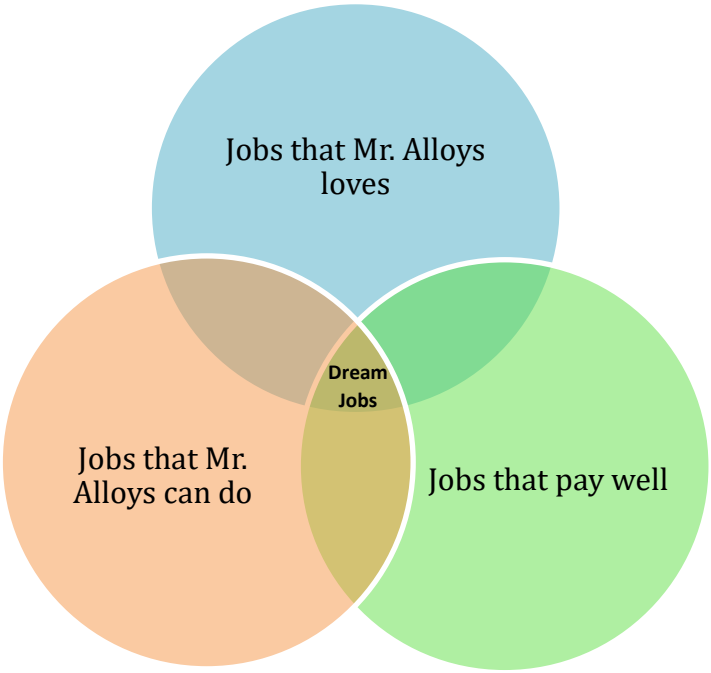
1. Ask trainees to read **Task 6**. As trainees read, sketch the Venn diagram on the blackboard so everyone in class can see the diagram at once. Discuss the diagram representing all 3 criteria for Mr. Alloys to find his dream job. Trainees should then work in pairs, and describe what they see to their partner then complete Task 5.

Topic 1.1 Task 6

Choosing a Dream Job: Mr. Alloys is a level 2 student. At the end of his studies he wonders what might be his dream job. There are 3 factors which matter to him when choosing a job:

- Jobs that Mr. Alloys loves to do,
- Jobs that Mr. Alloys is good at,
- Jobs that will pay good money to Mr. Alloys.

Mr. Alloys thinks about potential jobs and puts them into the proper spaces inside the following Venn diagram:



a. Describe each different part of the Venn diagram—and give examples of different jobs that may fit into the three sections. Write the answers in your notebook.

b. Decide if you agree with the following statements (or not) and justify your feeling:

i. Jobs that Mr. Alloys is good at would be the most practical choice.

ii. Jobs that Mr. Alloys loves to do would be the most practical choice.

iii. Jobs with good earning potential would be the most practical choice for Mr. Alloys.

iv. A job which includes all of these three criteria would be the dream job for Mr. Alloys.

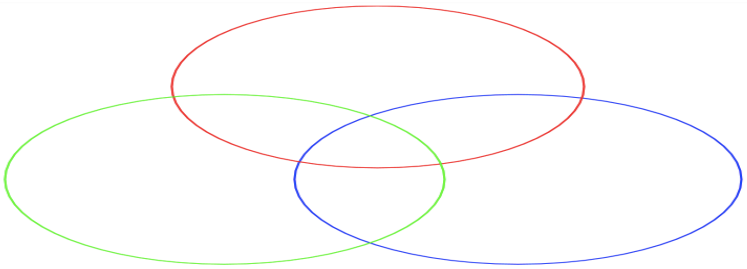
c. Identify all jobs that satisfy the following criteria FOR YOU! List all jobs for each category and then think if there are any jobs that you have listed which fit into more than one of the categories and then write them inside the spaces on a Venn diagram.

i. Set A contains "jobs that you think you are (or will be) good at", A= {……}

ii. Set B contains "jobs that you would love to do", B= {……}

iii. Set C contains "jobs that you think will pay well", C= {……}

d. After you have identified jobs, already organized into 3 sets, create a Venn diagram as was used for Mr. Alloys. Note: each job should only be written once—if it is found in two sets then it is placed in the overlapping area, if it is all three sets put it in the very middle where all three circles overlap.



Possible answers:

a. List the colours of each section, describe how some jobs may be in more than one zone, etc.

b.

i. **Possible answer:** you may find more success if you follow a career you are good at.
ii. **Possible answer:** you would be happy even if you are not very successful.
iii. **Possible answer:** making good money will allow you to have added opportunities.
iv. **Possible answer:** yes, if all three factors are in place that is great…but it may be a dream.

c. and **d.** will vary

2. Ask a few volunteers to share their responses and discuss. Clarify any misconceptions about the Venn diagram and how to represent information using the Venn diagram.

3. Ask trainees to take 2 minutes to read Task 7 carefully then work in pairs to complete the task. Trainer walks around the class to help when needed and ensure all have understood the concept well.

Topic 1.1 Task 7
Using Venn diagrams to solve problems

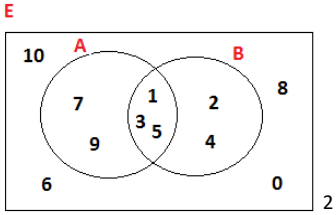
1. Consider all the integers (counting numbers) from 0 to 10. Because this is the complete set of elements considered, we call this our **universal set.**
2. Label the **Universal Set** using the letter E: E= {0,1,2,3,4,5,6,7,8,9,10}

Inside E there are two smaller sets: sets: set A = {1, 3, 5, 7, 9} and set B = {1, 2, 3, 4, 5}
Note that both A and B are **subsets** of set E so we could write: A⊂ E and B⊂ E.

3. Answers the following questions regarding the sets E, A and B:
a. Which elements are listed in both sets A and B? These are the common elements.
b. Find all the elements of set A which are NOT in set B:
c. Find all the elements of set B which are NOT in set A:
d. List the elements that are found in A as well as those that are found in B: (only list each element one time)
e. Try to represent this situation using a Venn diagram: (hint: pay attention to shared elements)

Answer:
The Venn diagram below shows the two sets A = {1, 3, 5, 7, 9} and B = {1, 2, 3, 4, 5}.

a. The elements that are in both A and B are: 1, 3 and 5.
b. The elements in A (but not B) are: 7 and 9.
c. The elements in B (but not in A) are: 2 and 4.
d. The elements in both A and B are: 1,2,3,4,5,7,9
e. The Venn diagram representing this situation is the following:
NOTE: Pay close attention to those elements that are OUTSIDE both A and B
NOTE: Pay close attention to those elements that are in BOTH A and B



[2]

[2] Ndyabasa, E. et al (2016). Mathematics for Rwandan Schools Senior 1, Student's Book. Kigali, Rwanda: Longhorn Publishers Limited.

4. Write the answers on the board and explain each step. Ensure that all trainees have noted the following (say the following aloud in the class):

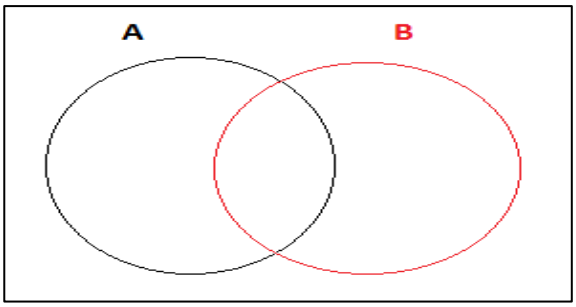
- The overlapping region is the INTERSECTION. A ∩ B = {1, 3, 5}.
- The two circles together represent the UNION, written in symbols: A ∪ B = {1, 2, 3, 4, 5, 7, 9}
- 3 elements in E are not in A and also not in B. Place them outside the circles that represent those two sets, but inside the universal set E.

5. Ask trainees to carefully read Task 8 then work in pairs to complete the Venn diagram.

Topic 1.1 Task 8 Cities that people have visited

Travel agents surveyed 100 people to find out how many of them had visited the cities of Kibuye and Huye. They were told 31 people visited the city of Kibuye, 26 people visited Huye, and 12 people had visited both cities. Some people had not visited either city.

f. Let set A represent the people who visited Kibuye, and set B the people who visited Huye. Fill in the Venn diagram below with the number of people who visited each of the cities. Take care with the number that visited both cities, and the number that visited neither city.



g. Based on the information in your Venn diagram, find the number of people who had visited:

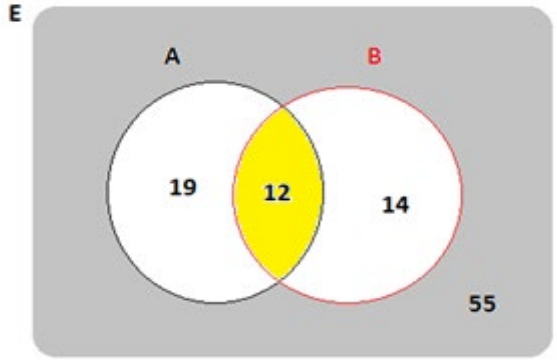
i. The city Kibuye or Huye but not both.
ii. The city Huye but not Kibuye.
iii. Either no cities or both cities.
iv. The number who have not visited Huye.

h. Identify which of the parts of their Venn diagrams represent the following:

i. Intersection of A and B
ii. Union of A and B
iii. The complement of set B

Answer:

a. See notes below on making this Venn diagram.



[3]

b. i) 33 people ii) 14 people iii) 67 people iv) 74 people

c. i) the part that shows 12 ii) the parts that show 19, 12, and 14 iii) the parts that show 55 and 19

Notes on making this Venn diagram:

- Numbers represent how many people are in each set—in A are all people that visited Kibuye: a total of 31. Among that 31, 12 also visited Huye—thus the people that visited ONLY Kibuye is 31 – 12 = 19
- Similarly, we know that 26 people visited Huye, but among those, 12 have also visited Kibuye. To find the number of people that ONLY visited Huye we take 26 – 12 = 14
- To figure out how many people visited NEITHER city (the 55 in the lower right corner) consider that a total of 100 people responded to the survey. This is our universal set E. People who visited either city (or both) are counted INSIDE the two circles. We add those numbers then subtract our sum from the total in our Universal set. 19+12+14=45 so only 45 people have visited our cities. 100 – 45 = 55. In our universal set but NOT in set A or B we have 55 people.

6. When all trainees have completed the task, ask trainees to compare their Venn diagrams. Do they have similar information? Discuss the methods trainees used to complete (fill in) the Venn diagrams. Discuss all answers and ensure all trainees have understood how to make a Venn diagram and how to calculate the missing bits of information. If needed, do more examples.

7. Explain that algebra is also useful in using Venn diagrams. Ask trainees to read Task 9 individually then to suggest how to proceed. Trainer will work problem on board using trainee suggestions. Describe each step and thinking carefully and take all questions while working.

Topic 1.1 Task 9

There are 17 trainers in a school. 10 teach Economics and 9 teach Mathematics. There are some trainers that teach no classes this term, and there are some trainers that teach both Mathematics and Economics. Use X to represent the number of trainers that are NOT teaching. There are two times as many trainers that teach both subjects as there are that teach none, so use 2x for the number of teachers teaching both.

Make a Venn diagram to represent this situation, then figure out the actual values for each area in the Venn diagram and answer the following questions:

How many trainers teach:

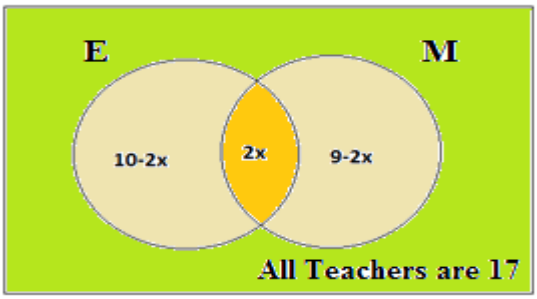
[3] Adapted from Ndyabasa, E. et al (2016). Mathematics for Rwandan Schools Senior 1, Student's Book. Kigali, Rwanda: Longhorn Publishers Limited.

a) Both subjects?
b) None of the subjects?
c) Only one subject?

Answer:
To create the Venn diagram, keep in mind the following information given:

- Number of all trainers is 17
- Number of Economics trainers is 10 total including only Economics and both E and M
- Number of Mathematics trainers is 9, including only Mathematics and both M and E
- Use x to represent the number of trainers who teach neither M or E
- The number of trainers who teach both M and E is 2x.

Using the above information, we get the following diagram:

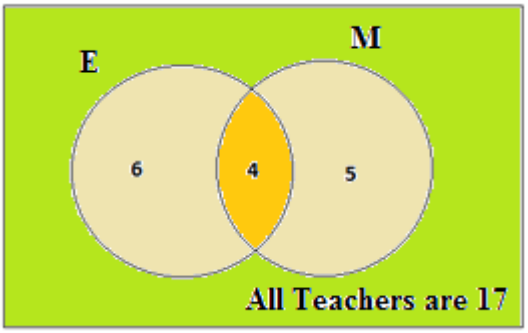


[4]

If we add all the sections together, we will get: $(10 - 2x) + (2x) + (9 - 2x) + (x) = 17$

Grouping the like terms, we get: $19 - x = 17 \rightarrow -x = 17 - 19 \rightarrow -x = -2 \rightarrow x = 2$

Knowing x=2 we can complete the Venn diagram below and then answer the questions



[5]

Answer: a) 4, b) 2, c) 11

8. Refer trainees to **1.1 Key Facts** as a form of review of the terminology used so far regarding real numbers and venn diagrams.

Application Activity

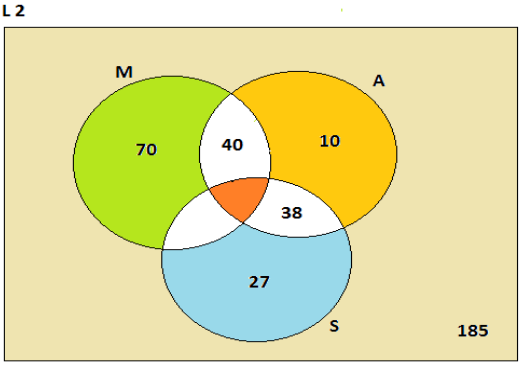
[4] Adapted from Ndyabasa, E. et al (2016). Mathematics for Rwandan Schools Senior 1, Student's Book. Kigali, Rwanda: Longhorn Publishers Limited.

[5] Adapted from Ndyabasa, E. et al (2016). Mathematics for Rwandan Schools Senior 1, Student's Book. Kigali, Rwanda: Longhorn Publishers Limited.

1. Ask trainees to work in groups of 4. In pairs, trainee will be assigned one problem – Task 10 or Task 11 - to work on, thus as a group both problems will be solved. Trainees will have 3-5 minutes to complete their problem. Then trainees will let each person explain their problem to other group members. Groups are responsible to ensure understanding by every member of the group for both problems as each trainee presents their answers to the other pair in the group. Trainer confirms correct responses in each group.

Topic 1.1 Task 10

A school in Gisenyi has 185 trainees in level two. 110 trainees study the Mathematics module, 88 study the Agriculture module, and 65 study the Science module. 40 trainees study both Mathematics and Agriculture modules, 38 study Agriculture and Science modules.[6]



Use the given venn diagram to answer the following questions regarding numbers of students in group:

a. Find the number of trainees who study Science module only. (S)
b. Find the number of trainees who study Mathematics module only. (M)
c. Find the number of trainees who study Agriculture module only. (A)
d. Find the total number of trainees who study Mathematics.
e. Find the total number of trainees who study Agriculture.
f. Find the number of trainees who study Mathematics and Agriculture. (M ∩ A)
g. Find the number of trainees who study Science and Agriculture. (A ∩ S)
h. Find the total number of trainees who study Science.
i. Find the number of trainees who study Science and Mathematics. (M ∩ S)
j. Find the total number of trainees who study all three. (S ∩ M ∩ A)

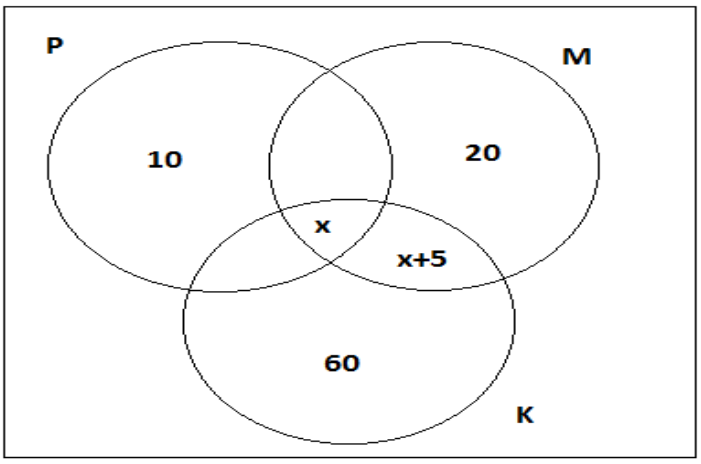
Answer:

a. The number of trainees who study Science only is 27, shown in the blue section
b. The number of trainees who study Mathematics only is 70 shown in the green section
c. The number of trainees who study Agriculture only is 10 shown in the orange section
d. The total number of all trainees who study Mathematics is 70 + 40 = 110
e. The total number of trainees who study Agriculture is 40 + 38 + 10 = 88
f. The number of trainees who study Mathematics and Agriculture is 40 (a white area)
g. The number of trainees who study Science and Agriculture is 38 (other white area)
h. The total number of all trainees who study Science is 38 + 27 = 65
i. No trainees study both math and science
j. No trainees study all three

Topic 1.1 Task 11

The Venn diagram below shows the number of Senior 1 students who like Mathematics (M), Physics (P) and Kinyarwanda (K). Some like more than one subject. In total 55 students like Mathematics.

[6] Adapted from Ndyabasa, E. et al (2016). Mathematics for Rwandan Schools Senior 1, Student's Book. Kigali, Rwanda: Longhorn Publishers Limited.



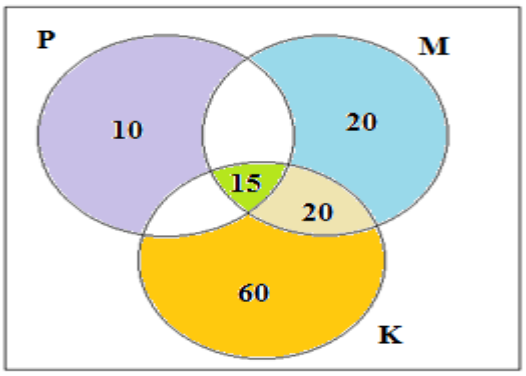
a. How many trainees like the three subjects?
b. How many trainees like Physics?
c. How many trainees like Kinyarwanda?
d. How many trainees like both Kinyarwanda and Physics?
e. Find the total number of Senior 1 trainees in the school.

Answer:

Start by solving for the value of x. We know that 55 students like mathematics so: (20) + (x + 5) + (x) = 55 → 2x + 25 = 55 → 2x = 30 → x = 15

Use the value for x to calculate the missing values in the Venn diagram.

a. Trainees who like three subjects (Mathematics, Kinyarwanda and Physics) is x so 15
b. Trainees who like Physics includes 10 + x so 10 + 15 = 25
c. Trainees who like Kinyarwanda is 60 + (x + 5) + x= 65 + 2x so 65 + 2 (15) = 65 +30 = 95
d. Trainees who like Kinyarwanda and Physics both is x so it is 15
e. The total number of Senior 1 students in the school is 125, which is given by the sum of all students in these three sets. i.e., 10+15+20+20+60=125



[7]

Points to Remember

- In Mathematics there are different types of numbers which we can consider as sub-sets of the Real Numbers. These are: Natural numbers, Integers, Rational numbers, Irrational

[7] Adapted from Ndyabasa, E. et al (2016). Mathematics for Rwandan Schools Senior 1, Student's Book. Kigali, Rwanda: Longhorn Publishers Limited.

numbers. All sub-sets of Real numbers can be found on a number line.

- Venn diagrams are used to illustrate and solve real life problems involving relationships between sets.

Formative Assessment

1. Read aloud the objectives for this Learning Outcome. Ask trainees if they feel they have accomplished these objectives poorly, average, or very well.
Each trainee should draw ☹, 😐, or ☺ at the top of the page in their notebook to indicate their level of confidence in the understanding of this material. If there are a lot of ☹, the trainer should plan to review the concepts of this lesson again during the next class. If only a few trainees need more support, trainer can make time to work with the trainees individually.

Ask class if there are questions that have not been covered. Give more examples if needed.

2. Trainees complete the formative assessments individually. Explain this is an opportunity for trainees to test themselves and put the new concepts to work. Give approximately 10 minutes to complete the following three problems, then reads answers aloud so trainees self-check.

Formative Assessment Questions

1. Consider the following sets:

The universal set being considered is E= {1,2,3,4,5,6,7,8,9,10,11,12} Inside E we have:
A = {6, 7, 8, 9, 10} B = {1, 2, 3, 4, 5, 6, 7} C= {2, 4, 6, 8, 10, 12} and D= {1, 3, 5, 7, 9}

List out the elements that are found in the following:

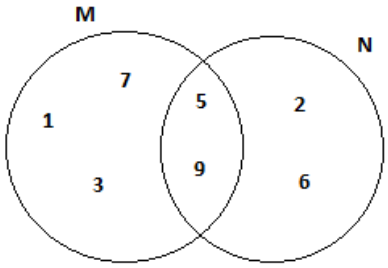
- **a.** The INTERSECTION of A and B, that is $A \cap B$
- **b.** The UNION of B and C, that is $B \cup C$
- **c.** The INTERSECTION of C and D, that is $C \cap D$
- **d.** The complement of set C, that is C' and
- **e.** The INTERSECTION of sets A, B, and C, that is $A \cup B \cup C$

Answer:

- **a.** $A \cap B = \{6,7\}$
- **b.** $B \cup C = \{1,2,3,4,5,6,7,8,10,12\}$
- **c.** $C \cap D = \{\} = \varnothing$, or in words, the empty set or the null set
- **d.** $C' = \{1,3,5,7,9,11\}$
- **e.** $A \cap B \cap C = \{6\}$

2. Given sets M= {1, 3, 5, 7, 9} and N= {2, 5, 6, 9};
Represent set M and N on a Venn diagram then list out the elements in $M \cap N$

Answer:



M ∩ N = {5, 9}

3. A survey was carried out. 50 people were asked about where they had eaten lunch over the past month. Among the Hilltop, Serena, and Lemigo hotels, it was found out that 15 people had eaten at Hilltop, 30 people had eaten at Serena, and 19 people had eaten at Lemigo, 8 people had eaten at both Hilltop and Serena, 12 people had eaten at both Hilltop and Lemigo, and 7 people had eaten at Serena and Lemigo. Just 5 people had eaten at all three: Hilltop, Serena, and Lemigo.

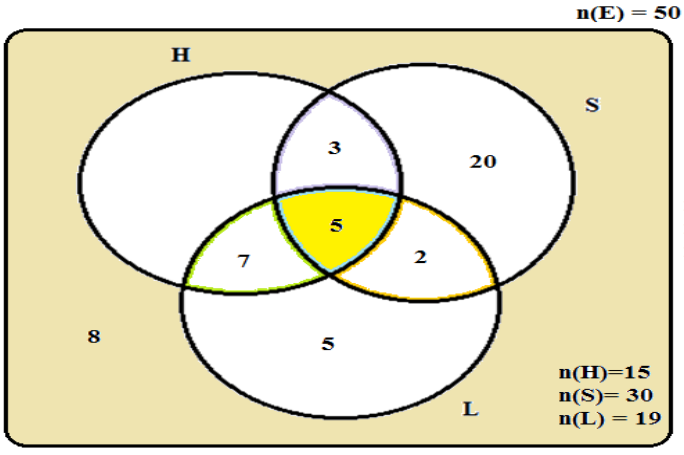
a. Represent the information on a Venn diagram.
b. How many people ate only at Hilltop?
c. How many ate at Hilltop and Serena but not at Lemigo?
d. How many people did not eat from any of these three hotels?

Answer:

a. The Venn diagram is as follows:

Let H represent Hill-Top hotel, S represent Serena hotel, L represent Lemigo hotel.

To make a Venn diagram first put the intersection of all three hotels, so a 5 in the yellow zone. Hilltop: a total of 15 people ate at Hilltop, but given the 5 who at all three and a total of 8 people who ate at Serena so there are three in the zone that overlaps between H and S.
Similarly, we can figure out that there should be 7 in the overlap of H and L.
Using the same processes, the other sections of the Venn diagram can be completed.



[8]

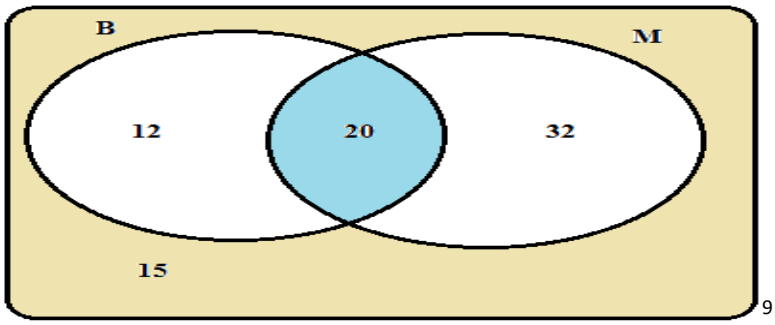
b. The number of people who took lunch at Hilltop only is zero.
c. The number of people who take lunch at Hilltop and Serena but not Lemigo is 3.
d. The number of people who did not take lunch from any hotel is 50-20-5-3-2-7-5=8

[8] Adapted from Ndyabasa, E. et al (2016). Mathematics for Rwandan Schools Senior 1, Student's Book. Kigali, Rwanda: Longhorn Publishers Limited.

4. A store surveyed all their customers during one day—there were 79 customers total. The survey asked how many of those customers bought bread, milk, both, or neither. Let x be the number of customers who bought both bread and milk; 52-x is the number of customers who bought milk only; 32-x is the number of customers who bought bread only. 15 customers bought neither bread nor milk.

Draw a Venn diagram to represent this situation—then replace the value of x with the actual number that is found from the information given.

Answer:
From given information, we know there were 79 customers total. We add all areas on the Venn diagram to find x: $(52 - x) + (32 - x) + x + 15 = 79$ which gives us $x = 20$, so we have this:



[9]

Further Information for the Trainer

Mathematical Definition of a set: a collection of well-defined, distinct objects is known as a set. The word 'well-defined' means easy to tell if the object belongs in the set, or not. The word 'distinct' means that the objects of a set must be all different from one another. For example: The collection of children in class who weight more than 35 kg is a set.

Notation for Sets and elements: A set is usually denoted by capital letters; small letters denote elements. If x is an element of set A, then we write $x \in A$ and say "x belongs to A." If x is not an element of set A, then $x \notin A$. [x does **not** belong to A]. If B is a subset of A then all elements of set B are also inside the set A and we write it $B \subset A$ [B is a sub-set of A].

Finite Set: A set which contains a countable number of elements is called a finite set. The set of Real Numbers is NOT a finite set--there are infinite elements; we cannot count them.

Empty Set or Null Set: A set, which does not contain any element, is called an empty set or the null set. It is denoted by $\emptyset$, say "the empty set." In list form, $\emptyset$ is denoted by {}. An empty set is a finite set, since the number of elements in an empty set is finite that is, 0.

Venn diagrams: a pictorial representation of sets represented by ovals is called a Venn diagram. Venn diagrams can be used to illustrate various operations like union and intersection.

[9] Adapted from Ndyabasa, E. et al (2016). Mathematics for Rwandan Schools Senior 1, Student's Book. Kigali, Rwanda: Longhorn Publishers Limited.

A Mathematician John Venn introduced the concept of representing sets pictorially which are useful in solving simple logical problems. In Venn diagrams, the Universal Set E is represented by a rectangle and all other sets under consideration by circles or ovals within the rectangle.

For more information on sets and Venn diagrams, consult the following link: https://www.math-only-math.com/intersection-of-sets-using-Venn-diagram.html

Learning Outcome 1.2: Calculate proportions, ratios, percentages, and mixtures in workplace and real-life situations

Objectives:
By the end of the learning outcome, trainees will be able to:

a. Express quantities as percentages, and percentages as fractions
b. Increase and decrease quantities by percentages
c. Solve everyday life problems involving direct or indirect proportions
d. Solve everyday problems related to mixtures

Time Required: 2 hours

Learning Methodologies: pair work, discussions

Materials Needed: Standard materials, student workbooks/notebooks, calculators if available

Preparation:

- ❑ Prepare by reviewing problems/solutions so trainer can explain them to trainees

Cross Cutting Issues:

- ✔ **Gender:** trainer will provide equal opportunities to both boys and girls during the lesson as well as balance gender when forming groups
- ✔ **Peace/value education:** trainer promotes team work and respect for others' views and work
- ✔ **Inclusive education:** trainer is sensitive to differences in learning abilities and uses a variety of activities and materials to cater to different learning styles

Prerequisites: Trainees should already be comfortable with and able to:

- Use addition, subtraction, multiplication, and division
- Do calculations with fractions, and change decimals into fractions (and back)
- Solve word problems that involving basic operations

Key Competencies:

1. Explain decimals, fractions, and percentages and how to use them in life contexts	**1.** Convert decimals into fractions and fractions into percentage and verse versa	**1.** Persistent
2. Describe ratios and proportions, both direct and indirect in practical and everyday contexts.	**2.** Calculate percentage increase and decrease in quantities, and find percentage profit or loss	**2.** Creative
3. Understand the concept of calculating mixtures and explain it to others	**3.** Solve every day problems related to percentages, ratios, proportions and mixtures	**3.** Accurate

Steps:

Getting Started: What do we know and where are we going?

1. Ask trainees to open their workbooks and look at the illustration in **Topic 1.2 Task 1** and make some observations about the illustration. Trainer will lead a discussion with class to ensure that the class has a common understanding of the details illustrated in the table.

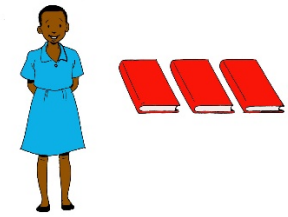
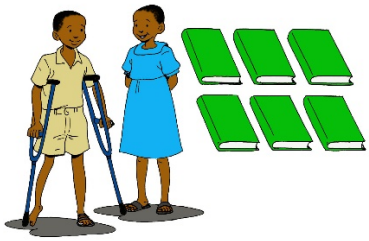
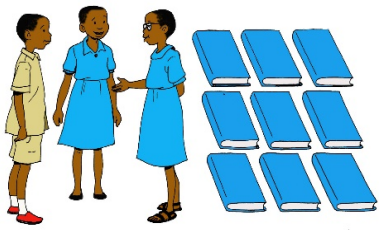
Questions to lead the discussion will include the following (and possibly others):

- How many children are shown in total? (6 children: Mary, Jane, Peter, Sarah, Steve, Ruth)
- How many books are shown in total? (18 books total: 3 red + 6 green + 9 blue)
- As you see, in the table, one child has 3 books, two children have 6 books and 3 children have 9 books. If we have a group of 4 children, how many books do you predict they will have in total? (12 books)
- Can you predict how many books we will have in total if we have 6 children? (18 books) how many books for 9 children? (27 books)
- If you have 24 books—how many children do they belong to? (8 children)
- Explain how you determine that number. (You can note there is a pattern: 3 books per child. Use number of children multiplied by 3 to find the number of books, or books divided by 3 to find numbers of children.)

2. After class brainstorms observations, ask trainees to think about the relationships that are shown in the table. Explain that in mathematics relationships between two numbers can be written as ratios which compare the items. For example: the number of red books compared to the total of books can be written as a ratio. Write 3/18 or 3:18 and say “three to eighteen.” Note: the order of a ratio is important! The ratio of green books to red books is 3/6 or 1/3—but that is different from the ratio of red books to green books which is 6/3 or 2/1. 3/6 is not 6/3.

3. Ask trainees to complete the task individually then take a volunteer to share answers.

Topic 1.2 Task 1

	**Mary has 3 books.**	**Mary has:** **3 of 18 books**
	**Jane and Peter have 6 books.**	**Jane and Peter have:** **6 of 18 books**
	**Sarah, Steve and Ruth have 9 books.**	**Sarah, Steve and Ruth have:** **9 out of 18 books**

a. What is the ratio of red books to the total number of books? (**Answer:** 3/18 or 1/6)
b. What is the ratio of blue books to red books? (**Answer:** 9/3 or 3/1)
c. What is the ratio of green books to blue books? (**Answer:** 6/9 or 2/3)
d. What is the ratio of blue books to green books? (**Answer:** 9/6 or 3/2)

4. Ask trainees to silently read **Topic 1.2 Task 2** then work out the answers individually. After 3 minutes ask for a volunteer to share answer and to explain processes used to find answers.

Topic 1.2 Task 2
Observe the table showing the number of pens owned by students in a middle school class:

Number of students	Number of pens
1	5
2	10
3	15
4	(a)
5	(b)
10	(c)
(d)	85

Complete the table by filling in the spaces libelled by (a), (b) and (c). Then explain a pattern you see which will help you solve these problems.

Answer: (a) 20 pens (b) 25 pens (c) 50 pens (d) 17 students
The pattern is that for every student added, the number of pens goes up by 5. To find the number of pens you take the number of people and multiply that number by 5.
If you are given the number of pens you take the number of pens and divide by 5 (since each student has 5pens) to get the number of students for any given number of pens. 85/5=17

5. Note: The number of pens is proportional to the number of students—that is: there is a mathematical relationship that exists between the two variables (students and pens).)

6. Tell trainees that we are now going to move to a related but different topic which is percentages. Brainstorm what trainees remember about percentages. Write all answers on the blackboard (if they are sharing something correct).

Possible answers: Percentage means part of 100.

- Percentages can be written as a decimal (move the decimal point two places to the left (35% = 0.35 and 5%=0.05)
- Percentages can be written as a fraction 75% = 0.75 = 75/100
- Percentages can be many things in real life: sale prices, taxes, ways to share, etc.
- To find 70% of a number, multiply the decimal (.70) by your number

(Trainees may have other answers that they remember about fractions—there are many good answers!)

7. Review the correct knowledge about percentages that came from the brainstorm by asking trainees to read aloud what trainer wrote on board. Then trainees work in pairs to complete Task 3. Pairs will have 3 minutes to complete task. Ask a volunteer to share answers.

Topic 1.2 Task 3
Convert the following decimal numbers and percentages into fractions and explain how.

a. 0.5
b. 0.25
c. 0.95
d. 85%
e. 30%
f. 6%

Answer:

a. 0.5 = 5/10 = 1/2 (One digit after decimal point means tenths)
b. 0.25 = 25/100 = ¼ (two digits after decimal point means hundredths)
c. 0.375 = 375/1000 = 3/8 (three digits after decimal points means thousandths)
d. 85% = 85/100 = 17/20
e. 30% = 30/100 = 3/10
f. 6% = 6/100 = 3/50

Problem Solving Activity

1. Ask trainees to read **Topic 1.2 Task 4** and then carefully answer the related questions. After 3 minutes, ask two trainees to present their responses. Give trainees guidance if needed.

Topic 1.2 Task 4

In a school, there are 56 female trainers out of a total of 70 trainers.

- **a.** What is the ratio of female trainers to the total number of trainers?
- **b.** What is the ratio of male trainers to the total number of trainers?
- **c.** What is the ratio of female to male trainers?
- **d.** What percentage of the trainers are female?
- **e.** What percentage of the trainers are male?
- **f.** What is the relationship between the two percentages we found in (a) and (b)?

Answer:

- **a.** Remember that a ratio is a relationship between two numbers. The ratio of female to total trainers is 56/70 or (when fraction is simplified) 8/10 or 4/5. This can also be written as 56:70 or 8:10 (or 4:5 if simplified further but we are interested in 8/10 for questions below).
- **b.** There are 70 total trainers. 56 are female. 70-56=14 gives us the number of male trainers. The ratio of males to the total number of trainers is 14/70 or (when fraction is simplified) 2/10 (or 1/5 if simplified further).
- **c.** The ratio of female trainers to male trainers is 56/14 or 4/1
- **d.** To find percentage, re-write the ratio as a fraction with 100 as the denominator (because percentages are parts of 100). We know from (a) the ratio of female trainers to total trainers is 8/10. Multiply numerator and denominator by 10: (8 x 10)/ (10 x 10) = 80/100 or 80%.
- **e.** Use the same method as in (d) to find percent of male trainers, OR think of the whole situation: 80% of trainers are women, the remaining trainers are male. All the trainers is 100%, so 100% - 80% = 20% thus 20% of the trainers must be male.
- **f.** Add the percentages of female and male trainers to get ALL trainers, or 100%. Or consider the ratio of women to men trainers: 56/14 or 4/1. For 5 trainers we will have 4 women and 1 man.

Guided Practice Activity

1. Ask a trainee to read aloud **1.2 Key Facts** to prepare the class for the next activities.

2. Trainer and trainees work the following tasks together – with trainer writing the problems on the board as a trainee reads the problem aloud and offer ideas on how to proceed.

3. Trainer explains all steps and focuses on the process. Trainees participate by offering ideas, asking questions, and thinking about task while Trainer guides and leads the trainees' understanding. Ensure that boys AND girls are being responsive. Give trainees time to copy the answers when a task is complete rather than copying while working on task together.

Topic 1.2 Task 5

In a classroom of 50 trainees, there are 30 girls and 20 boys.

- **a.** Give a ratio for girls in the class, and for boys in the class using fraction notation.
- **b.** You are asked to represent the number of boys out of 100, how should you proceed?

c. What is another word for the number of boys out of 100?

d. One day, 5 trainees are absent. What percentage are absent from the class?

Answer:

a. $\frac{30}{50}$ are girls or 30 out of 50 are girls while $\frac{20}{50}$ are boys or 20 out of 50 are boys.

b. $\frac{20}{50} = \frac{x}{100} \Leftrightarrow \frac{20}{50} \text{x} \frac{2}{2} = \frac{40}{100}$, or 40% are boys.

c. A ratio out of 100 is also called a percentage.

d. $\frac{5}{50} = \frac{1}{10}$, then $\frac{1}{10} \text{ X} \frac{10}{10} = \frac{10}{100}$ or 10% of the trainees are absent.

Topic 1.2 Task 6

Study the following table showing the number of trainers to trainees in a school in Burundi. Fill in the missing numbers. State the ratio used and explain the pattern used to find that ratio.

Number of trainers	1	2	3	4	5	-------	7
Number of pupils	30	60	----	-----	150	180	-----

Answer:

Number of trainers	1	2	3	4	5	**6**	7
Number of pupils	30	**60**	**90**	**120**	150	180	**210**

The ratio is 1:30 (1 trainer for 30 trainees) and the number of trainees is increasing proportionally to the number of trainers (if the number of trainers double, the number of trainees doubles too), if the number of trainers triple, the number of trainees increases by a factor of three too, and so on), also for every trainer there are 30 pupils.

Topic 1.2 Task 7

Find the price of 8 pens given the price of 10 pens is 1 500 Frw.

Answer:

The price of 1 pen is $\frac{1500}{10}$ Frw

The price of 8 pens is $\frac{1500}{10} \text{x } 8 = 1200 \text{ Frw}$

Or use a proportion to solve this problem:

cost / pen gives us 1500/10 = x/8. Solve for x. Multiply both sides of the equation by 8: (1500x8)/10=x so 12 000/10=x or 1 200=x which tells us that 8 pens will cost 1 200 Frw.

Topic 1.2 Task 8

At a special school of 300 trainees, all trainees perform well in national examinations. This makes the school more popular so they must register more trainees the next year. The number of existing trainees was increased by 40%. Find out the number of newly registered trainees.

Answer:
The number of newly registered trainees is 40% or 0.40 of 300 which is 0.40 x 300 = 120. Thus, the number new registered trainees is 120. The new total number of trainees is 300+120=420.

Topic 1.2 Task 9
A business man mixed 200 kg of one kind of soya beans which cost 600 Frw per kg with 300 kg of another type of soya beans which cost 800 Frw per kg, the reason being that the soya beans costing 800 Frw were not preferred by the people because of the high price. What is the average price of 1kg of the mixed soya beans?

Answer:
200 kg of soya beans cost: 600 Frw × 200 kg= 120 000 Frw
300 kg of soya beans cost: 800 Frw × 300 kg= 240 000 Frw
Total mass: 200 kg + 300 kg = 500 kg
Total price: 120 000 Frw + 240 000 Frw = 360 000 Frw
1kg of mixture cost: 360 000 Frw / 500 kg= 720 Frw /kg

Application Activity

1. Ask trainees to read each task carefully then work independently on the following 5 tasks. Trainer should walk around the classroom to review the work being done by individuals and respond to questions, provide guidance, and support those trainees who are struggling.

2. After the individual work, ask trainees to compare their solutions with a partner. Let pairs work together to find correct answers by explaining their processes to one another until both agree.

3. Ask for volunteers to write solutions to each task on the board, while explaining thinking. Trainer facilitates to ensure correct answers. Each trainee corrects their own work.

Topic 1.2 Task 10
4 dresses can be made from 6 meters of cloth.

a. How many meters of cloth must I buy for 6 dresses?
b. If I have 15 m of cloth, how many dresses can I make?

Answer:

a. Set up the ratio needed which is dresses: cloth so we are given 4:6 or 4/6 or 2/3
Using that ratio, we now use a proportion to find the number of dresses, that is: 2/3=6/x
Multiply both sides by x to get 2x/3=6. Multiply both sides by 3 to get 2x = 18
Divide both sides by 2 to get x=9
That is, we need 9 meters of cloth to make 6 dresses

b. Using the same ratio of dresses: cloth, or 2/3, we have the proportion 2/3=x/15
Multiply both sides by 15 to get 30/3=x or 10=x
We can make 10 dresses out of 15 m of cloth

Topic 1.2 Task 11
On a scale drawing, 1.5 cm represents 20 km of road.
What length of road is represented by 6 cm?

Answer:
Make a ratio of cm: km to get 1.5/20 or 3/40. Using this ratio, set up a proportion: 3/40 = 6/x
Multiply both sides by x to get 3x/40=6 THEN multiply both sides by 40 to get 3x = 240
Divide both sides by 3 to get x = 80. So 6cm on a map represents 80km on a road.

Topic 1.2 Task 12
A cooperative named "ABOROZI B'INKOKO" sold 500 eggs a week. On Monday, they sold 25% of the eggs, on Tuesday they sold 30% of the eggs, and on Friday, they sold 5% of the eggs. On Saturday, they sold the rest of the eggs.

- **a.** What percentage of eggs is left to be sold on Saturday?
- **b.** How many eggs were sold on:
  Monday? On Tuesday? On Friday? On Saturday?

Answer:

- **a.** Percentage of eggs to be sold on Saturday: 100% - 25% - 5% - 30% = 40%
- **b.** Eggs sold on Monday: 25% x 500 = 0.25 x 500 = 125 eggs.

On Tuesday: 30% x 500 = 0.30x 500 = 150 eggs, on Friday: 5% x 500 =0.5 x 500 = 25 eggs,
Eggs sold on Saturday: 40% x 500 = 0.4x 500 = 200 eggs

Topic 1.2 Task 13
For purposes of sales promotion, the price of a book has been reduced by 20% to a sale price of 3 600 Frw. What was the price before the reduction?

Answer:
Suppose that the price of a book was X. The price reduction is 20% of X.
This means, that the remaining cost (3 600 Frw) is 80% of the original cost (100%-20%=80%)
80% = 0.80 so make the equation: X x 0.80 = 3 600 Frw or X = 3 600/0.80 = 4 500 Frw
X= 4 500 Frw is the original price of the book.
OR consider that the discount is 20% of X so X - (0.20X) = 3 600 Frw so we have 0.80 x X = 3 600.
Additionally, 4 500Frw – 3 600 Frw = 900 Frw so the discount was 900 Frw.

Topic 1.2 Task 14
A mixture of two types of juice is made from 120 l of 1st quality juice costing 150 Frw per litre and 80 l of a 2nd quality type (diluted).
Find the price of 1 litre of the second type if 1 litre of the mixture was 138 Frw per litre.

Answer:
For 1st quality juice, the price is 150 Frw per litre. Total cost is 150 Frw × 120 l = 18 000 Frw
Inside the mixture the cost of first quality juice is 18 000 Frw.
We are told that 120 l of 1st quality is mixed with 80 l of 2nd quality: 120 l + 80 l = 200 l
So Total quantity of mixed juice is 200 l
Price of mixed juice is 138 Frw per litre. There are 200 l.
Total cost of mixed juice is 138Frw × 200 l = 27 600 Frw

Remove the Price of 1st quality juice 27600 Frw – 18 000 Frw = 9 600 Frw to get cost for the second quality juice inside the 200 l of mixed juice.
We have 80 l of the 2nd quality juice inside the mixture so 9 600 Frw / 80 l = 120 Frw/litre.
Thus, the price for the 2nd quality juice is 120 Frw per litre.

Points to Remember

- A ratio is a mathematical statement, which shows how two quantities compare.
- In simplifying ratios, the two quantities of a ratio may be multiplied or divided by the same number without changing the value of the ratio
- When using percentages, you can calculate either using a fraction or a decimal that is equivalent to the percentage. For example: 75% = 0.75 = 75/100 = 3/4

Formative Assessment

1. Ask trainees to complete the following 5 exercises individually then to share their responses with a neighbour. In pairs trainees explain process and thinking to arrive at the solutions and to determine if they have understood the concept. Trainer walks around and offers hints and support to trainees that are struggling.

2. At the end of the time allotted, read out correct answers, and if needed, work the problems on the chalkboard, explaining each step and all thinking aloud. Trainees correct their own work.

Formative Assessment Questions

1. A company produced 23 000 shirts in September. This was 8% less than the August production. How many shirts did the company produce in August?

Answer:
Suppose that the production of shirts in August was X which is 100% of the amount.
If we reduce that amount by 8% then 100% – 8% = 92%
For August amount, take 92% of the September amount or 0.92 x X = 23 000
Divide both sides by 0.92 to get X = 23 000/0.92 = 25 000 shirts which is the shirts made in Aug.

2. A farmer bought a cow at 55 000 Frw and sold it a year later for 42 900 Frw. Find her percentage loss when she sold her cow.

Answer:
49 900 is smaller than 55 000 so the farmer sold the cow for less than she bought it for so she lost money on that sale.
55 000 – 42 900 = 12 100 loss

$\frac{12100}{55000}$ = 0.22 to turn this decimal into a percent we multiply by 100: 0.22 x 100= 22% loss

3. The price of 8 small tables for nursery schools is 36 000 Frw. How many tables are you going to produce if you are paid 99 000 Frw by the school administration?

Answer:
Set up a ratio showing number of tables: cost so 8 : 36 000
Re-write the ratio as a fraction then set up a proportion:
Let X be the number of tables we can produce for 99 000 Frw
8/36 000 = X / 99 000
Now solve for x: Multiply both sides by 99 000 to get (99 000 x 8)/36 000 = X
792 000/36 000 = X or 22 = X so we produce 22 tables if administration gives us 99 000 Frw.

4. A business woman mixed 250 kg of fine rice which cost 660 Frw per kg with 125 kg of broken-up rice which cost 360 Frw per kg, the reason was that the business woman wanted to lower the total price so many people could still eat rice. What is the price of 1kg of the mixed rice?

Answer:
250 kg of fine rice cost: 660 Frw × 250 kg= 165 000 Frw
125 kg of broken rice cost: 360 Frw × 125 kg= 45 000 Frw
Total mass: 250 kg + 125 kg = 375 kg
Total price: 165 000 Frw + 45 000 Frw = 210 000 Frw
1kg of mixed rice should cost: 210 000 Frw / 375 kg = 560 Frw

5. A man's daily wage was increased by 25% to 5 000 Frw. Find his salary before the increase.

Answer:
Let the man's original salary be X.
His original salary was 100% (all of it) and it was INCREASED (which means added to) by 25%.
100% + 25% =125% so the new salary is 125% his old salary.
Remember: 125% in decimal form is written as 1.25
Make the equation: 1.25 x X = 5 000 to get X = 5 000/1.25 or X = 4 000 Frw
The man's original salary was 4 000 Frw per day.

ⓘ Further Information for the Trainer

Ratio: A ratio defines a relationship between two quantities. It is expressed as x: y or a minimized fraction (no common factor between the numerator and the denominator) x/y.

Example: the ratio of the age of a father and a son might be: $\frac{50}{20} = \frac{5}{2}$

Proportion: Proportion is a way to work with ratios to solve problems by making two ratios equal to one another. It is critical that the same sort of quantity is in the same position in both ratios. Problems involving proportions are best handled by manipulating equivalent fractions where one of the values is missing.

A direct proportion describes a relationship that when one variable increases the other variable increases too. **Example:** food served by a restaurant is directly proportional to the number of guests: as guests increase the amount of food served increases too.
An **indirect proportion** describes the relationship when one quantity increases the other decreases. **Example:** Team tasks are often an example of an indirect proportion. As the number of people helping increases then the time taken to do a job usually decreases. In this situation, number of people is indirectly proportional to time it takes to complete a task.

Percentages: 'Per cent' means 'per hundred' and is denoted by the symbol %. 100% is the same as the whole population. Percentages can be used to indicate parts of the whole (as in '64% of the population voted' which is for every 100 people, 64 voted or 64/100 voted). Percent can also indicate changes (as in 'an increase of 4%'). Our everyday experience of percentages includes percentage increases (like VAT at 17% added to a cost, or a service charge of 15% added to your hotel bill) and percentage decreases (such as a discount of 15% on a sale item at a store). Percentages can be considered as proportions or fractions or decimals—all in relationship to 100.

- http://www.suresolv.com/efficient-math-problem-solving/how-solve-arithmetic-mixture-problems-few-simple-steps-2
- https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratio-proportion/cc-7th-write-and-solve-proportions/a/multi-step-ratio-and-proportion-problems
- https://www.open.edu/openlearn/ocw/mod/oucontent/view.php?id=6491&printable=1

Learning Outcome 1.3: Calculate discount, commission, profit, loss, interest, taxes

Objectives: By the end of the learning outcome, trainees will be able to:

a. Calculate percentage discounts and commissions
b. Solve everyday life problems involving profit, loss, and taxes
c. Discuss and calculate amounts related to interest

Time Required: 2 hours

Learning Methodologies: pair work, small group work, and group discussions

Materials Needed: Standard materials, trainee workbook/notebook, calculators if available

Preparation:

- ❑ Prepare additional examples should trainees need extra support

Cross Cutting Issues:

- ✓ **Gender:** trainer will provide equal opportunities to both boys and girls during the lesson as well as balance gender when forming groups
- ✓ **Peace/value education:** trainer promotes team work and respect for others' views and work
- ✓ **Inclusive education:** trainer is sensitive to differences in learning abilities and uses a variety of activities and materials to cater to different learning styles

Prerequisites: Trainees should already be comfortable with and able to:

- Use addition, subtraction, multiplication, and division
- Do calculations with fractions, and change decimals into fractions (and back)
- Solve word problems that involving basic operations

Key Competencies:

1. Explain discounts, profits and losses and how to calculate them	**1.** Solve everyday problems involving discounts, profits and losses	**1.** Persistent
2. Describe times when a commission might be provided, or a tax applied and explain how to calculate both	**2.** Given percentage rates, calculate commissions and calculate the taxes applied	**2.** Creative
3. Understand the concept of interest and explain how to calculate it	**3.** Solve every day problems of simple interest	**3.** Accurate

Steps:

Getting Started: What do we know and where are we going?

1. Ask trainees to open their manuals and read **Topic 1.3 Task 1** carefully and then complete the task. Explain that each trainee will have their own answers based on their own experiences.

2. When all trainees have completed the task, facilitate common definitions for each vocabulary word. Encourage trainees to use local language to ensure clear understanding of all the words and guide discussion to correct and clear meanings of the underlined words. Ask for different examples from trainees' real lives for each word. Build on trainees' knowledge or experiences.

Topic 1.3 Task 1
Read the following paragraph. In your own works, write a definition for each underlined word. You may use examples from your own life to help explain particular concepts.

There is a new shopkeeper named Mary in the neighbourhood. She sells different types of grains and beans, including: red beans, green peas, sorghum, maize, soya beans, and rice. The Rwandan government charges a **tax** on imported grains which make the cost of those grains more expensive, but some people still prefer those items. Mary also sells locally grown grains. To make her business more productive, Mary is advised to use different agents and pay the agents a **commission** when they make a sale for her. Mary will determine which grains and beans make the most **profit** and also calculate if she is taking a **loss** on the sale of any of her products. If some of her grains are not selling well Mary will consider introducing a **discount** for some items.

Answer:
Responses from trainees may vary but will have the following basic information:

Commission is the money paid to sales representatives or agents for the sales made it is usually calculated as a percentage of the value of the sales, and usually on a monthly or annual basis. The

more the sales, the more the commission thus this may encourage the sales representative to get more sales.

The **list price**, is the price initially quoted by the seller to the buyer. However, there are cases where the seller may sell the good or service to a buyer at a price lower than the list price. The amount reduced from the list price is known as the **discount**. The exact price for which an item is sold after a discount is known as the **sale price**.
Discount = marked price – sale price.

Profit is the extra amount gained after selling a commodity at a price higher than the buying price.
Profit = selling price – buying price
Loss refers to the amount of money lost when a commodity is sold below the actual buying price.
Loss = buying price– selling price

Simple interest is the amount charged when one borrows money or takes out a loan. This interest is a fixed percentage of the amount that is borrowed (called the principal)

Problem Solving Activity

1. Gather trainees into groups of three to read and discuss Task 2. Each trainee will represent one of the businesses, and will answer the questions for their own business. Trainees will try to find both positive and negative points for their own business then together they will choose the best option.

2. Upon completion of the task in all groups, facilitate a discussion to find the best solution.

Topic 1.3 Task 2
You are starting a printing business and you need to buy paper from a local supplier. You talk to representatives from three businesses in town. The following are prices and conditions they quote. Review the three quotes then answer the following questions.

Business 1:
The standard price for a ream of paper is 7 500 Frw
BUT if you buy 10 or more reams, they will give you a 2% discount.
Business 2:
The price for a ream of paper is the basic cost of 6 700 Frw BUT because they import the paper from Tanzania, they need to add the cost of import tax which is 11%.
Business 3:
The standard price for a ream of paper is 7 450 Frw. No discounts or taxes.

- **a.** For each of the business offers above, calculate the cost of 1 ream, 5 reams and 10 reams of paper. Keep in mind the percentage problems that we did during the last Learning Outcome 1.2.
- **b.** For each of the business offers above, list at least one reason why it would be a good choice—and one reason why it may not be a good choice.

Answer:
- **a.**

Business	Cost for 1 ream	Cost for 5 reams	Cost for 10 reams
1	7 500	37 500 = 7 500 x 5	73 500=7 500 x 10 x 0.98
2	7 437=6 700 x 1.11	37 185 = 7 437 x 5	74 370=7 350 x 10
3	7 450	37 250 = 7 450 x 5	74 500=7 450 x 10

b. Answers may vary. Possible answers may include:

Business 1:
Good choice because it is cheapest if you buy 10 (or more reams)
Bad choice: most expensive if you buy less than 10 reams—and may not have money for 10 as the business is starting up.
Business 2:
Good choice: if you are buying less than 10 reams this is the cheapest option
Bad choice: if you find out that you need more then you are not using the cheapest option
Business 3:
Good choice because the price is the clearest and easiest to figure out
Bad choice: it is not the cheapest option

3. Note: trainees used percentages to figure out discounted prices – when the cost is reduced. Percent is also used when tax is added to a product so cost increases. Note aloud for the students that while discounts are taken away from costs, taxes are often added on and should also be considered. And the class has also figured out list price and profits. Trainer announces that we will be working with these concepts that are important for understanding how businesses work for the remainder of this learning outcome so their personal experiences and lives may provide some other good examples which trainees should be encouraged to share. Ask trainees to complete Task 3 in their groups.

Topic 1.3 Task 3
Consider the same printing business. Assume that you have chosen Business 1 and have bought 12 reams of paper. The cost of colour printing on that paper adds 6 250 per ream.
The reams are used to make monthly picture calendars. 150 calendars have been printed total.

a. Calculate the cost of each calendar.
b. If you want to make 50 Frw profit on each calendar you sell, what will the selling or LIST PRICE be for each of your calendars?

Answer:

a. If you buy 12 reams from Business 1 the cost will be 12 x 7350 = 88 200 Frw.
The cost for printing on 12 reams of paper will be 12 x 6250 = 75 000 Frw.
Total cost for all calendars is 88 200 + 75 000 = 163 200
Cost for each calendar is 163 200 / 150 = 1 088 Frw
In order to make back the costs spent you need to sell each calendar for 1 088 Frw.

b. To make a profit of 50 Frw per calendar the list price should be 1 088 + 50 = 1 138 Frw.

4. Let volunteers give their answers and explain their thinking and mathematical processes used. Trainer will correct if mistakes are made and respond to questions from trainees.

Guided Practice Activity

1. Ask trainees to review the idea of a commission (which was discussed at the introduction to this Learning Outcome). Explain that the concept is a way to motivate a salesperson to sell more product—which will then bring higher profits (from more sales) to the business owner so both the owner and the seller will benefit. Ask trainees to work in pairs on Tasks 4 and 5.

Topic 1.3 Task 4

Mary is offered a part-time job selling restaurant equipment. She will be paid Frw100 000 per month plus a 6% commission on her sales. The sales manager tells Mary she can expect to sell between Frw 800 000 and Frw 1 500 000 worth of equipment per month. To help her decide whether to accept the job, Mary does a few calculations.

Based on the sales manager's estimate, what monthly income can Mary expect from this job? What annual salary would that provide? Compute monthly and annual incomes for each sales total shown in the table. The first two examples of monthly income have been done for you. Discuss if this is a good job for Mary to take.

Sales Amount	**Calculation: 6% of Sales (6%=0.06)**	**Mary's Monthly Income**	**Annual Income (monthly x 12)**
500 000	500 000 x 0.06 = 30 000	100 000 + 30 000 = 130 000	
800 000	800 000 x 0.06 = 48 000	100 000 + 48 000 = 148 000	
1,000,000			
1,500,000			
2,000,000			
3,000,000			

Answer:

Sales Amount	**Calculation: 6% of Sales (6%=0.06)**	**Mary's Monthly Income**	**Annual Income (monthly x 12)**
500 000	500 000 x 0.06 = 30 000	100 000 + 30 000 = 130 000	1 560 000
800 000	800 000 x 0.06 = 48 000	100 000 + 48 000 = 148 000	1 776 000
1,000,000	1 000 000 x 0.06 = 60 000	100 000 + 60 000 = 160 000	1 920 000
1,500,000	1 500 000 x 0.06 = 90 000	100 000 + 90 000 = 190 000	2 280 000
2,000,000	2 000 000 x 0.06 = 120 000	100 000 + 120 000 = 220 000	2 640 000
3,000,000	3 000 000 x 0.06 = 180 000	100 000 + 180 000 = 280 000	3 360 000

This seems to be a good job, assuming that Mary can actually sell at the level expected. She can more than double her base salary if she is a good sales person.

Topic 1.3 Task 5

A company's sales representative sold goods worth 6 760 000 Frw in a certain month. The representative earns a base salary of 150 240 Frw and gets a commission of 10% on the sales that are above 5 200 000 Frw. Calculate how much the sales representative earned that month.

Answer:

The commission is given on sales OVER 5 200 000 so to find how much commission we first find the following amount (6 760 000 Frw – 5 200 000 Frw = 1 560 000 Frw)
Representative earns commission on 1 560 000 Frw. Commission is 10% = 0.10
1 560 000 Frw x 0.10 = 156 000 Frw which is the month's commission added to the base salary.
150 240 Frw + 156 000 Frw = 306 240 Frw which is the total monthly salary for the representative.

2. Walk around classroom while pairs are working on the commission problems to offer support to those struggling and to ensure that all understand the concept well. When complete, ask volunteers to explain their processes and give their solutions. Trainer guides to ensure correct.

3. Ask trainees to read silently **1.3 Key Facts** that are found in their workbooks. If trainees have questions regarding these key facts, trainer will respond and will provide additional examples.

4. Note that the class has already solved problems that relate to Commission, Profit and Loss, and discounts in this as well as the last learning outcome. Working with Interest rates is new. Ask trainees if any have had experiences borrowing money that they would like to share. In general, to borrow money (take a loan) you must pay for that service. The payment is called 'interest' and is usually based on the amount of money that is borrowed and the length of time that you have to repay the loan.

5. Discuss and use examples to illustrate process to calculate interest using the formula: Interest = % Rate x Principal (amount borrowed) x time (in years) or I = R x P x T

6. Read aloud Task 6 and identify the three important parts (Rate, Principal and Time) then put those numbers into the formula to demonstrate how to solve this sort of problem. While working Task 6 on the board, explain the reasoning at each step.

Topic 1.3 Task 6
Robert borrowed 200 000 Frw from a bank for 2 years at a rate of 8% per year. How much interest did he pay for the service of the loan?

Answer:
The formula we have is: **I = R x P x T**
In this problem: R=8%= 0.08, P=200 000 Frw, and T=2 years
So I = 0.08 x 200 000 x 2 = 32 000 Frw
Robert will have to pay the bank 32 000 for the service of his loan after 2 years.

7. Explain to trainees that you are paying the bank for the service of providing you with a loan. BUT to get money to loan out to people, the bank will pay YOU to save your money with them. You PAY interest on a loan, but you GET interest on savings accounts. That is how banks work. To figure out how much interest you get on a savings account, use the same formula: **I = P x R x T**.

8. Ask trainees to complete Task 7 then trainer writes the answer on the board.

Topic 1.3 Task 7
How much simple interest will a savings account with 50 000 Frw earn at a rate of 4% per year in 5 years?

Answer:
The formula is **I = R x P x T**
We are looking for the interest amount (I) that is paid into this savings account.
R=4%= 0.04, P=50 000 Frw, and T=5 years
So I = 0.04 x 50 000 x 5 = 10 000 Frw
We will get 10 000 Frw from the bank for keeping this savings invested for 5 years.

Application Activity

1. Divide the class into two groups (left side of the class and right side of the class) trainees on the right side of the class will work on the even numbered tasks and trainees on the left side of the class will work on the odd numbered tasks.
2. Let trainees work independently on the tasks for 5 minutes. Walk around and help any trainee that has questions and to ensure that the trainees are arriving at the correct answers. THEN pair up trainees—one from the left side and one from the right side—and have the two trainees share their work with each other. After a sharing time, all trainees will have all answers for even and odd tasks. Trainer will read aloud the correct answers and trainees will self-correct and take notes to ensure clear understandings.

Topic 1.3 Task 8
Calculate the percentage loss for an item bought at 2 000 Frw and sold at a price of 1 850 Frw.

Answer:
Cost is 2 000 Frw
Selling price is 1 850 Frw
Loss is 2 000 Frw – 1 850 Frw = 150 Frw
Percentage loss is obtained by putting Loss over the Cost to get a fraction, turn the fraction into a decimal, and then turn the decimal into a percentage. 150/2000 = 0.075 which is 7.5% when we move the decimal point two places to the right, so percentage loss is 7.5%

Topic 1.3 Task 9
Find the sale price of a watch whose marked price is 30 000 FRW if 20% discount is given.

Answer:
Marked price is 30 000 Frw. Discount is 20%.
We take 20% from 100% so the remaining price is 100-20 = 80% of the marked price. 80%=0.80
30 000 Frw x 0.80 = 24 000 Frw.
Sale price is 24 000 Frw and the discounted amount is 30 000 – 24 000 = 6 000 Frw.

Topic 1.3 Task 10
A trader bought a pair of shoes at 3 600 Frw and sold them for 4 500 Frw.
Find her profit or loss and then find the percentage profit or loss.

Answer:
She sold for more than the cost so she made a PROFIT.
Her profit is calculated 4 500 – 3 600 = 900 Frw

Percentage loss is obtained by putting PROFIT over the Cost to get a fraction, turn fraction into a decimal then change decimal into percentage. 900/3 600 = 0.25 = 25% percentage profit.

Topic 1.3 Task 11
The percentage profit on each cell phone bought at an original cost of 48 000 Frw is 20%. At what price does the business person sell each cell phone to make that profit?

Answer:
Percentage profit = 20%, which means that the phone is sold at 20% ABOVE the cost which means that you add 20% to the original cost of 100% to get 100 + 20 = 120% = 1.20
Given that the cost is 48 000 Frw, the selling or list price will be 120% of 48 000 Frw,
48 000 x 1.2 = 57 600 Frw. A selling (list) price per phones of 57 600 Frw makes a 20 % profit.

Topic 1.3 Task 12
A saleslady received a commission of 5% for the first sale of 80 000 Frw and 6% for sales above 80 000 Frw. In one month she made sales amounting to 168 000 Frw. Find her total commission that month.

Answer:
Commission for the first sale is 5% of 80 000 Frw, which is 4 000 Frw
Total sale was 168 000 Frw so excess after 80 000 Frw is 168000 Frw – 80000 Frw = 88000 Frw
6% of 88 000 Frw is 5 280 Frw. Total monthly commission is 4 000 Frw + 5 280 Frw = 9 280 Frw

Topic 1.3 Task 13
A company makes a product at a cost of 120 000 Frw which is sold at a list price of 130 000 Frw. The Value Added Tax (VAT) is a tax on manufactured goods. The tax is calculated ONLY on the amount of profit added to the cost when list price is determined. Currently the standard rate of VAT in Rwanda is 18%. This extra 18% will be paid to the tax authority after the sale of the product. Find the amount of tax that the company must pay to the government.

Answer:
Cost of manufacturing and materials is 120 000 Frw
List price of the product is 130 000 Frw
Value added is 130 000 Frw – 120 000 Frw = 10 000 Frw
Therefore, 18% VAT is charged on 10 000 Frw. Remember that 18% = 0.18
Therefore, VAT is 10 000 x 0.18 = 1 800 Frw
They will pay a Value Added Tax or VAT of 18% which is equal to 1 800 Frw on this product.

Points to Remember

- **Percentages** are used frequently in every-day life, including discounts, profits/losses, commissions, interest rates, and taxes. It is often easiest to calculate percentages if they are turned into decimal numbers which can be calculated directly.

- **Interest** is the amount **charged for borrowing** money—and Interest is also the amount that is **earned for saving** money in bank accounts.
  You **get** money for saving but you **pay** money to take out a loan.

Formative Assessment

1. Ask trainees to complete the following exercises individually then to share their responses with their neighbour. In pairs they can explain how they arrived at the answers to determine if they have understood the concept.

2. At the end of the work time allotted, read out the correct answers.

Formative Assessment Questions:

1. Find the simple interest earned from 3 400 Frw saved in a bank account for 3 years at the rate of 10% per year. How much will the entire account have in it at the end of this period?

Answer:
Use the formula: $I = P \times R \times T$ so we have $I = 3\,400 \times 0.10 \times 3 = 1\,020$ Frw
The total interest is 1 020 Frw which is added to the principal in the account so the total in the account will be: 1 020 + 3 400 = 4 420 Frw.

2. Peter borrowed 100 000 Frw from a bank to start a business making greeting cards. The bank charged him an interest rate of 12% per year. How much interest did he pay after 2 years?

Answer:
Simple interest I is obtained as follows, $SI = P \times R \times T$
We have $I = 100\,000 \times 0.12 \times 2 = 24\,000$ Frw
The total interest is 24 000 Frw which is added to the principal amount borrowed so the total that Peter must return to the bank will be 100 000 + 24 000 = 124 000 Frw.

3. A company of weavers produces baskets at a cost of 20 000 Frw each and sells each basket for the price of 28 000 Frw. Calculate the Value Added Tax (VAT) of 18% that the company will pay to the tax authority after the sale of each product, keeping in mind that the VAT is calculated only on the amount of the profit that is added to the cost of the basket.

Answer:
Cost of manufacturing each basket is 20 000 Frw. Selling price of the product is 28 000 Frw.
Value added is 28 000 Frw – 20 000 Frw = 8 000 Frw. VAT is charged at the rate of 18% or 0.18.
Therefore, VAT is 8 000 x 0.18 = 1 440 Frw. They will pay 18% VAT of 1 440 Frw for each basket.

4. Find the sale price of a watch whose marked price is 30 000 Frw if 10% discount is given.

Answer:
Marked price is 30 000 Frw
Discount is 10% = 0.10. We calculate 30 000 Frw x 0.10 = 3 000 Frw
Sale price will be 30 000 Frw – 3 000 Frw = 27 000 Frw
OR
We calculate directly that a 10% discount will give us 100%-10%=90% = 0.90
30 000 Frw x 0.90 = 27 000 Frw

5. An auto parts sales person received a commission of 4% for sales up to 50 000 Frw and 5% for sales above 50 000 Frw. One month her sales amounted to 88 000 Frw. Find her commission.

Answer:
Commission for the first part is 4% of 50 000 Frw, is 50 000 x 0.04 = 2 000 Frw
Commission for the second part of 88 000 is for the extra amount 88000 - 50000 = 38000 Frw the extra commission is calculated at 5%. This means 38 000 Frw x 0.05 = 1 900 Frw
The total commission earned that month is 2 000 Frw + 1 900 Frw = 3 900 Frw

ⓘ Further Information for the Trainer

Profit and loss: A simple business model is to consider sales and costs. Costs may include the cost of the production (labour and raw materials) as well as wages and rent and other things. Sales refer to the income of the business. Thus, we can define "Sales − Costs = Profit/Loss".

Simple interest: When money is borrowed, the borrower repays the amount they have borrowed plus another amount that is called the interest which is a payment for the service. The amount of interest depends on the amount borrowed, called the principal, the rate at which the interest is charged, and the time for which the money is borrowed.
Thus, we have the interest formula I = P x R x T
Note: interest is EARNED when you save money in a bank—again use the formula I = P x R x T

Discount: It is common for a shop to discount the price of an item. Discounts are normally expressed as a percentage of the original price. Finding the price after a discount is the same as decreasing a quantity by a given percentage.

http://amsi.org.au/trainer_modules/consumer_arithmetic.html

Learning Outcome 1.4:
Solve simple algebraic equations and inequalities

Objectives: By the end of the learning outcome, trainees will be able to:

a. Form equations or inequalities with variables

b. Solve simple algebraic equations and inequalities

Time Required: 2 hours

Learning Methodologies: individual work, pair work, group work, group investigation, discussion

Materials Needed:

- Calculator if available and standard materials, trainee workbook
- Ruler (if available)

Preparation:

❑ Draw a large number line across the top of the blackboard in the classroom

❑ Review methods for solving algebraic equations and inequalities

Cross Cutting Issues:

✔ **Gender:** trainer will provide equal opportunities to both boys and girls during the lesson as well as balance gender when forming groups

✔ **Peace/value education:** trainer promotes team work and respect for others' views and work

✔ **Inclusive education:** trainer is sensitive to differences in learning abilities and uses a variety of activities and materials to cater to different learning styles

Prerequisites: Trainees should already be comfortable with and able to:

- Use addition, subtraction, multiplication, and division
- Manipulate algebraic expression with one or more variables.

Key Competencies:

1. Describe the concept of simple algebraic equations with one unknown	**1.** Form and solve equations with variables	**1.** Persistent
2. Explain the concept of simple algebraic inequalities in one unknown	**2.** Form and solve inequalities with variables	**2.** Proactive
		3. Accurate

Steps:

Getting Started: What do we know and where are we going?

1. As a warm up exercise as trainees to complete Task 1 in their manuals in 3 minutes. When completed, reads answers aloud and trainees will self-correct.

Topic 1.4 Task 1
Solve the following given the values for the different variables:

- **a.** If a = 4, b = 8 and c = 2, find the value of a + b + c
- **b.** If x = 3 and y = 4, find the value of (2 + x)(5 + y)
- **c.** If x = z, y = -8 and z = -2, find the value of the product of the three terms: xyz

Answer:

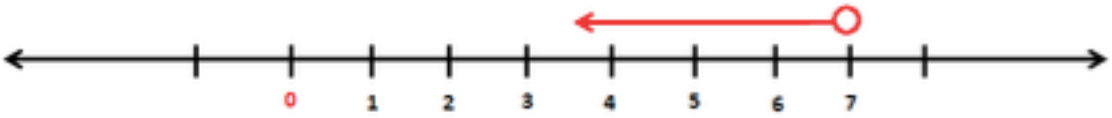
- **a.** If a = 4, b = 8 and c = 2, find the value of a + b + c, then a + b + c = 4 + 8 + 2 = 14
- **b.** If x = 3 and y = 4, find the value of (2 + x)(5 + y) , then (2 + 3)(5 + 4) = (5)(9) = 45
- **c.** If x = z, y = -8 and z = -2, find the value of xyz , then -2 x -8 x -2 = -32 (Remember, when you multiply two negatives together you get a positive and you get a negative when you multiply a positive number by a negative number).

2. While trainees work on Task 1, draw a big number line on the blackboard that goes from one edge of the classroom to the other. Ask a trainee to choose a number on the number line and then go to the board and put their hand under the chosen number.

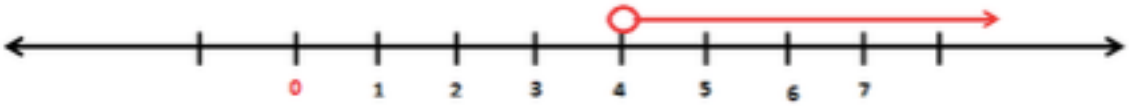
3. Ask another trainee to give a number that is LESS THAN the chosen number. Trainees will offer possibilities then go up and put their hand UNDER the number they represent.

4. Invite as many trainees as possible to stand at the board in the "less than" area—to the left of the chosen number.

5. Note aloud there are MANY answers that are correct when we are finding numbers that are LESS than another number on a number line. Remind trainees that the symbol $<$ means "less than" and that we can show such inequalities on a number line in the manner of the example: (trainer should use the example chosen by trainee—this example shows $x<7$.)



6. Ask standing trainees to sit down. Let another trainee choose a number and go to the board to show their number. This time, ask examples of numbers that are GREATER than the chosen number. Trainees can go to the board to touch numbers greater than original number, again note that there are many correct answers, so again we must write a solution using a number line in the manner of the following example for $x > 4$.



7. Ask trainees to sit down and thank them for their participation in this class activity.

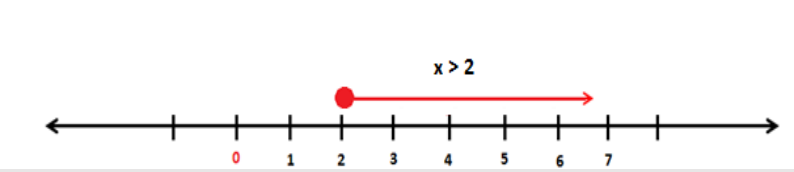
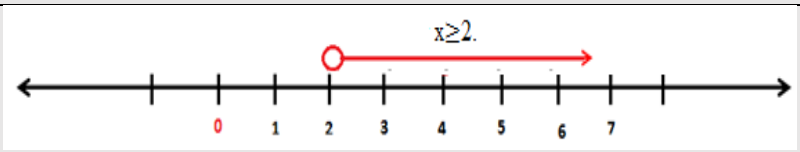
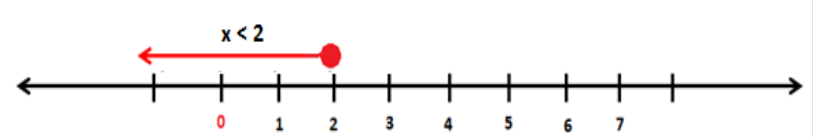
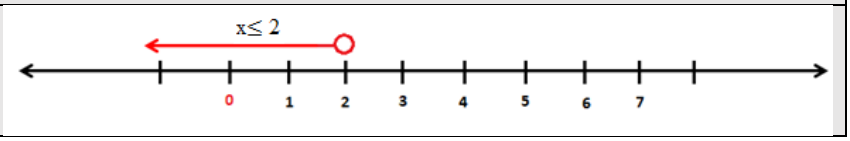
8. Ask trainees to carefully complete Task 2 in their manuals. When completed, then pair with a neighbour to check their work and confirm correct answers.

Topic 1.4 Task 2

Use a mathematical sentence (inequality) AND a number line to show and write the following numbers: (Hint: pay close attention to the difference between $<$ and $\leq$ or $>$ and $\geq$. Note the circle on the number line is either filled in or left empty!)

- **a.** All numbers greater than 2
- **b.** All numbers greater or equal to 2.
- **c.** All numbers less than 2.
- **d.** All numbers less or equal to 2.

Answer:

Sentence	Mathematical sentence	Number line
a) All numbers greater or equal to 2	$x \geq 2$	
b) All numbers greater than 2	$x > 2$	
c) All numbers less or equal to 2	$x \leq 2$	
d) All numbers less than	$x < 2$	

9. Ask volunteers to put answers on the board. Guide their presentation by identifying errors and correcting them. Note: we use X (or another letter) for an unknown number – a variable.

Problem Solving Activity

1. Explain that for this activity, trainer reads aloud a math sentence, then trainees write the sentence as an equation, and then solve the equation. That is: Form an equation and solve.

2. Once trainees have found an answer, they put pencils/pens down. Choose one boy and one girl to write answers on the board. Verify answers are correct before moving to next sentence.

Topic 1.4 Task 3

Trainer will read each sentence aloud. Trainee writes it using math symbols and solves

- **a.** I have a number. If I subtract 20 from my number, my answer is 6. What is my number?
- **b.** Think of a number, double it and subtract 6 to get 10. What is the number?

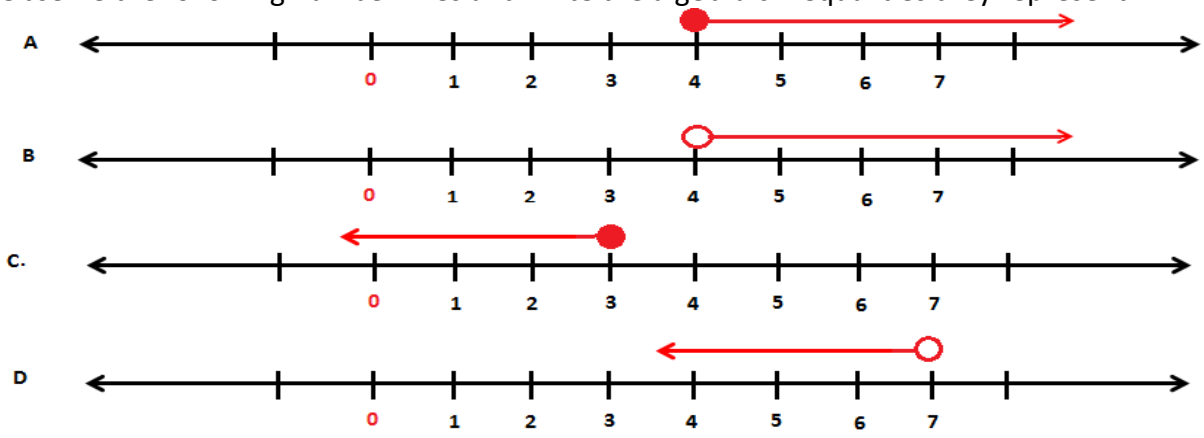
Answer:

- **a.** x – 20 = 6 x – 20 + 20 = 6 +20 x = 26 The number is 26.
- **b.** 2x – 6 = 10 2x – 6 + 6 = 10 + 6 2x = 16 2x/2 = 16/2 X = 8 The number is 8.

3. Draw the following number lines and given ranges on the black board. Ask trainees to write the algebraic inequality represented by each of the number lines.

Topic 1.4 Task 4

Observe the following number lines and write the algebraic inequalities they represent:



Answer:

- **a.** $x \geq 4$ (all numbers greater or equal to 4)
- **b.** $x > 4$ (all numbers greater than 4)
- **c.** $x \leq 3$ (all numbers less or equal to 3)
- **d.** $x < 7$ (all numbers less than 7)

Guided Practice Activity

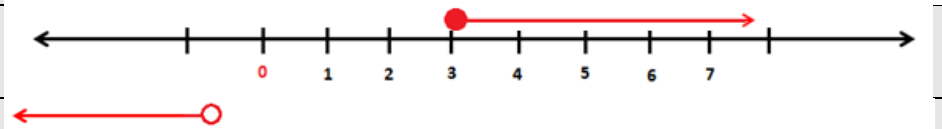
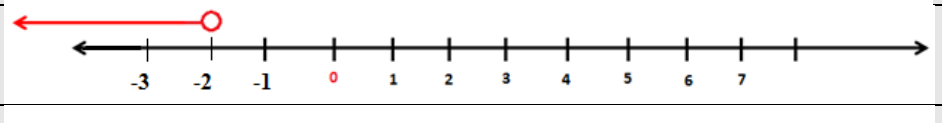
1. Solve the following problems on the board explaining each step so trainees can observe and ask questions thus becoming more confident and able to solve similar problems independently.

Topic 1.4 Task 5

For each of the following inequalities, draw number lines to illustrate the solution set:

(a) $x > 3$
(b) $x \geq 3$
(c) $x < -2$
(d) $x \leq -2$

Answer:

Inequality	Number line
(a) $x > 3$	
(b) $x \geq 3$	
(c) $x < -2$	
(d) $x \leq -2$	

2. Remind trainees that we have been working with inequalities, but similar to equalities, where the two sides of the equality have the same value, we can also work with these inequality math sentences to find solutions.

3. Ask trainees to remember how to work with algebraic equations by reviewing the processes needed to solve the equations in the Task 6. Trainees should work individually, but if they are unsure how to proceed, they may consult with the trainer or with their neighbour. Read answers aloud when all have completed their work.

Topic 1.4 Task 6

Solve for x:

a. $x + 2 = 10$
b. $x - 4 = 7$
c. $5x = 25$
d. $5(x + 3) - 3(x + 4) = 9$

Answer:

a. Subtract 2 from each side of the equation: $x + 2 - 2 = 10 - 2$ thus $x = 8$
b. Add 4 to each side. $x - 4 + 4 = 7 + 4$ thus $x = 11$

c. Divide both sides by 5. $\frac{5x}{5} = \frac{25}{5}$ and thus x = 5

d. Remove brackets, combine like terms, subtract 3 from each side and divide both sides by 2:

5x + 15 – 3x-12 = 9 then 2x + 3 = 9 then $\frac{2x}{2} = \frac{6}{2}$, thus x = 3

4. Note: the same processes used to solve EQATIONS are used to solve INEQUALITIES too. EXCEPT (this is very important!) if we multiply or divide both sides by a negative number then we MUST change the direction of the inequality symbol.

5. Demonstrate this change of symbol by working out the following task on the blackboard while trainees pay close attention and ask questions as needed. Explain each step carefully and fully.

Topic 1.4 Task 7

Solve the following inequalities and represent your solution on a number line.

a. 2 (x – 1) ≥ x + 2

b. x – 1 ≥ 3x + 2

Answer:

a.

2 (x – 1) ≥ x + 2 multiply to remove the parentheses

2x – 2 ≥ x + 2 add 2 to both sides

2x ≥ x + 4 subtract x from both sides

2x – x ≥ x – x + 4 combine like terms

Then x ≥ 4

b.

x – 1 ≥ 3x + 2 add 1 to both sides

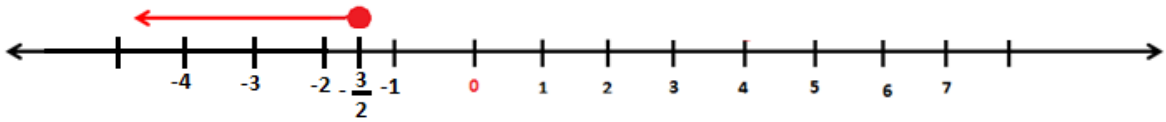
x ≥ 3x + 2 + 1 combine like terms

x - 3x ≥ 3 subtract 3x from both sides

-2x ≥ 3 combine like terms

2x ≤ -3 multiply both sides by **negative 1**—CAREFUL!! You MUST switch the inequality symbol!

x ≤ -3/2 divide both side by 2 (no change of symbol because we divide by a **positive** number!)



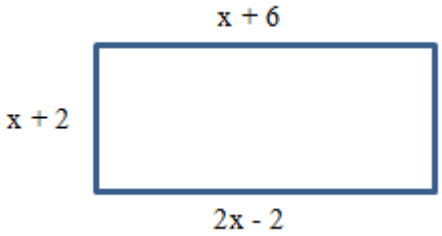
6. Ask a volunteer to read aloud **1.4 Key Facts** in the student manual. Explain as needed.

Application Activity

1. Trainees work in pairs to solve tasks in the application section. Hint: read carefully, there are many different steps for solving equations and inequalities. Refer to **1.4 Key Facts** if helpful. Trainer will walk around the class to help any trainees that are having trouble.

Topic 1.4 Task 8

A rectangular plot of land has dimensions in meters. Observe the diagram below:



a. The 2 parallel sides of a rectangle are equal. Use that property to find the value of x.
b. Find the length and the width of the plot of land.
c. Calculate the perimeter of the plot of land in meters.
d. Calculate the area of the rectangular plot of land in meters squared.

Answer:

a. Find x: $2x - 2 = x + 6$ $2x - x = 6 + 2$ $x = 8$

b. Find the length and the width Length = x+ 6 Substitute 8 in for x Length = 8 + 6 = 14
Or use L = 2x – 2 L = 2(8) – 2 L = 16 – 2 = 14
width = x + 2 Substitute 8 in for x width = 8 + 2 = 10

c. Calculate the perimeter Perimeter = 2 (l + w) P = 2 (14 + 10)
Substitute numbers for length and width P = 2 (24) = 48

d. Calculate the area Area = l x w Area = 14 x 10 = 140
Substitute numbers for length and width

Topic 1.4 Task 9

Solve the following inequalities and represent the solutions on number lines.

1. (a) $x + 4 > 11$ (b) $x - 6 \leq 5$
2. (a) $2x - 8 \leq 4$ (b) $3x + 4 > 19$
3. (a) $3 > 4x - 2$ (b) $7 \leq 5x + 12$
4. (a) $3 - 2x < 5$ (b) $4 - 5x \geq -11$
5. (a) $13 - x - 3 > 4$ (b) $15 - x + 2 < 1$

Answer:

	Inequality	**Solution**
1.	(a) $x + 4 > 11$ (b) $x - 6 \leq 5$	$x > 7$ $x \leq 11$
2.	(a) $2x - 8 \leq 4$ (b) $3x + 4 > 19$	$x \leq 6$ $x > 5$
3.	(a) $3 > 4x - 2$ (b) $7 \leq 5x + 12$	$5/4 > x$ or $x < 5/4$ $-1 \leq x$ or $x \geq -1$

4.	(a) $3 - 2x < 5$ (b) $4 - 5x \geq -11$	$x > 1$ $x \leq 3$ NOTE BOTH SYMBOLS CHANGED!!
5.	(a) $13 - x - 3 > 4$ (b) $15 - x + 2 < 1$	$x < 6$ $x > 16$ NOTE BOTH SYMBOLS CHANGED!!

Points to Remember

- A letter such as x used above is called a **variable**. Variables can be used in both **equations** and in **inequalities** to represent an unknown value (or many unknown values) which we find by using ALGEBRA and ARITHMETIC.

- **We use arithmetic steps to solve equations. We can:**
  Add the same number to both sides or subtract the same number from both sides.
  Multiply both sides by the same number or divide both sides by the same number.

- **We use the same steps to solve inequalities EXCEPT watch out for negative numbers!**
  The direction of the inequality symbol changes when we multiply or divide an inequality by a negative number!

Formative Assessment

1. Ask trainees to work out the following activities individually, then to share their responses with their neighbour. In pairs they can explain what they did and how they arrived at their answers to determine if they have understood the concept of equation and inequality. Trainer should walk around and offer hints and support to trainees that are struggling.

Formative Assessment Questions

1. Solve the following equations:

(a) $x + 6 = 20$
(b) $16 + x = 25$
(c) $x - 3 = 12$
(d) $12 - x = 10$
(e) $x + 9 = 23$
(f) $17 + x = 25$
(g) $3x - 5 = 1$
(h) $14 - x = 8$

Answer:

a) X = 14
b) X = 9
c) X = 15
d) X = 2
e) x = 14
f) x = 8
g) x = 2
h) x = - 6

2. Solve the following inequalities and represent the solutions on number lines:

(a) $2(1 + x) + 3(x – 2) \geq 25$
(b) $3(4 – 3x) – (5x – 3) \leq 2$

Answer:

a) $2 + 2x + 3x – 6 \geq 25$	$5x – 4 \geq 25$	$5x \geq 29$	$x \geq 29/5$
b) $12 – 9x – 5x – 3 \leq 2$	$-14x \leq -7$	$x \geq -7/-14$	$x \geq 1/2$

Further Information for the Trainer

In daily life situations, people are faced mathematical problems that require both simple and complex calculations. Some people prefer to use only numbers (arithmetic methods) while others prefer to use numbers mixed with variables/letters (algebraic methods). Most problems require both algebra and arithmetic to solve effectively.

Equations and inequalities:
An **equation** says that two things are equal. It will have an equal sign "=." An equation is like a statement saying "this equals that."
For example: $x – 2 = 4$ and this means that what is on the left $(x – 2)$ is equal to what is on the right (4).
A solution for an equation is a value that can replace a variable to make the equation true.

An **inequality** is a way to represent the solution to an inequality is to graph it on a number line. We use an open circle for < and > and a closed circle for ≤ and ≥ then an arrow to represent the direction where the solution is found along the number line.

While solving inequalities, pay attention to the direction of inequality.

ALWAYS REMEMBER:
If you **multiply or divide by a negative number you MUST change the direction of the symbol** (while you work toward the solution).

Related to this, be very careful that you do not multiply or divide by a variable (unless you know it will always have the value of a positive number). Best to not multiply or divide by a variable!

1. https://www.mathplanet.com/education/algebra-1/linear-inequalitites/solving-linear-inequalities
2. https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq
3. http://elearning.reb.rw/course/view.php?id=203&section=11
4. https://www.mathsisfun.com/algebra/inequality-solving.html

Learning Outcome 1.5: Plot and interpret graphs of linear and quadratic functions

Objectives: By the end of the learning outcome, trainees will be able to:

a. Use an x-axis and a y-axis to plot and name points on the Cartesian plane
b. Plot and interpret graphs of linear functions
c. Plot and interpret graphs of quadratic functions

Time Required: 2 hours

Learning Methodologies: individual work, pair work, group work, discussions

Materials Needed:

- Standard materials, trainee workbook/notebook, calculator and rulers (if available)

Preparation:

- ❑ Prepare manila papers or any other chart showing different values of m and c in a function of the form of y=mx +b.
- ❑ Prepare manila papers or any other chart showing different values of a, b and c in a function of the form $y= a x^2+ b x + c$.
- ❑ Prepare manila paper showing graphical solutions of linear function and quadratic function in a Cartesian plane.

Cross Cutting Issues:

- ✓ **Gender:** trainer will provide equal opportunities to both boys and girls during the lesson as well as balance gender when forming groups
- ✓ **Peace/value education:** trainer promotes team work and respect for others' views and work
- ✓ **Inclusive education:** trainer is sensitive to differences in learning abilities and uses a variety of activities and materials to cater to different learning styles

Prerequisites: Trainees should already be comfortable with and able to:

- Use addition, subtraction, multiplication, and division
- Plot points in a Cartesian plane
- Simplify and solve algebraic expressions with one or more variables

Key Competencies:

Knowledge	Skills	Attitudes
1. Explain the difference between a linear and a quadratic function	**1.** Plot and name points on a Cartesian plane	**1.** Accurate
2. Describe a Cartesian plane (x-axis and y-axis with ordered pairs as Points)	**2.** Plot linear and quadratic functions and interpret the graphs of those functions	**2.** Innovative
3. Explain how to use the plot of quadratic or linear functions to provide solutions.	**3.** Solve linear and quadratic functions using a graph	**3.** Persistent

Steps:

Getting Started: What do we know and where are we going?

1. Ask trainees to draw 2 perpendicular lines and name the intersection point, the origin, with the coordinates (0, 0). Put arrows at the ends of the vertical and the horizontal lines respectively. Name the vertical line Y and the horizontal line X. As the trainees are working on this in their notebooks, draw an example on the board so they can confirm their work.

2. Ask trainees to mark 1 cm intervals on the X axis and on the Y axis, which is to graduate the lines in all 4 directions. Label each mark with numbers (positive when moving to the right of 0 on the x axis and upward from 0 on the y axis and negative when moving left from 0 on the x axis and downward, below 0 on the y axis). Each trainee has now drawn a Cartesian plane in their notebook.

3. Remind trainees that points on a plane are given by two numbers, called coordinates—one for x and another for y – written (x, y), which tells where to put the point. Use the plane on the board to find the point (3, 2): move three units to the right of the origin (in the positive direction) on the x axis and two units upward (in the positive direction) from the origin on the y axis. The point is then placed where those two numbers meet. Show students how to find the following points: (5, 2) (-2, 5) (-3, -2) (2, -3) as well as a few other examples.

While the trainer is demonstrating this on the blackboard, trainees should be finding the same points on the grid they have drawn in their notebooks as individual practice.

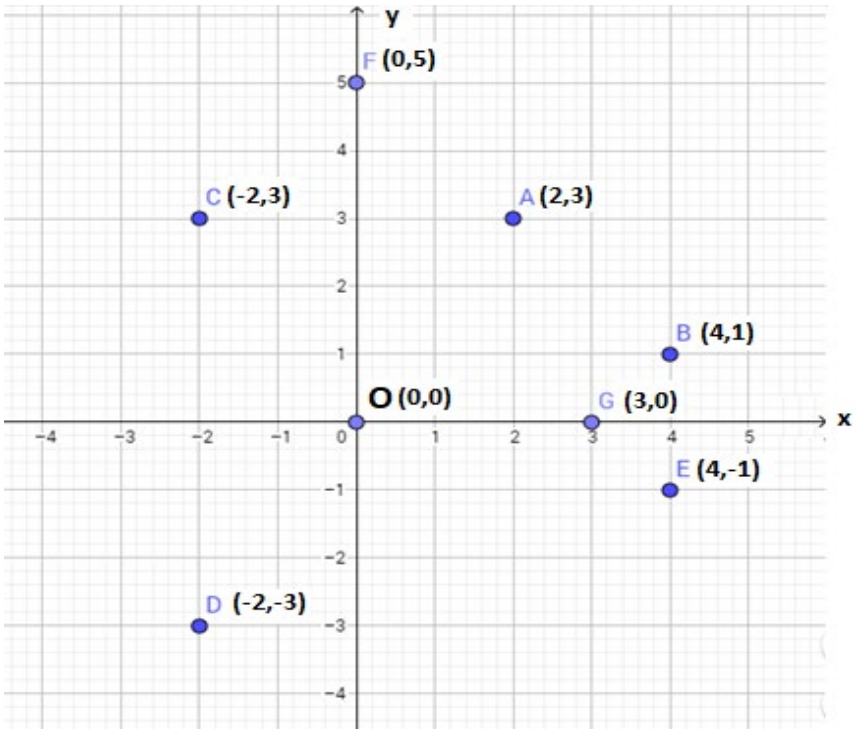
4. After this review warm-up activity, Pair up trainees and ask them to complete Task 1.

Topic 1.5 Task 1

Plot the following points on your Cartesian plane. Label each point with a letter and the ordered pair – the coordinates—on the grid:

a) (2, 3) b) (4, 1) c) (-2, 3) d) (-2, -3) e) (4, -1) f) (0, 5) g) (3, 0)

Answer:



5. Ask a volunteer to sketch the given points onto the Cartesian plane on the board. Trainer will ensure answers are correct—and provide encouragement.

6. Note: any two points can be joined by straight lines. Use a long ruler or a straight stick, show how lines might be drawn that could connect 2 (or sometimes 3) points on the existing diagram.

7. Tell trainees we will work with lines as well as one type of curve called a parabola. We will look at equations and the related graphs. For all equations with x and y, we can find points, then plot them to see the related graph (a line or curve). On the graphs we will note INTERCEPTS (where the graph crosses the x-axis or the y-axis).

Problem Solving Activity:

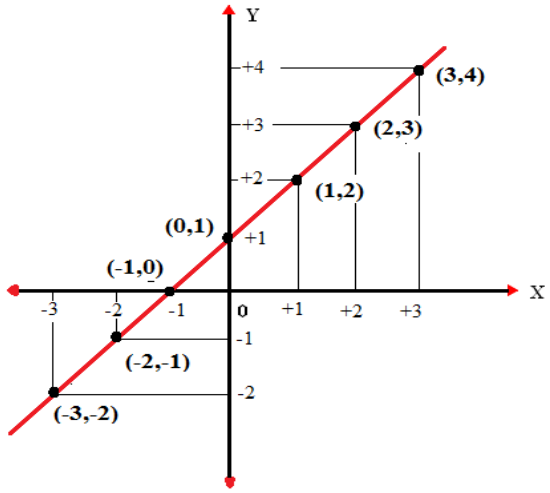
1. Ask trainees to complete Task 2 individually—then raise their hand when completed. Walk around class to verify correct responses and to provide guidance, support and encouragement.

Topic 1.5 Task 2

- **a.** Plot the coordinates (-3, -2); (-2, -1); (-1, 0); (0, +1); (+1, +2); (+2, +3) and (+3, +4) on a plane drawn in your copy book then join all the points by drawing one straight line.
- **b.** Find a relationship between the x and the y values for the points given in (a)—that is, what math is done to the x value to equal the y value?

Answer:

a. Plotting and joining the points in a Cartesian plane we find a straight line.



b. The relation or pattern between the X and Y values is add 1 to the value of x to equal y. Written mathematically this is: y = x + 1. This equation gives ALL the points on the line, for example if x=75 we can figure out the y value (75 + 1) = 76 so the point (75,76) is on the line.

To organize and help us think about this pattern, we can put the values into a table:

x	-3	-2	-1	0	1	2	3
Y = x + 1	-2	-1	0	1	2	3	4

2. Trainer explains that when given an equation with variables x and y, we can use different values of x to find related values for y. Once we have those linked values for x and y, we have a set of points which can then be graphed.

Guided Practice Activity:

1. Write the equation y = 3x – 1 on the black board. Show trainees that if let x= -2 then we get the equation: y = 3 x (-2) -1 = -6 -1 = -7 so we get the point (x, y) = (-2, -7).
On the same line (because it is the same equation) we can find points for other values of x. Ask different trainees to figure the y values for when x = -1, 0, 1, 2 and 3…as they get the answer then plot the points on the board.

Note: when x = 0, then y = -1 to get the point (0, -1). This is where the graph crosses the y axis—the y-intercept. Similarly, when the graph of an equation crosses the x-axis, we call it the x intercept.

2. Ask trainees complete Task 3 and 4 in pairs. Walk around the class to support and provide guidance then let a volunteer explain answers to the class. Trainer verifies and reinforces main points.

Topic 1.5 Task 3

Complete the following tables, list out the points given, then make a sketch of the graphs.

a.

x	-3	-2	-1	0	1	2	3
Y = 2x + 2							

b.

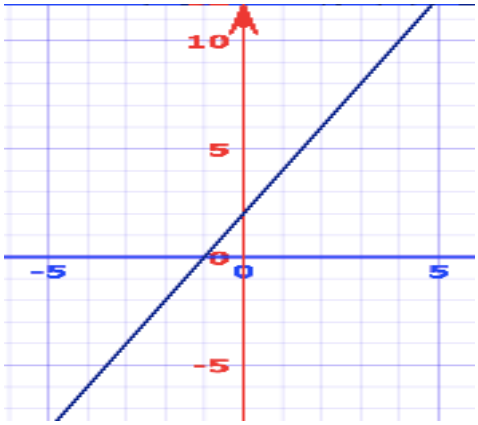
x	-1	0	1	2	3	4	5
$Y = x^2 - 4x + 3$							

Answer:

a.

x	-3	-2	-1	0	1	2	3
Y = 2x + 2	-4	-2	0	2	4	6	8

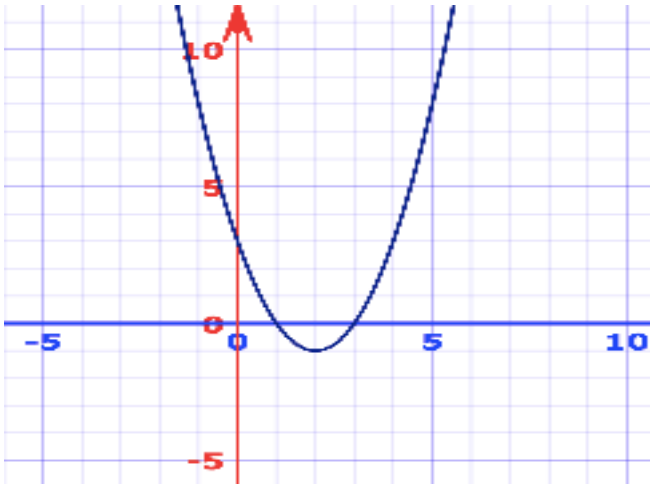
Points on the graph are: (-3,-4), (-2,-2), (-1,0), (0,2), (1,4), (2,6) and (3,8)



b.

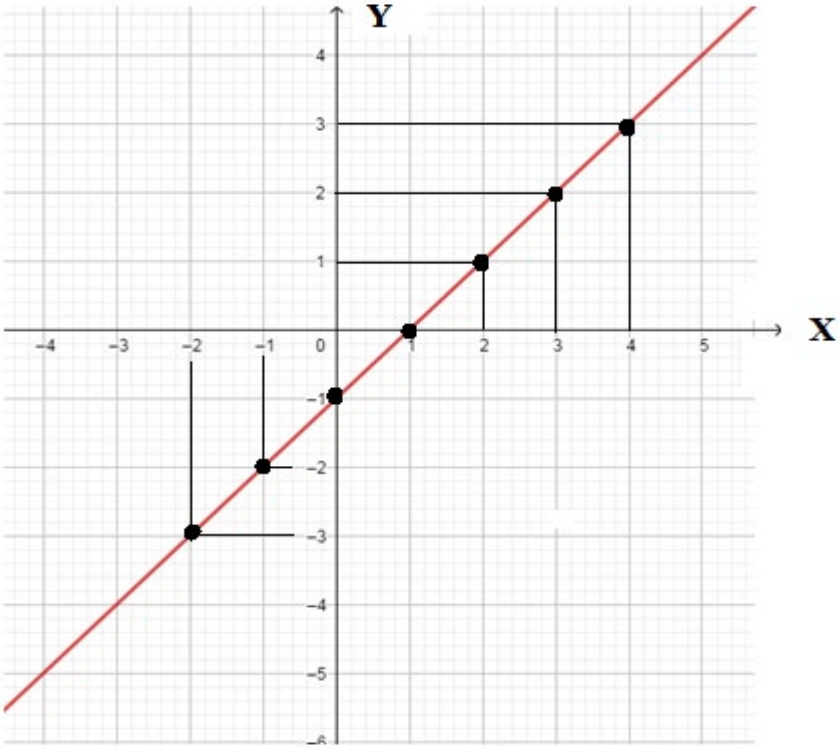
x	-1	0	1	2	3	4	5
$Y = x^2 - 4x + 3$	8	3	0	-1	0	3	8

Points on the graph are: (-1,8), (0,3), (1,0), (2,-1), (3,0), (4,3) and (5,8)



Topic 1.5 Task 4

Given the following graph, write out coordinates for at least 5 points. Then try to find an equation that lets you find y for any given value of x.



Answer:
Points on this line have the coordinates: (-2, -3), (-1, -2), (0, -1) (1, 0), (2, 1), (3, 2), and (4, 3).
To find the relationship between x and y, use the table below:

x	-2	-1	0	1	2	3	4
y	-3	-2	-1	0	1	2	3

Looking at the numbers in the table, to get from x to the related y we subtract 1, that is, y = x -1

3. Announce that the same pairs will now work with a QUADRATIC equation—that is, an equation with one variable raised to the 2nd power—so the graph will be a curve called a parabola. Trainees will work together to explore the given function and the related graph while trainer walks around the class to help and guide as needed.

Topic 1.5 Task 5
Consider the function: $y = 2x^2 + 3x - 2$

a. Use the equation given to complete the following table of values:

x	-3	-2	-1	0	1	2
$Y = 2x^2 + 3x - 2$	7	0	-3	-2	3	12

b. List the coordinates in ordered pair format (x, y) for all the values that you have found.
c. Plot the graphs on Cartesian plane.
d. List the points where the graph crosses the x-axis. These points are called the X-INTERCEPTS.
e. Give the coordinates for the Y-INTERCEPT is where the graph crosses the y axis.
f. Describe the shape of the curve you find.

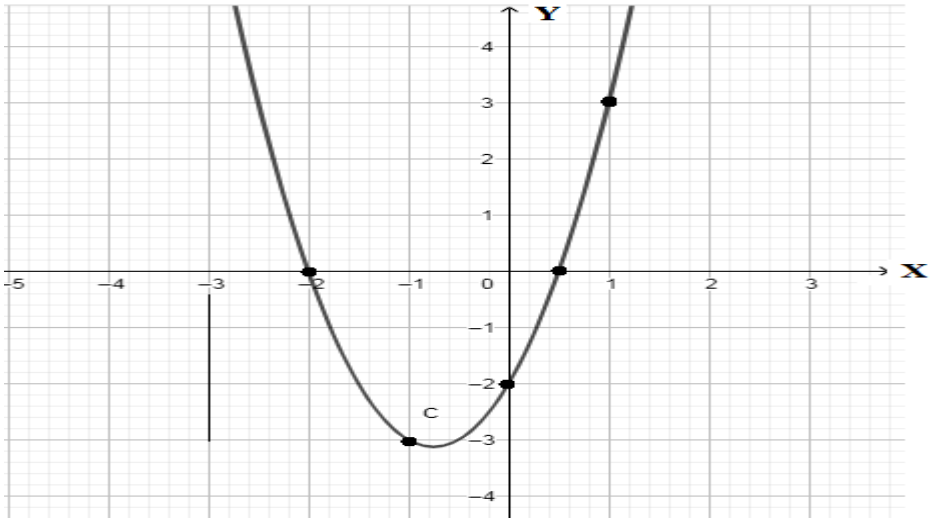
Answer:

a.

x	-3	-2	-1	0	1	2
$Y = 2x^2 + 3x - 2$	7	0	-3	-2	3	12

b. (-3, 7), (-2, 0), (-1, -3), (0, 2), (1, 3), (2, 12)

c. Graph of $y = 2x^2 + 3x - 2$:



d. Coordinate(s) of the point(s) where the graph cuts x-axis are (-2, 0) and (0.5, 0).

e. Coordinates of the point where the graph crosses the y-axis are (0, -2).

f. The graph is a curve, opening upward, which is called a parabola.

4. Write the following four equations across the top of the blackboard so that each equation has space for workings underneath the equation:

(a) $y = 3x + 6$ (b) $y = x^2 - x - 2$ (c) $y = x - 4x + 9$ (d) $y = x^2 — 9$

5. Ask volunteers to read each of the equations aloud to the class. Then brainstorm the following:
-- How are these equations similar? How are they different?
-- Can we tell which will give lines and which will give curves?

Possible answers may include:

Same because:
--all have x and y as variables, and all have y all alone on one side of the equation.
Different because:
--some equations have variables that are squared—and others just have plain variables.
--some will have linear graphs and others will have curved graphs (the ones that have exponents will give a curved graph).

6. Divide the class into four groups—a, b, c, and d. Ask each group to go to the section of the board, find a few points on their function, graph the function, and find the x and the y intercepts.

Topic 1.5 Task 6

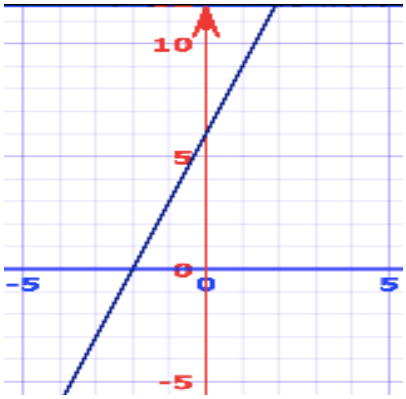
Find a few points on the given function's graph. State if the function is a line or a curve. Sketch the graph for the function. And find the x- and y-intercepts.

a. $y = 3x + 6$

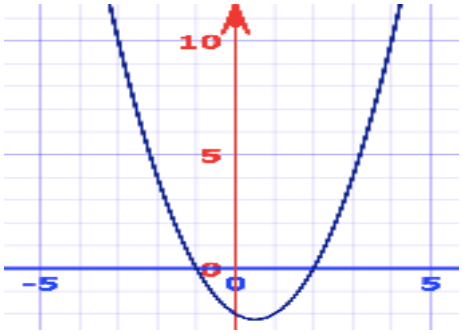
b. $y = x^2 – x – 2$
c. $y = x – 4x + 9$
d. $y = x^2 — 9$

Answer:

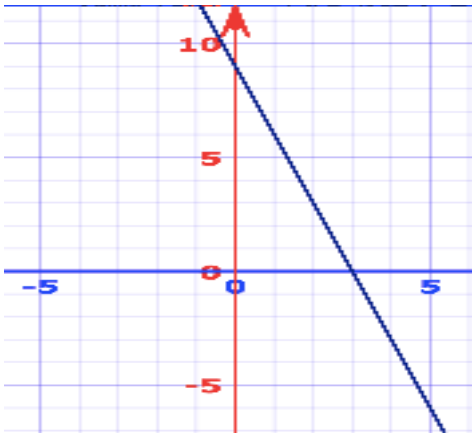
a. $y = 3x + 6$
x-intercept: (-2, 0) y-intercept: (0, 6)



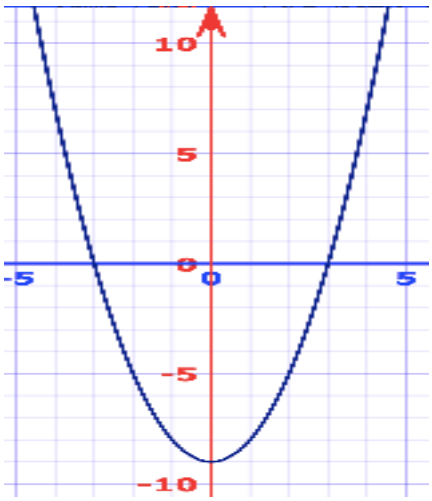
b. $y = x^2 – x – 2$ is quadratic because the power of x is 2,
x-intercepts: (-1, 0), (2,0) y intercept:(-2, 0)



c. First this equation must be simplified by combining like terms: $y = x – 4x + 9 = -3x + 9$
$y = -3x +9$ shows us that this is a line. X-intercept is (3, 0) and y intercept is (0, 9)



d. $y = x^2 - 9$, is quadratic function because the power is 2—thus the graph will be a curve.
x-intercepts are (-3, 0) and (3, 0) and y-intercept is (-9, 0)



7. Ask a trainee to read the Key Facts in the Trainee's Manual aloud. Respond to questions and clarify any concepts not yet clear by providing additional examples as needed for explanation and for practice.

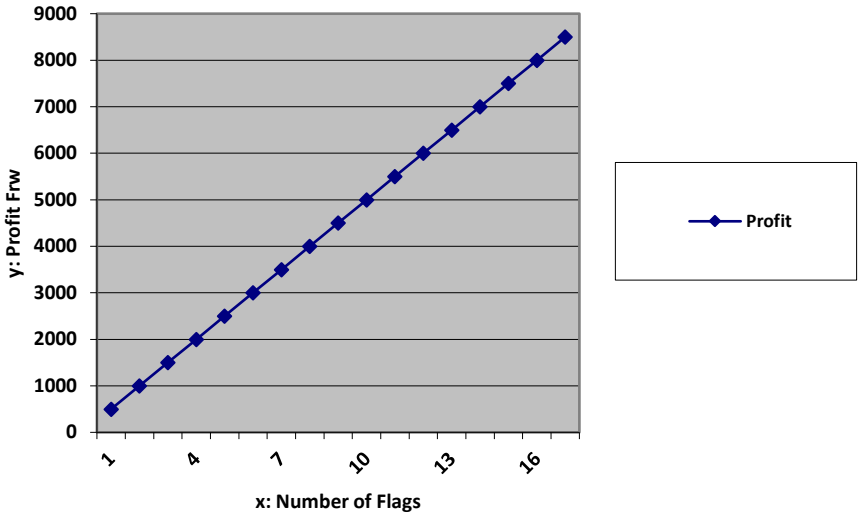
Topic 1.5 Task 7

The local football team is trying to make money to cover costs of a trip to Goma to participate in an international tournament. They are selling flags. For each flag they sell they make 500Frw profit, each team member needs to make 8 500 Frw to cover the cost of their trip. If they sell 0 flags, they make 0 profit. If they sell one flag, they make 500 Frw profit and so forth. This gives the points (0, 0) and (1, 500). Find a few more points so you can plot the graph.

a. Plot the graph which shows the number of flags sold as the x value and the total profit as they value.
NOW use your graph to determine:
b. how many flags they need to sell to make a profit of 8 500 Frw
c. how much profit is made if they sell 7 flags

Answer:

a. Note: This graph is showing the first quadrant (upper right) of the Cartesian plane only. You can extend the x axis to the left of 0 and the y axis below 0 to show the Cartesian plane but often, as is in this case, the information we are interested in is in the first quadrant.



b. Each team member needs to sell 17 flags (or more!) to cover trip costs.
c. If a team member sells 7 flags they will make a profit of 3 500 Frw.

Application Activity:

1. Ask trainees to complete the three application tasks independently then share their answers with a neighbour. When confident their answers are correct, trainer will verify the answers OR to provide support and hints to help student find the correct answer.

Task 1.
Topic 1.5 Task 8
Mary is offered a part-time job selling restaurant equipment. She will be paid Frw100 000 per month plus a 6% commission on her sales. The sales manager tells Mary she can expect to sell about Frw 800 000 worth of equipment per month. Draw a graph to represent this situation to help Mary understand her potential income for different levels of sales, so x will be sales and y will be income for your points. The values are given in the table below.
Use your graph to determine how much Mary will have to sell to make 200 000 Frw.

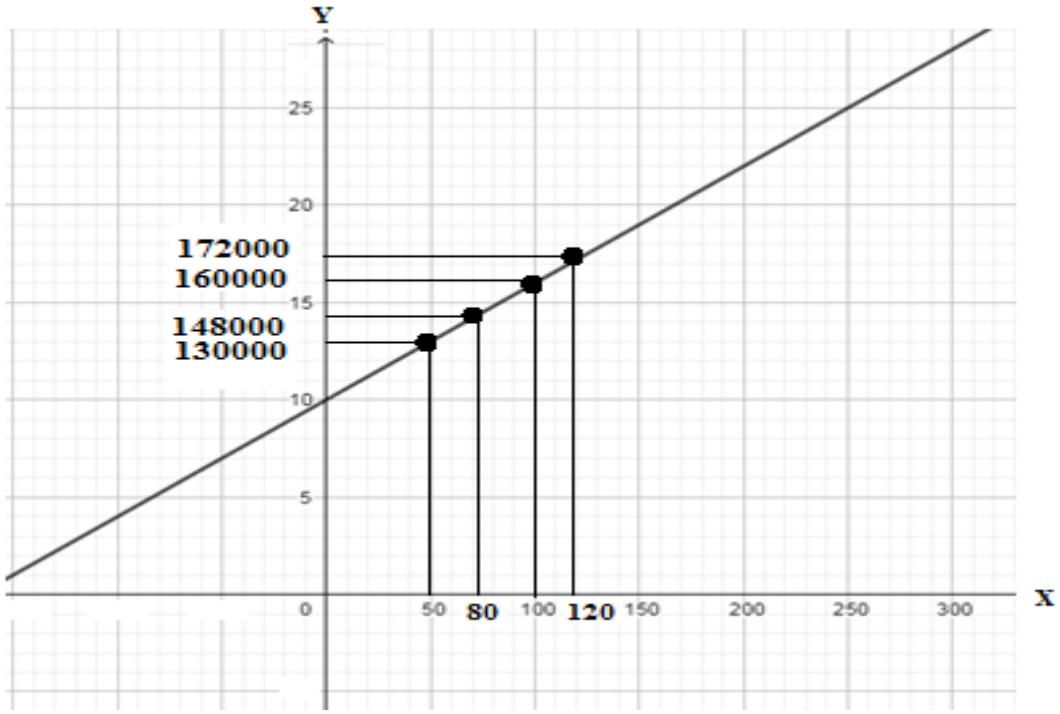
Monthly incomes for each sales total.

Sales (x-value)	6% of Commission	Mary's Monthly Income (y-value)
500000	$\frac{6 \text{ x } 500000}{100} = 30000$	100000 + 30000 = 130000
800000	$\frac{6 \text{ x } 800000}{100} = 48000$	100000 + 48000 = 148000
1,000,000	$\frac{6 \text{ x } 1000000}{100} = 60000$	100000 + 60000 = 160000
1,200,000	$\frac{6 \text{ x } 1200000}{100} = 72000$	100000 + 72000 = 172000

Answer:

Use Sales as your x-value and Monthly income as your y value to find points to plot.

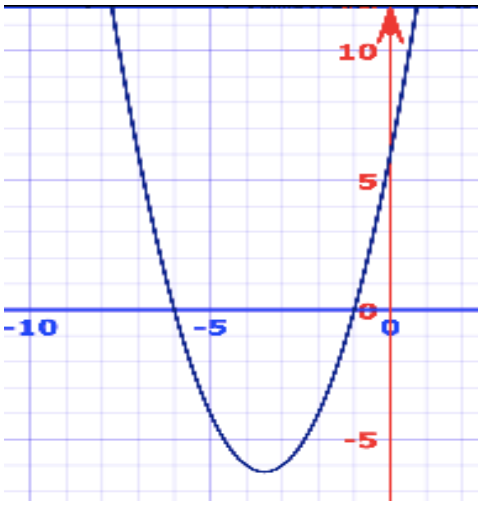
Plotting data points to show Mary's monthly income for all possible monthly sales totals.



Note: on this graph 10 stands for 100, 000 Frw and 50 stands for 500, 000Frw
Mary will have to sell 1 700 000Frw to make an income of 200 000 Frw.

Topic 1.5 Task 9
Graph the quadratic equation given below.
$x^2 + 7x + 6 = y$

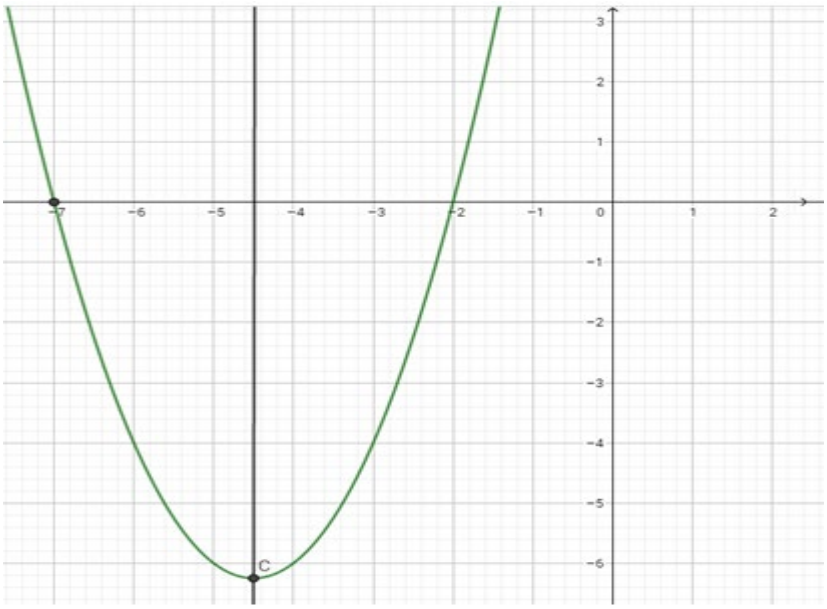
Answer:
The graph of the quadratic function $y= x^2 + 7x + 6$ is plotted below basing on the calculated points such as vertex (- 2, -4) and (-5,-4) and intercepts: (-1, 0) and (-6, 0) and y intercept (0,6).



Topic 1.5 Task 10
Given function: $y = x^2 + 9x + 14$

Find coordinates for points by putting values for x into the equations to find related y values. Plot the graph of the function.
Find the solutions to the equation $0 = x^2 + 9x + 14$, that is, find the x intercepts of the graph.

Answer:
By graphing this function, we see that the x-intercepts are at (-7, 0) and (-2, 0).



Points to Remember

- Coordinates that define points on a Cartesian plane are always in the order of (x, y) so
  - X-intercepts will always have a zero in the y position.
    Ex.: The x intercept for the graph of the equation y=3x – 6 will be (2, 0)
  - And y intercepts will always have a zero in the x position
    Ex.: The y intercept for the graph of the equation y=–2x + 5 will be (0, 5)

- Graphing a function can be done by plotting points on a grid—the more points that you plot the more accurate the sketch of your graph will be.

- A linear equation will give the graph of a straight line—a quadratic function (with a variable that is squared) will give the graph of a parabola which is a curve.

Formative Assessment

1. Ask trainees to individually work out the tasks in the formative assessment section of their workbooks.

2. When all trainees have completed the tasks, ask trainees to share their responses with their neighbour. When trainees are confident that they have good answers, ask for volunteers to explain what they did and how they arrived at their answers. Trainer will verify if correct.

Formative Assessment Questions

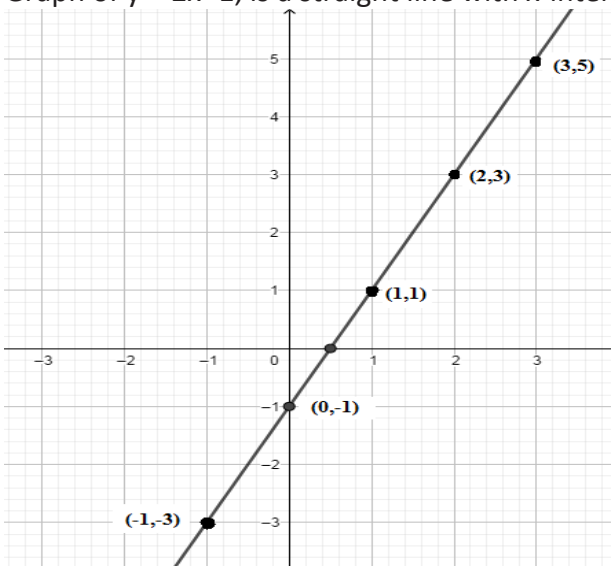
1. Find coordinates of x and Y for the given functions, sketch the graph, and give coordinates for the x- and the y-intercepts.
   a. $y = 2x - 1$
   b. $y = x^2 - 1$

Answer:

a. $y = 2x - 1$

x	-3	-2	-1	0	1	2	3
Y=2x-1	-7	-5	-3	-1	1	3	5

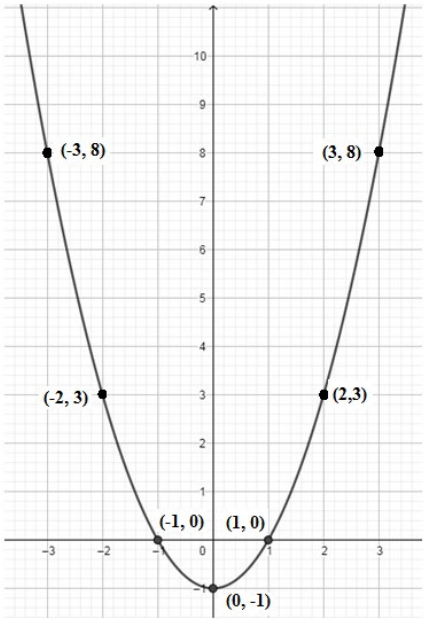
Graph of y = 2x -1, is a straight line with x-intercept at (0.5, 0) and y intercept at (0, -1).



b. $y = x^2 - 1$

x	-3	-2	-1	0	1	2	3
Y= x^2-1	8	3	0	-1	0	3	8

Graph of $y = x^2-1$, is a quadratic function, x-intercepts are (-1, 0) and (1, 0), y-intercept is (0, -1).



Further Information for the Trainer

Linear functions are typically in the form of y = mx + b where m stands for the slope, or rate of change, and b gives the y intercept (0, b).

Linear functions are graphed as straight lines because the x variable is not raised to any exponent.

When **graphing a linear function,** it is a good idea to start with the y-intercept. This is the point where the graph crosses the vertical y-axis. When you have a linear function in the form of y = mx + b, use the b value to find out where the graph crosses the vertical axis. Next, we can use the slope or the m value to find another point. The m value is right in front of the x.

Quadratic functions are typically in the form $y = ax^2 + bx + c$. Quadratic functions will always have the x variable to the second power, that is, an x is squared. This makes for a symmetrical, curved graph called a parabola.

With quadratic function, the best practice is to start with the maximum or minimum point- the graph's vertex. This is the spot point where the graph curves. Use the following formula to find the value of x at that point: $x = -\frac{b}{2a}$ and once the x-value is known, plug it into the function to find the y- value.

- https://study.com/academy/lesson/comparing-graphs-of-quadratic-linear-functions.html
- http://jwilson.coe.uga.edu/EMT668/EMAT6680.Folders/Barron/unit/Lesson%206/6.html

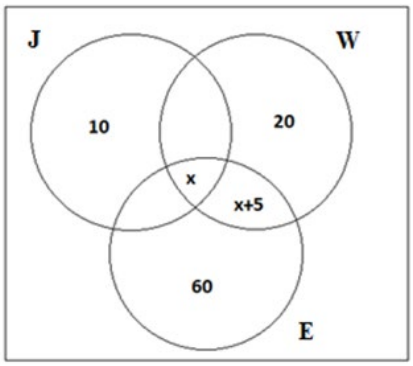
Summative Assessment—Unit 1

Version for the trainees:

Read the following linked situations carefully and answer the questions carefully. Show your work.

1. At the end of level 2 courses, a group of trainees, boys and girls, decide to start a handicraft association. They decide to start with making Woven baskets (W), Jewelry (J) and Embellishments for parties (E). All of these products are made from natural raw materials that can be found and grown in Rwanda such as sisal fibres, banana fibre, and herbs.

The Venn diagram below shows the number of trainees who make Woven baskets (W), Jewelry (J) and Embellishments (E). Some make more than one product. In total, 55 trainees make Woven baskets.



Answer the following questions regarding the scenario described above:

a. How many trainees like to make the following products:
i) Woven Baskets? ii) Jewelry? iii) Embellishments? iv) all three

b. Find the total number of trainees in the group that started this handcraft association.

c. How many trainees like to make Jewelry only?

2. After one week, their products were ready to compete with other products from different associations on the market. The association decided to employ a part-time seller.

The seller will be paid 8% of commission for each sold product.

The table below shows the number of produced products, the number sold, and the unit price:

Produced Products	**Sold products**	**Unit price**
120 Woven baskets	70 woven baskets	5000 Frw
250 Jewelry	200 Jewels	600 Frw
100 Embellishments	50 Embellishments	4000 Frw

After paying the commission to the seller, the remaining money was saved in a bank at the simple interest of 18% a year.

Answer the following questions regarding the information above:

a. In the first month, the seller hired by the association sold 50 Embellishments, 200 Jewelry pieces, and 40 Woven baskets.

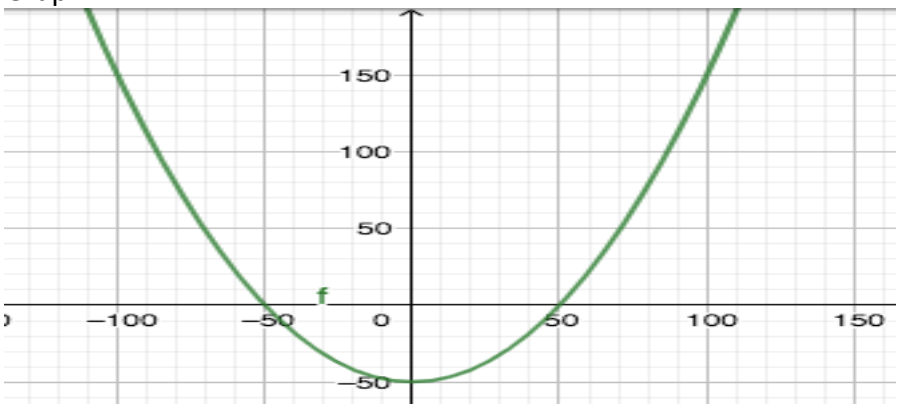
i. How much commission (at 8% of total sales) was earned by the seller?

ii. How much money did the association make as profit this month?

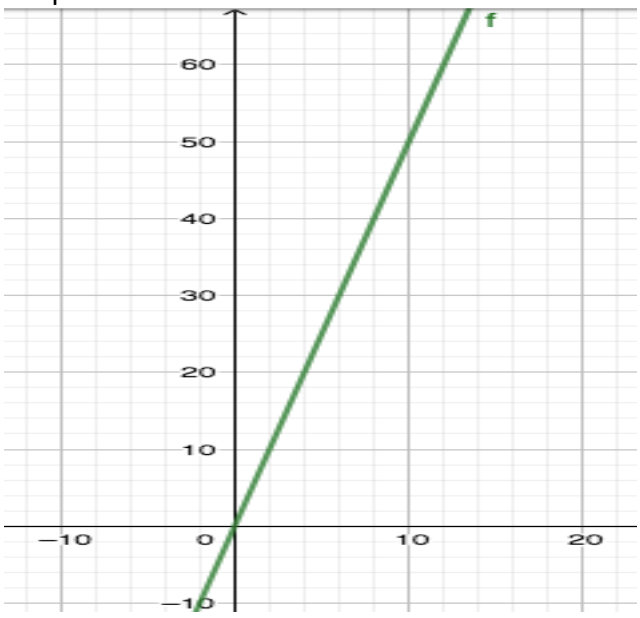
b. After paying the commission to the seller, the remaining money was saved in a bank account at the simple interest of 18% a year. Calculate the interest to be given to the association after 2 years.

3. Observe the following two graphs:

Graph A shows **profits for baskets made in a factory** by a machine.
X-axis shows numbers of baskets sold. Y-axis shows profits (x 1 000Frw) for sales of the baskets.
Graph A:



Graph B shows **profits made for hand woven baskets** in our trainee's association.
X-axis shows numbers of baskets sold. Y-axis shows profits(x 1 000Frw) for selling those baskets.
Graph B:



a. Identify which graph shows a liner function and which shows a quadratic function.

b. We know that for each basket woven by our association, it is sold for 5 000 Frw, which is what we see in Graph B. This explains why the y intercept is at the origin (0, 0)—if no basket is woven, no profit is made. In Graph A, however, at a factory, the y intercept is at (0, -50) which means that if no baskets are made, the factory LOSES money.

i. Give a reason why a factory will lose money if they do not make baskets.

ii. How many baskets must the factory make to break even—not lose money?

iii. How many baskets must the factory make to get a profit of 50 000 Frw?

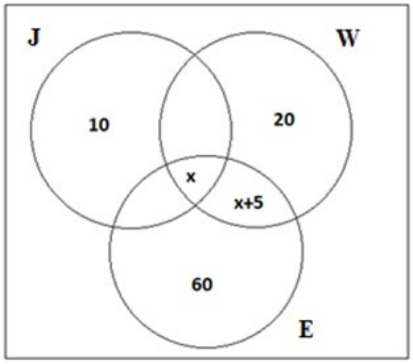
iv. How many baskets must the association sell to get a profit of 50 000 Frw?

Version for the trainers (with answers):

Read the following linked situations carefully and answer the questions carefully. Show your work.

1. At the end of level 2 courses, a group of trainees, boys and girls, decide to start a handicraft association. They decide to start with making Woven baskets (W), Jewelry (J) and Embellishments for parties (E). All of these products are made from natural raw materials that can be found and grown in Rwanda such as sisal fibres, banana fibre, and herbs.

The Venn diagram below shows the number of trainees who make Woven baskets (W), Jewelry (J) and Embellishments (E). Some make more than one product. In total, 55 trainees make Woven baskets.



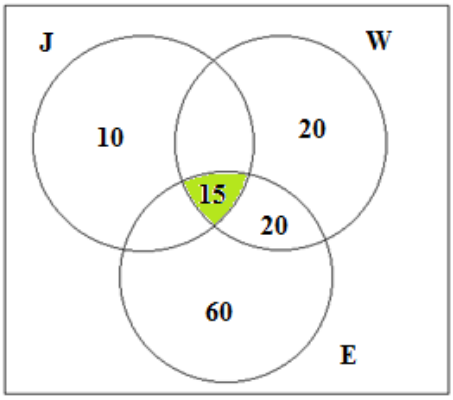
Answer the following questions regarding the scenario described above:

a. How many trainees like to make the following products:
i) Woven Baskets? ii) Jewelry? iii) Embellishments? iv) all three

Answer:
Number of trainees who like to make Woven baskets is 55= 20 + x + (x + 5) so X =15
Using the Venn diagram shows that trainees who like to make the three categories are the following:

i. Woven Baskets = **55** (this number was given to us)
ii. Jewelry = 10 + X = 10 + 15 = **25**
iii. Embellishments = 60 + X + (X+5) = 60 +15 + (15 + 5) = 75 + 20 = **95**
iv. All three is the intersection of all three sets which is **15** trainees.

b. Find the total number of trainees in the group that started this handcraft association.
Answer:
The total number of all trainees who started the association is 125 as show in the Venn diagram below is 10 + 20 + 15 + 20 + 60 = **125** trainees.



c. How many trainees like to make jewelry only?
Answer:
10 trainees like to make jewelry only per the Venn diagram above.

2. After one week, their products were ready to compete with other products from different associations on the market. The association decided to employ a part-time seller.

The seller will be paid 8% of commission for each sold product.

The table below shows the number of produced products, the number sold, and the unit price:

Produced Products	**Sold products**	**Unit price**
120 Woven baskets	70 woven baskets	5000 Frw
250 Jewelry	200 Jewels	600 Frw
100 Embellishments	50 Embellishments	4 000 Frw

After paying the commission to the seller, the remaining money was saved in a bank at the simple interest of 18% a year.

Answer the following questions regarding the information above:

c. In the first month, the seller hired by the association sold 50 Embellishments, 200 Jewelry pieces, and 70 Woven baskets.
- **i.** How much commission (at 8% of total sales) was earned by the seller?
- **ii.** How much money did the association make as profit this month?

Answer:
The money remaining and the commission are shown in the table below:

Produced Products	Sold products	Unit price	Money in Frw after selling	Commission in Frw
120 Woven baskets	70 woven baskets	5 000 Frw	70 x 5 000 Frw = 350 000	8% x 350 000 = 28 000
250 Jewels	200 Jewels	600 Frw	200 x 6 000 = 120000	8% x 120 000 = 9 600
100 Embellishments	50 Embellishments	4000 Frw	50 x 4 000 = 200 000	8% x 200 000 = 160 000
		Total:	670 000	53 600
Remaining money (profit) after paying commission is 670 000 Frw – 53 600 Frw = 616 400				

Frw

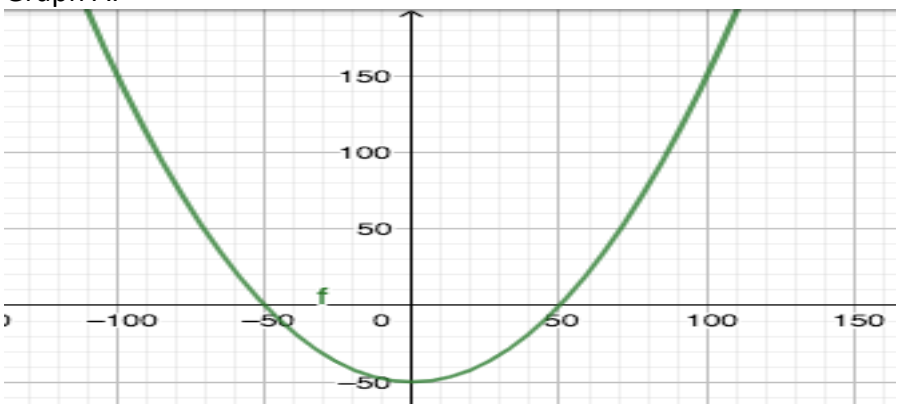
So the seller was paid **53 600 Frw as her commission**.
The **money remaining (profit) was 616 400 Frw** which will be deposited in a savings account in a bank.

d. After paying the commission to the seller, the remaining money was saved in a bank account at the simple interest of 18% a year. Calculate the interest to be given to the association after 2 years.

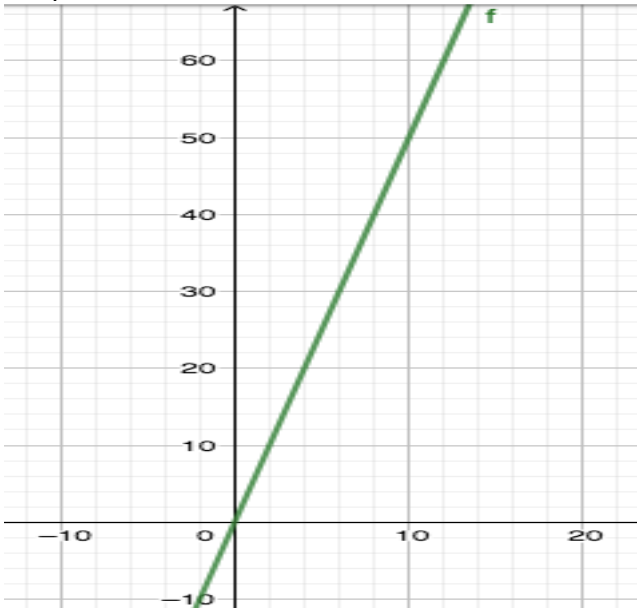
Answer:
The formula for simple interest is I = R x P x T.
Then the interest after 2 years is (616 400 Frw X 0.18 X 2) = **221 904 Frw**

3. Observe the following two graphs:

Graph A shows **profits for baskets made in a factory** by a machine.
X-axis shows numbers of baskets sold. Y-axis shows profits (x 1 000Frw) for sales of the baskets.
Graph A:



Graph B shows **profits made for hand woven baskets** in our trainee's association.
X-axis shows numbers of baskets sold. Y-axis shows profits(x 1 000Frw) for selling those baskets.
Graph B:



a. Identify which graph shows a liner function and which shows a quadratic function.

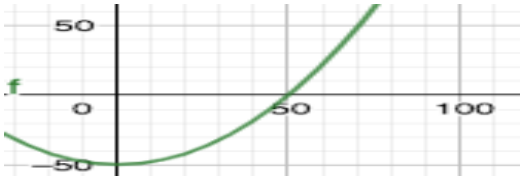
Answer: Graph A shows a quadratic function (a curve) while graph B is a liner function (a line).

b. We know that for each basket woven by our association, it is sold for 5 000 Frw, which is what we see in Graph B. This explains why the y intercept is at the origin (0, 0)—if no basket is woven, no profit is made. In Graph A, however, at a factory, the y intercept is at (0, -50) which means that if no baskets are made, the factory LOSES money.

i. Give a reason why a factory will lose money if they do not make baskets.
Answer: The factory loses money if nothing is produced because they had to buy machines to make the product so until that cost is made up they will lose money. Also, factories must pay their workers, electricity and other services like insurance even when the machines stop.

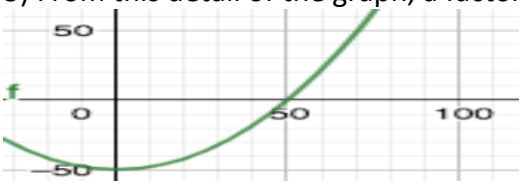
ii. How many baskets must the factory make to break even—not lose money?
Answer: The factory must sell 50 baskets to break even (make no profit –but not lose money).

iii. How many baskets must the factory make to get a profit of 50 000 Frw?
Answer:
From this detail of the graph, a factory must sell 70 baskets to make a profit of 50 000 Frw.



iv. How many baskets must the association sell to get a profit of 50 000 Frw?
Answer:
From the linear graph we see that the association must sell only 10 baskets to make a profit of 50 000 Frw.

f) (i) Graph A shows a quadratic function (a curve) while graph B is a liner function (a line).
(ii) 1) The factory loses money if nothing is produced because they had to buy machines to make the product so until that cost is made up they will lose money. Also, factories must pay their workers, electricity and other services like insurance even when the machines stop.
2) The factory must sell 50 baskets to break even (make no profit –but not lose money).
3) From this detail of the graph, a factory must sell 70 baskets to make a profit of 50 000 Frw.



4) From the linear graph we see that the association must sell only 10 baskets to make a profit of 50 000 Frw.

Self-Reflection

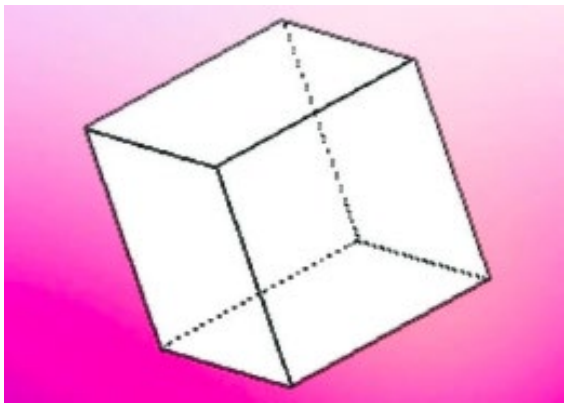
1. Take few minutes and ask trainees to re-take the self-assessment they took at the beginning of the Learning Unit 1. Trainees will fill in the table with an X to mark where they feel they have understood the concepts now that they have completed during the unit of study.

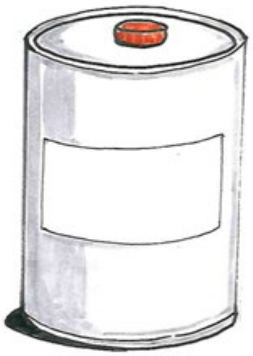
2. Explain that this self-reflection is to help trainees identify areas of strength and areas for improvement as well as actions they can take to improve their learning for the next unit. After completing the table trainees should complete the brief table below to focus their intentions for the next unit of study and to guide review of this unit.

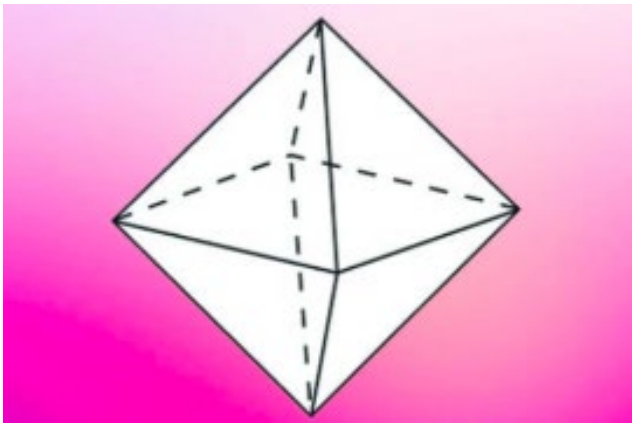
Areas of strength	Areas for improvement	Actions to be taken to improve
1. **2.**	**1.** **2.**	**1.** **2.**

3. Discuss individual results with each trainee privately. Identify the areas that are giving many trainees difficulties and plan to provide additional mini-lessons for extra support those topics that trainees found difficult. (For example, trainers can use class time before you begin the next learning outcome to go through commonly identified difficult concepts).

Learning Unit 2: Geometric shapes and angle properties







Learning Outcomes

By the end of the Learning Unit, trainees will be able to:

2.1 Sketch shapes and angles to illustrate relevant relationships and properties

2.2 Draw regular and irregular shapes as well as angles to represent properties of the given figures

2.3 Measure lengths and calculate areas of shapes using formulas, theorems, and Thales theorem

2.4 Construct mathematical arguments using the angle properties of parallel lines Transformation properties

2.5 Calculate, using given formulas, the surface area and volume of common geometrical solids

Learning Unit 2: Self-Assessment

1. Ask trainees to look at the illustration at the beginning of the unit in their Trainee Manual. Choose a trainee to describe the illustration in detail. Allow other trainees to share observations too. Based on these observations of the illustrations, ask trainees to guess the topics that will be included in this unit. After some brainstorming, share the learning outcomes and objectives.

2. Explain that the purpose of the self-assessment is to become familiar with the topics in the unit and think about what they already know (or do not know) at the beginning so the trainees can build on what they already know and focus their learning. At the end of the unit, they will do the same self-reflection again, to review their progress and identify areas of strength and areas that need improvement as well as to think of actions to take to improve their learning overall. The self-assessment is not a test!

3. Ask trainees to fill out the self-assessment found at the beginning of the unit in their Trainee Manuals. This should take 2 or 3 minutes.

Learning Outcome 2.1: Sketch shapes and angles to illustrate relevant relationships and properties

Objectives: By the end of the learning outcome, trainees will be able to:

a. Draw two parallel lines with a transversal, name all equal, vertically opposite, corresponding, alternate, and supplementary angles

b. Sketch triangle, square, rectangle, and circle based on their properties

Time Required: 3 hours

Learning Methodologies: pair work, group work, discussion

Materials Needed:

- Standard materials, trainee workbook, calculators if available
- Scissors and geometric instruments: ruler, protractor, compass, T-square

Preparation:

❑ Draw different shapes in different colours (triangles, rectangles, squares, circles).

Cross Cutting Issues:

✔ **Gender:** trainer will provide equal opportunities to both boys and girls during the lesson as well as balance gender when forming groups

✔ **Peace/value education:** trainer promotes team work and respect for others' views and work

✔ **Inclusive education:** trainer is sensitive to differences in learning abilities and uses a variety of activities and materials to cater to different learning styles

Prerequisites: Trainees should already be comfortable with and able to:

- Describe properties of obtuse, acute, right, and supplementary angles
- Explain basic properties of 2-D figures (circle, triangle, square, rectangle)

Key Competencies:

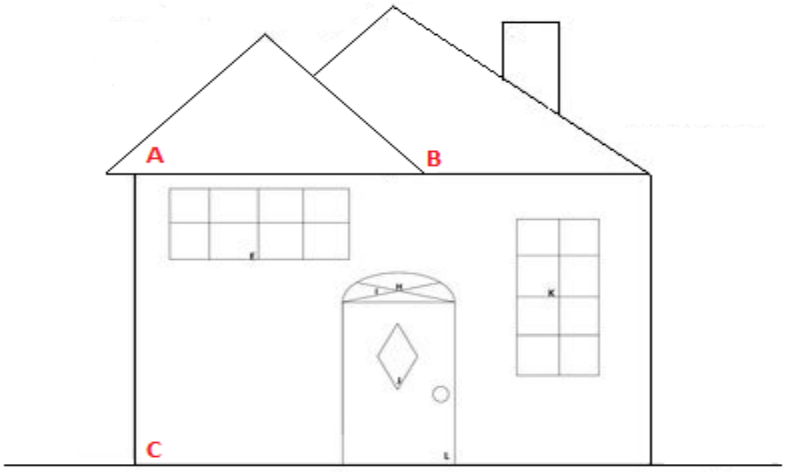
Knowledge	Skills	Attitudes
1. Classify shapes and angles based on their properties (triangle, square, rectangle, circle)	**1.** Sketch 2-d shapes, measure sides and angles, and calculate area and perimeter	**1.** Persistent
2. Based on parallel line properties, identify equal angles, vertically opposite angles, corresponding angles, alternate, and supplementary angles	**2.** Analyse properties of parallel lines and the angles	**2.** Attentive to details

Steps:

Getting Started: What do we know and where are we going?

1. Look at the picture in **Topic 2.1 Task 1** in Trainee Manuals. In pairs, trainees should write down as many responses as they can. Circulate in the class to help. When completed trainees should share their answers aloud with the class.

Topic 2.1 Task 1



a. Observe the picture carefully. List out all the shapes you see in the picture.
b. Look at the different angles formed in the picture: Identify examples of:

i. right angles (90 degrees—the intersecting lines are perpendicular)

ii. acute angles (less than 90 degrees)
iii. obtuse angles (greater than 90 degrees)
iv. supplementary angles (angles that add up to be 180 degrees—or a straight line)

c. Label angles A, B and C as either right, acute, or obtuse.
d. Explain the use of the following geometry tools: Protractor, Ruler, T-square, Compass

Answer:

a. You can find squares, rectangles, triangles, circle, rhombus, trapezoid, etc.

b.

i. The corners of the doors and windows all form right angles. The top of the chimney has right angles, where the wall meets the ground a right angle is formed.
ii. The diamond on the door has acute angles at the top and bottom, the outside edges of the roof are acute angles.
iii. The diamond on the door has obtuse angles on the two sides.
iv. Angle B and the angle just to the left of it are supplementary, Angle C and the angle to the left of it are supplementary.

c. Angle A, less than 900, is an acute angle. Angle B, greater than 900, is an obtuse angle. Angle C is equal to 900, it is a right angle.

d. Protractors are used to measure angles, Rulers are used to draw line segments and to measure length, T-square is used to draw right angles, and compass is used to draw circles.

2. Tell trainees this has been a review of basic geometry then review the objectives for the LO.

Problem Solving Activity

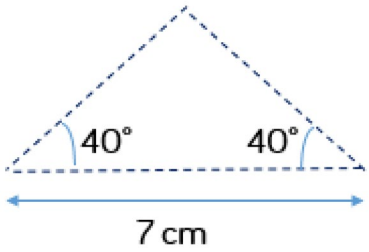
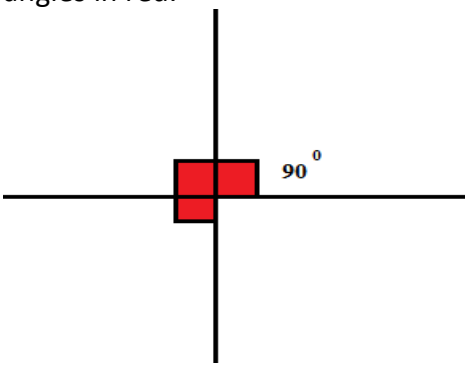
1. Organize trainees into groups of 3 trainees. Each trainee will get one of the three "Talk and Task" questions to respond to. Trainees work individually on their own task for 5 minutes then each trainee will explain their talk and task to others in their group.

Peers will support and correct. Encourage trainees to use their own words.

2. After groups have shared within the group, read correct answers to the class.

Topic 2.1 Task 2
Each trainee works on one talk-and-task then shares answers within their group:

Talk and Task A:	
Talk A: What do you know about the square and rectangle? How are they the same? How are they different? What is a parallelogram?	Task A: On a piece of paper, accurately draw: A square with perimeter 16 cm A rectangle with an area of 20 cm^2

Talk and Task B	
Talk B: What do you know about triangle properties? What is an isosceles triangle? What is a right triangle? What is an equilateral triangle?	Task B: Aloys has been asked to re-draw the isosceles triangle below on a plain sheet of paper using only a protractor and a ruler. Create a step by step plan to show how to do this.  Now draw a right-angled triangle with a height of 8 cm and a base of 6 cm.
Talk and Task C	
Talk C: What are perpendicular lines? What are parallel lines? What angle is formed when two lines have a perpendicular intersection? What is the radius of a circle? What is the diameter of a circle? What is the circumference of a circle?	Task C: Observe the picture and find out the sum of the angles in red.  Now draw a circle with radius 2 cm.

Answer:

Talk-and-Task A:

Talk answer:

Square, rectangle, and parallelograms are all quadrilaterals –figures with 4 sides and 4 angles.
In a square all sides are equal—in rectangle and parallelogram the pairs of opposite sides are equal.
Squares and rectangles have all right angles—parallelograms have equal angles in opposite corners.

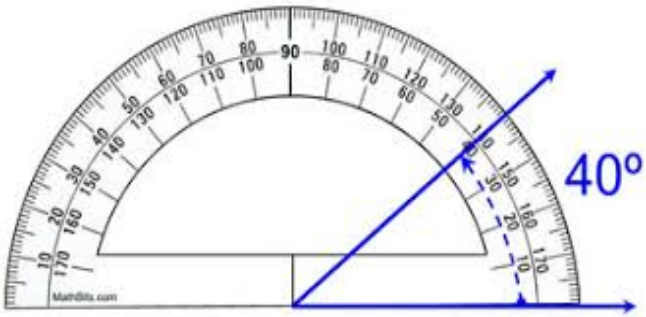
Task answer:

If a square has a perimeter of 16cm then each side is 16/4=4cm. To draw it, use a T-square to get a right angle. Measure 4cm in each of the two directions. Where the segments end, use the T-square again to get another right angle, again measure 4cm, which will be a square.
If a rectangle has an area of 20 cm^2 then we know Length x Width = Area so l x w = 20 so we may have a rectangle with (l=20 and w=1) or (l=10 and w=2) or (l=5 and w=4) draw as for the square but measured at the given l and w.

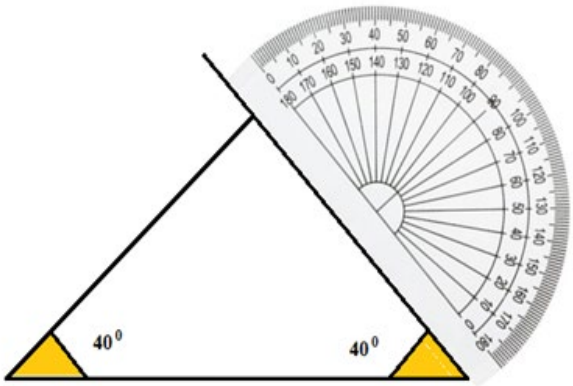
Talk-and-Task B:

Talk answer:
Triangle is a shape with 3 sides and 3 angles. The angles inside a triangle add up to 180 degrees.
An Isosceles triangle is a triangle with 2 equal sides and 2 equal angles.
An equilateral triangle is a triangle with 3 equal sides and 3 equal angles.
A right triangle is a triangle that has one right angle.

Task answer:
Given the base and adjacent angles, one can use a protractor to draw an isosceles triangle through the following steps.
Step 1- Draw a base 7 cm long, and then measure the two angles adjacent to the base with your protractor and put a mark at the point equal to 40^0. Draw a line through that point. Do the same thing on the other end of your 7cm base. Where the two lines intersect creates the apex of the triangle. The resulting figure is an isosceles triangle.[10]



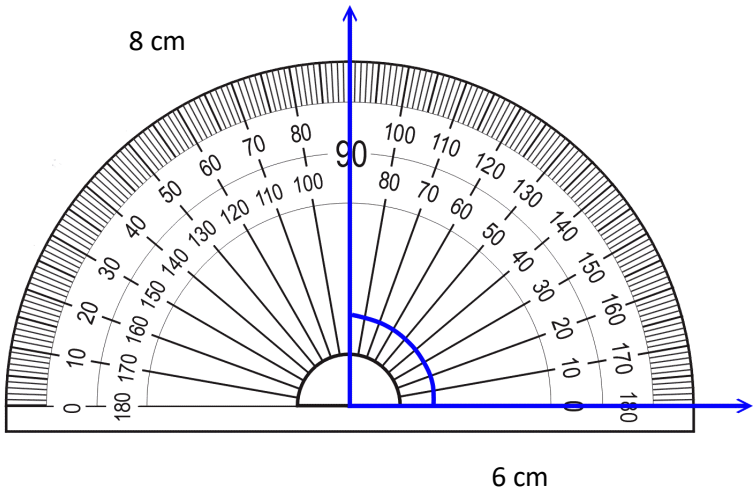
7 cm



7 cm

To draw the right triangle, start by drawing the base which is 6cm long.
Use the T-square to make a right angle and draw that segment (the height) to 8cm long.
Connect the ends of your two segments and you will have the required right triangle.

[10] Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher's Manual and Learner's Book*. Waltham: Education Development Center) Education Development Center.



Talk-and-Task C:
Talk answer:
Perpendicular lines are lines that intersect forming right angles. Four 90-degree angles are formed when perpendicular lines intersect.
Parallel lines will never intersect. The two lines will always be the same distance apart.
A circle is a shape with no side and no angle. It has only circumference (distance around the outside), radius (distance from centre to the circle itself), diameter (twice the radius—the length of a line reaching from one side through the centre to the other side of the circle).
Task answer:
The drawing shows perpendicular lines. Given that we know all angles are 90 degrees. Thus, there are 90 degrees in one right angle, two right angles will be 180 degrees (they are supplementary angles) and if we add together three right angles, we will get 270 degrees.

3. When groups have completed the exercise and sharing of answers, read aloud the correct answers so trainees can correct their own work.

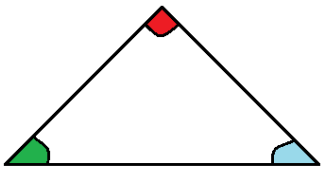
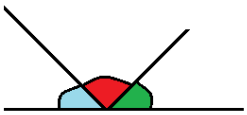
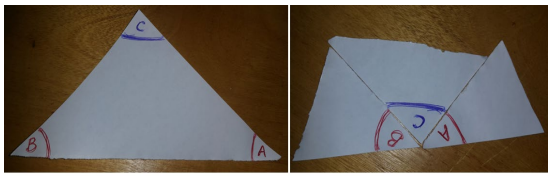
4. Choose a trainee to read aloud **2.1 Key Facts**. Respond to any questions posed.

5. After reading the key facts above, ask a trainee to explain the note at the bottom. Review the definition of regular polygon, then explain why each of the shapes is or is not regular.

Guided Practice Activity

1. To get started, trainer will read aloud the directions for Task 3 while trainees complete the task individually.

Topic 2.1 Task 3
Use a ruler to draw a triangle on the page of their notebook. Carefully cut out the triangle. THEN rip off the corners (which are called vertices). Place all vertices together so angles are adjacent (next to one another) but not overlapping as shown as is shown in the figures below:

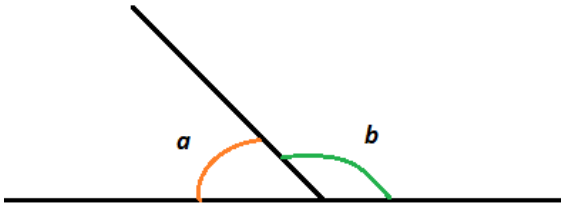




a. What do you observe?
b. Notice that each trainee made a slightly different triangle, have they found the same thing?
c. What does this tell us about ALL triangles?

Answer:
a. The three angles together make a straight line—which is 180^0.
b. All trainees' angles, form straight lines, although each triangle was different.
c. This tells us that when we add the three angles inside any triangle, we will get 180^0.

2. For Tasks 4, 5, and 6, trainees will work in pairs, supporting one another. When trainees have completed the three tasks, chose volunteers to write answers on the board. Make sure trainees explain the thinking and the methods used, not just the answers.

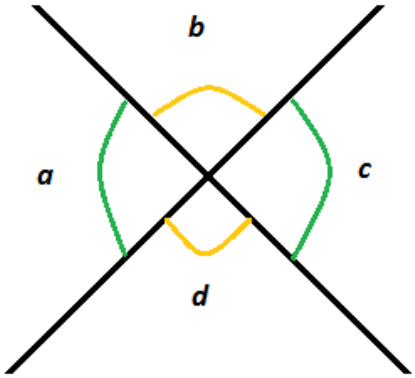
Topic 2.1 Task 4
Observe the following diagram carefully.



a. We can see that angles a and b, when taken together, form a straight line. What do we call two such angles?
b. What is the angle measure of a + b?
c. What angle will we get if we take 180^0 - a?

Answer:
a. Two angles that form a straight line when added together are called supplementary angles.
b. The angle measure of a + b is 180^0.
c. If we take 180^0 - a we will get angle b.

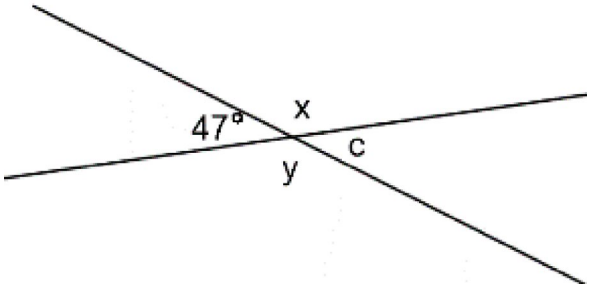
Topic 2.1 Task 5
Observe the following figure and respond to the questions below.



a. What do you notice about angles b and d?
b. What do you notice about angles a and c?
c. Is this always the case?
d. What do you notice about angles a and b?

Answer:
a. Angles b and d are equal.
b. Angles b and d are equal.
c. Yes, opposite angles are equal.
d. Angles a and b are supplementary.

Topic 2.1 Task 6
Observe the following figure and respond to the questions below.



a. Which angle is opposite to angle x?
b. Which angles are supplementary to angle x?
c. Find the angle measures for angle c, angle x, and angle y.

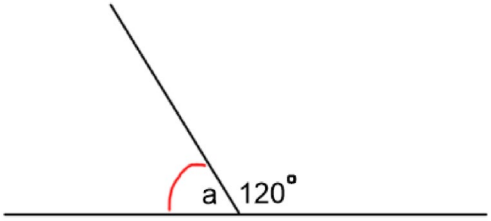
Answer:
a. Angle y is opposite to angle x
b. Angle C and the angle with measure 47^0 are both supplementary to angle x.
c. Angle C is opposite to angle 47^0, they are equal angles so angle C = 47^0.
d. 47^0 + angle x =180^0 because the two are supplementary. x = 180 – 47 = 133^0.
e. The angle x is equal to y because they are opposite angles so angle y = 133^0.

Application Activity

1. Trainees will work together. Observe the figures, and identify properties that will help answer the questions. Walk around the class. Provide additional examples, give more explanations, encourage trainees, and check answers and as needed.

Topic 2.1 Task 7

Observe the following figure carefully and find the measure of angle a.



a. What do we call angle a and angle 120 together?

b. Find the angle measure of angle a.

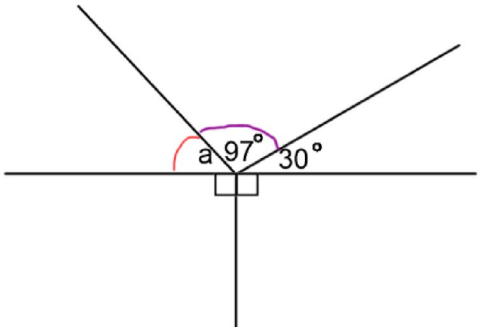
Answer:

c. Angle a and angle120 are supplementary angles. Together they make a straight line or 180^0.

d. The two angles are supplementary so the sum of the angles a and 120^0 is 180^0. The angle a = 180^0 - 120^0 Thus, the measure of angle a is equal to 60^0

Topic 2.1 Task 8

Observe the following figure carefully and find the measure of angle a.



Answer:

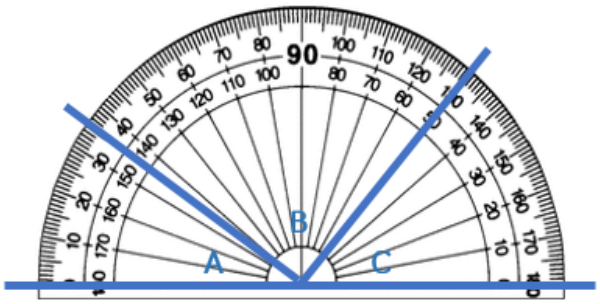
The sum of the three angles: a, 97^0, and 30^0 will be 180^0 because the three together make a straight line (which we see because the two angles on the bottom are supplementary (they are two right angles so 90 + 90 = 180).

If a + 97 + 30 = 180 then, a + 127 = 180 then a = 180 – 127 so the measure of angle a = 53^0 .

Points to Remember

- Two lines are **parallel** if they have no point of intersection.
- When two lines intersect at a point, four angles are formed.
- Two lines are **perpendicular** if their intersection forms right angles.
- To bisect an angle means divide the angle into two equal (congruent) parts.
- When measuring angles, make sure that:
  - The centre of the protractor is placed on the vertex of the angle.

- The zero of the protractor is placed on one side of the angle.



[11]

Formative Assessment

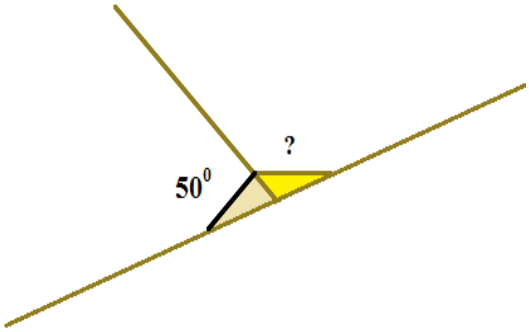
1. Explain that trainees will work individually. This formative assessment is designed so trainees can check their own learning and understandings

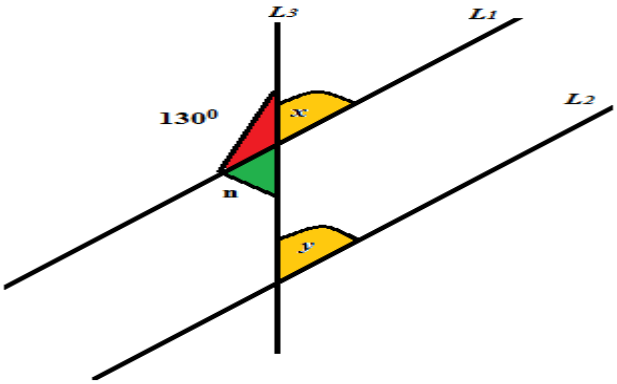
Formative Assessment

1.

Observe the figures, explain how the missing angle can be calculated, and find the angle measure for the following figures:

a) b)





b)

Answer:

a) The missing angle and 50^0 are supplementary angles so their sum is 180^0.
To calculate the missing angle: 180^{0v}- 50^0 = 130^0

b) The missing "x" angle and 130^0 are supplementary angles so their sum is 180^0. To calculate the value of x we take 180^0 - 130^0 = 50^0. Angle x = 50^0.
Angles x and n are opposite angles, so their measures are the same (x = n = 50^0).
The value of angles x and y are both equal to 50^0 because L_3 is an intersecting line of both

[11] Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher's Manual and Learner's Book*. Waltham: Education Development Center) Education Development Center.

Given that l_1 and l_2 are parallel, they make the same angles with the line that intersects them. We can see that angles x and y are at the same position of the lines l_1 and l_2 intersections so they must be equal. Thus $y = 50^0$.

2.
List quadrilaterals you know and their properties. Can you describe how properties can help you to sketch square, parallelogram, trapezium, rectangle or any other quadrilateral?

Answer:
Answers will vary—refer to **2.1 Key Facts** for details.

ⓘ Further Information for the Trainer

http://www.mrfischer.ca/geometry-and-spatial-sense/angles-and-shapes/

https://www.khanacademy.org/math/basic-geo/basic-geometry-shapes/basic-geo-quadrilaterals/v/quadrilateral-types-exercise

http://strader.cehd.tamu.edu/geometry/bisectangle1.0/bisectangle.html

Learning Outcome 2.2: Draw regular and irregular shapes as well as angles to represent properties of the given figures

Objectives: By the end of the learning outcome, trainees will be able to:

a. Calculate perimeter & area of a regular and irregular polygons
b. Investigate the sum of interior angles of regular polygons (equilateral triangle, square, pentagon, and hexagon, etc.)

Time Required: 4 hours

Learning Methodologies: pair and group work, investigations, discussions

Materials Needed: Standard materials, Trainee Manual, calculators if available
Ruler

Preparation:

- ❑ Prepare extra examples of perimeter and area problems to support trainees.

Cross Cutting Issues:

- ✔ **Gender:** trainer will provide equal opportunities to both boys and girls during the lesson as well as balance gender when forming groups
- ✔ **Peace/value education:** trainer promotes team work and respect for others' views and work
- ✔ **Inclusive education:** trainer is sensitive to differences in learning abilities and uses a variety of activities and materials to cater to different learning styles

Prerequisites: Trainees should already be comfortable with and able to:

- Use knowledge of different types of angles (obtuse, acute, right angle, supplementary angles)
- Use knowledge of the sum of angles in a triangle and polygon, as well as other basic properties of triangles, squares, rectangles.

Key Competencies:

Knowledge	Skills	Attitudes
1. State the properties of polygons	**1.** Draw regular and irregular polygons based on their properties and measures	**1.** Attentive to detail
2. Explain how to find the sum of interior angles of a regular polygon	**2.** Calculate the sum of interior angles of any regular polygon	**2.** Innovative
3. Explain how to find perimeter and area of regular and irregular shapes	**3.** Determine perimeter and area of a regular and irregular shapes	**3.** Accurate

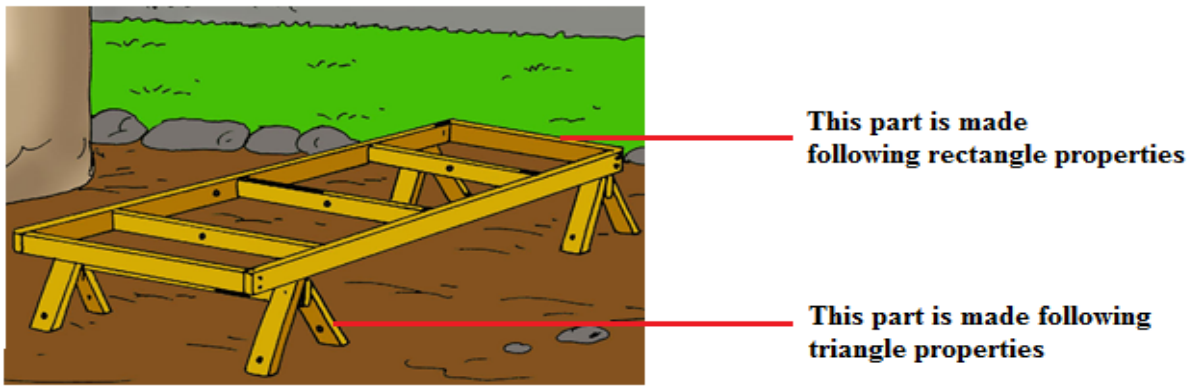
Steps:

Getting Started: What do we know and where are we going?

1. Ask trainees to think back on the work that was done during the first Learning Outcome of this Unit. They can work in small groups on the following two tasks as a review.

Topic 2.2 Task 1

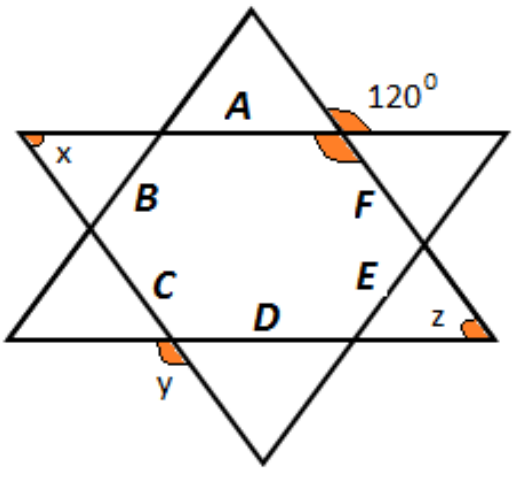
Observe the picture below. Name the angles and shapes used by a carpenter to make this table:



Possible answer: table top is made up of rectangles with right angles. The table legs form triangles with acute angles (angle less than 90 degrees) at the top.

Topic 2.2 Task 2

Observe the following illustration. Notice that the star is made up of three different sets of parallel lines.



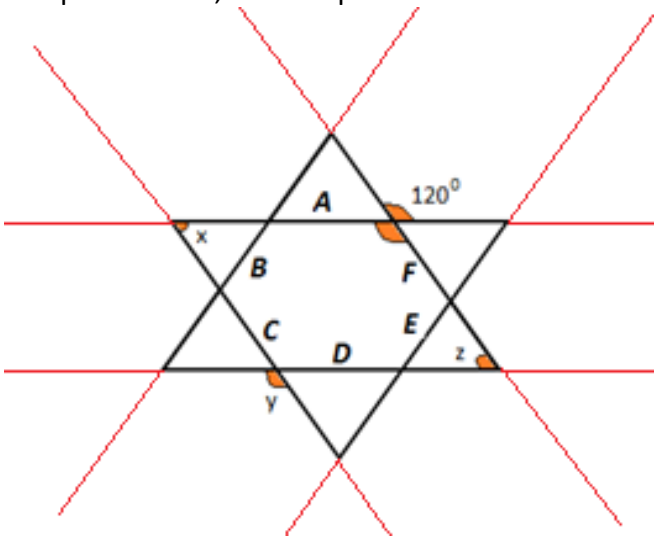
a. Extend the sides of the star to show better which pairs of lines are parallel.

b. Look carefully at the figure and see if you can find a parallelogram inside this figure. Outline that parallelogram with a thicker line.

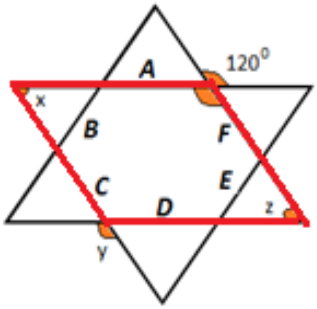
c. Now that we have several sets of parallel lines and also a parallelogram, and the measure of one angle, find the angle measures for the angles x, y, and z. Be sure to explain your logic.

Answer:

a. A is parallel to D, B is parallel to E, and C is parallel to F.



b.



c. Inside the parallelogram, the angle x and z are equal because they are opposite one another.
The top right angle of the parallelogram and the angle libelled 120^0 are equal because they are opposite angles where the two lines intersect.
Line A and Line D are parallel so the top right angle of the parallelogram and angle z are supplementary. 180 – 120 = angle z so angle z = 60^0.
Recall angles x and z are equal so angle x =60^0.

Now let's look at angle y. The lower left angle of the parallelogram and the top right angle of the parallelogram are equal because they are opposite one another. Further, angle y is equal to the lower left angle of the parallelogram because of the intersecting lines. Therefore, the lower left angle of the parallelogram and angle y = 120^0

2. Let groups have time to work out both problems. Walk around the class and observer who is doing the tasks well and helping trainees that are struggling.

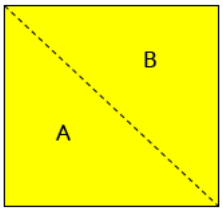
3. Brainstorm with the class a list of answers for Task 1. Choose the trainees who did Task 2 well to demonstrate their thinking, methods and answers on the board for the class.

Problem Solving Activity

1. Ask trainees to read the following task. They will work individually, then pair up to share and check responses. After peer correcting of answers, a volunteer to presents answer to the class.

Topic 2.2, Task 3
Review the activity in the last Learning Outcome: you drew a triangle and cut it out then placed the vertices together, finding there are 180 degrees total for the angles inside a triangle.
Keep that information in mind: The angles inside a triangle add to a total of 180 degrees.

Draw a square. Split the square into 2 triangles with a diagonal drawn between opposite vertices per the picture:



a. What do the angles of triangle A add up to?
b. What do the angles of triangle B add up to?
c. Given that the angles inside the square equal the sum of the angles from the two triangles, make a guess what the angles inside a square add up to.
d. Think about the properties of a square: we know a square has 4 right angles. Add up the total of 4 right angles to confirm your answer from (c).

Answer:

a. 180 degrees
b. 180 degrees
c. one triangle 180 + the other triangle 180 = 360
d. Each right angle is 90 degrees. 90 x 4 corners = 360 so we have confirmed our first answer.

In summary, we took a regular polygon (a shape with equal lengths of sides and equal angles) and we divided the shape into triangles. We know that the interior angles of a triangle add up to 180 and we can then use that to figure out the interior angles of regular polygons.

Topic 2.2 Task 4
Think about patterns made by dividing polygons into triangles.
We can use such a pattern to figure out the interior angle sum for any regular polygon.

a. Review the first two rows of the following table and then, complete the missing information by filling in the remaining information on the last three rows in the table below.

Shape	Number of sides	Number of triangles	180 × number of triangles	Sum of internal angles
 Square	4	2	180 × 2	360°
 Pentagon	5	3	180 × 3	540°
 Hexagon	**6**			
 Heptagon	**7**			
n sides Polygon	**n sides**			

b. How can we use this to work out the interior angles of regular polygons? Can we spot a pattern in the table? What predictions can we make?

Answer:

a.

Shape	Number of sides	Number of triangles	180 × number of triangles	Sum of internal angles
Square	4	2	180 × 2	360°
Pentagon	5	3	180 × 3	540°
Hexagon	**6**	**4**	**180 x 4**	**720**
Heptagon	**7**	**5**	**180 x 5**	**900**
n sides Polygon	**n sides**	**n-2**	**180 x (n-2)**	**180 x (n-2)**

b. A regular polygon can be divided into equal triangles. The number of triangles in a regular polygon multiplied by 180^0 can always give us the sum of interior angle of a regular polygon. The number of triangles in any regular polygon is found by subtracting 2 from the number of sides.

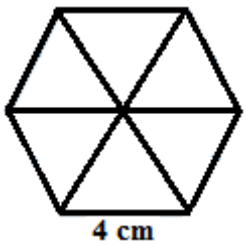
Guided Practice Activity

1. Trainees have explored how to find the measure of the sum of interior angles in a regular polygon. Explain that we will also want to find the measure of the perimeter, and the area. There are known formulae to find area of common shapes. For less common shapes we can often divide the figures into shapes we know then add up to find the total area.

2. Ask a volunteer trainee to read aloud **2.2 Key Facts** and respond to any questions posed.

Topic 2.2 Task 5

Observe the hexagon below. Each side measures 4 cm. Calculate the perimeter of the hexagon.

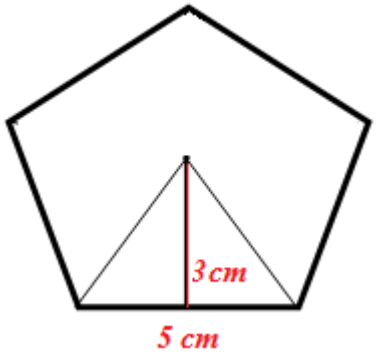


Answer:

A hexagon has 6 sides and each side measures 4 cm thus the perimeter is 6 x 4 cm = 24 cm.

Topic 2.2 Task 6

Observe the following regular polygon. Find the perimeter and calculate the area if the side is 5 cm and the height of the given triangle is 3 cm.

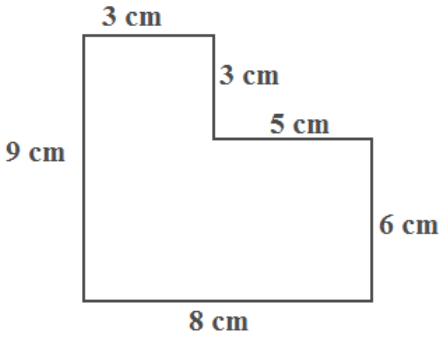


Answer:
The hexagon has 5 sides which measure 5cm. The perimeter is 5 x 5cm = 25cm.

To find the area we divide the figure into 5 equal triangles.
First, find the area of each triangle: base = 5 height = 3 so area = ½ x 5 x 3 = 7.5 cm^2.
Given that we have 5 triangles we find the area of our hexagon as 7.5cm^2 x 5 = 37.5 cm^2.

Topic 2.2 Task 7
Observe the irregular polygon below.

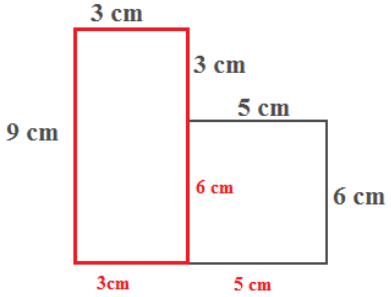
- **a.** How many sides does the figure have?
- **b.** Find the perimeter.
- **c.** Divide the polygon into already known shapes and calculate the area.



Answer:

- **a.** The polygon has 6 sides.
- **b.** The perimeter is the sum of lengths of all sides: 9 + 3 + 3 + 5 + 6 + 8 = 34cm
- **c.** To find the area, divide the polygon into two rectangles.

  The areas of the two rectangles are (9 x 3) cm^2 + (6 x 5) cm^2 which is 57 cm^2
  Note: there are many ways to divide up the polygon into rectangles. All give the same area.

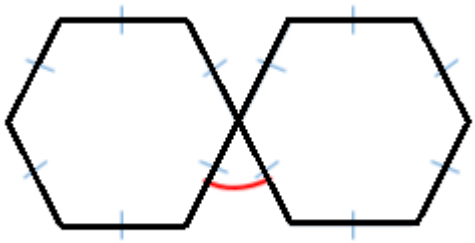


Topic 2.2 Task 8
Observe the two regular hexagons below. Each hexagon has 6 sides (n=6) and each side is 5 cm.

We know that the sum of the interior angles of a hexagon is 720°. (n-2) x 180 = 4 x 180 = 720^0.

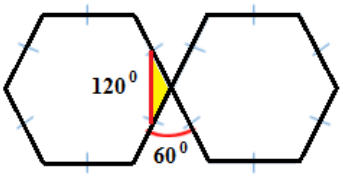
a. Find the missing angle marked between the two hexagons below.

b. Find the perimeter and area of the hexagon if each of the interior triangles has height of 3 cm.



Answer:

a. We know sum of interior angles of a hexagon is 720 degrees.
There are 6 interior angles in a hexagon. So, each interior angle measures 720^0 / 6 = 120^0.
As shown by the figure below, the missing angle and 120^0 are supplementary (make a straight line) thus the sum of the two angles is 180^0 Thus the missing angle 180^0 – 120^0 = 60^0.



b. The perimeter is 5 cm x 6 which is equal to 30 cm.
There are 6 interior triangles in a hexagon so area is the area of each triangle x 6.
Each internal triangle has an area of ½ x base x height = ½ x 5 x 3 = 7.5 cm^2.
6 x 7.5 = 45 cm^2.

Application Activity

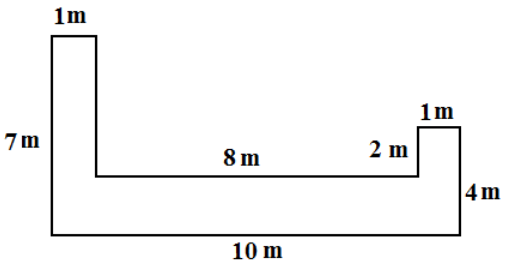
1. Ask trainees to work in pairs and use knowledge from guided practice activities to solve the tasks in the Application Section. Provide support to trainees who have questions or struggle.

Topic 2.2 Task 9

A plot of land has an irregular form with shape and dimensions shown below.

a. The farmer wants to put up a fence area. How many meters of fencing is needed?

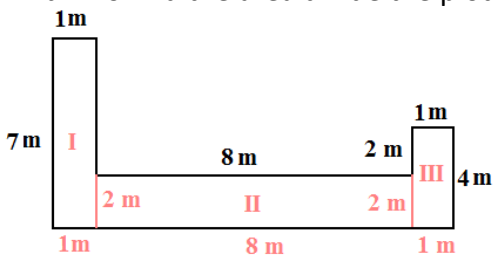
b. The farmer knows that sheep should each have 3m2 of pasture to graze well. How many sheep can the farmer put inside this enclosed land if they are to eat well?



Answer:

a. To find the length of fence needed we calculate the perimeter (the sum of all sides) as follows: Perimeter: 7 m + 1 m + 8 m + 2 m + 1 m + 4 m + 10 m = 33 m
The farmer will need 33 m of fencing to enclose this plot of land.

b. To find the area divide the plot into three rectangles as follows:



Area of rectangle I: 7 m × 1 m = 7 m^2
Area of rectangle II: 8 m × 2 m = 16 m^2
Area of rectangle III: 4 m × 1 m = 4 m^2
Total area of the polygon: 7 m^2 + 16 m^2 + 4 m^2 = 27 m^2
The field is 27m^2. Each sheep needs 3m^2 so 27 m^2/3=9 sheep. 9 sheep can graze on this field.

Topic 2.2 Task 10

1. A farmer has a plot of land in the form of a parallelogram whose base is 120 m. He needs to fence it. What is the length of the fencing he will need to surround his plot if the slanting side is 70 m long?

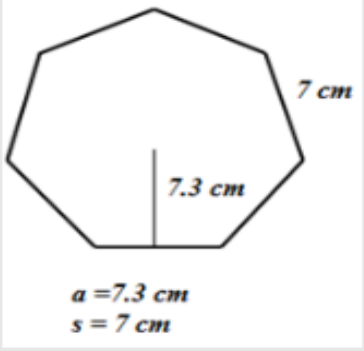
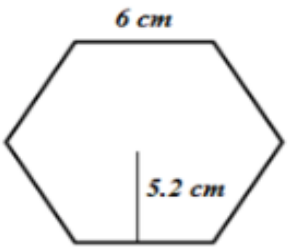
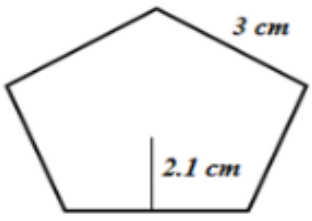
Answer:

a. The length of the fencing needed is 120 m + 120 m + 70 m + 70 m = 380 m

Topic 2.2 Task 11

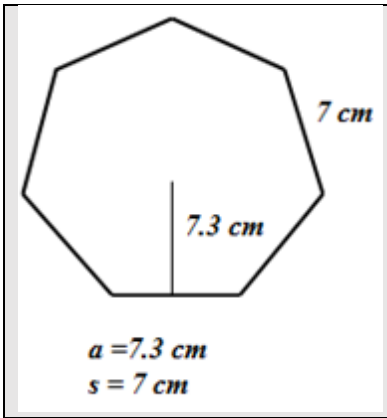
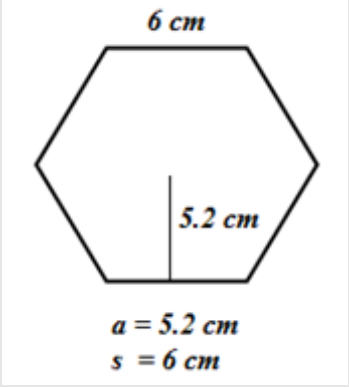
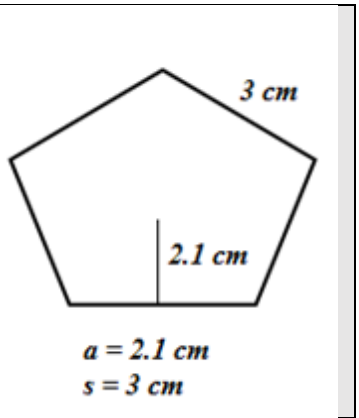
1. Ask trainees to look at the images of the three polygons. Ask what they think "s" and "a" are. After a few responses explain that "s" is side length and "a" is altitude which is the distance from the mid-point of one side to the centre of the figure. "s" and "a" are similar to "b" and "h" for base and height in triangles discussed earlier.

Calculate the area and perimeter for each polygon

Heptagon	Hexagon	Pentagon
		

Answer:

Heptagon	Hexagon	Pentagon

		
Area $A = n\left(\frac{1}{2}s.\mathrm{a}\right)$ $A = 7\left(\frac{1}{2}7x7.3\right)cm^2 = 178.85cm^2$	Area $A = n\left(\frac{1}{2}s.\mathrm{a}\right)$ $A = 6\left(\frac{1}{2}6x5.2\right)cm^2 = 93.6cm^2$	Area $A = n\left(\frac{1}{2}s.\mathrm{a}\right)$ $A = 5\left(\frac{1}{2}3x2.1\right)cm^2 = 15.75cm^2$

Points to Remember

Regular polygons are often defined by the number of sides that they have, these can be divided into triangles that all meet at the centre point.

To find **perimeter** of a shape we add up the lengths of all the sides

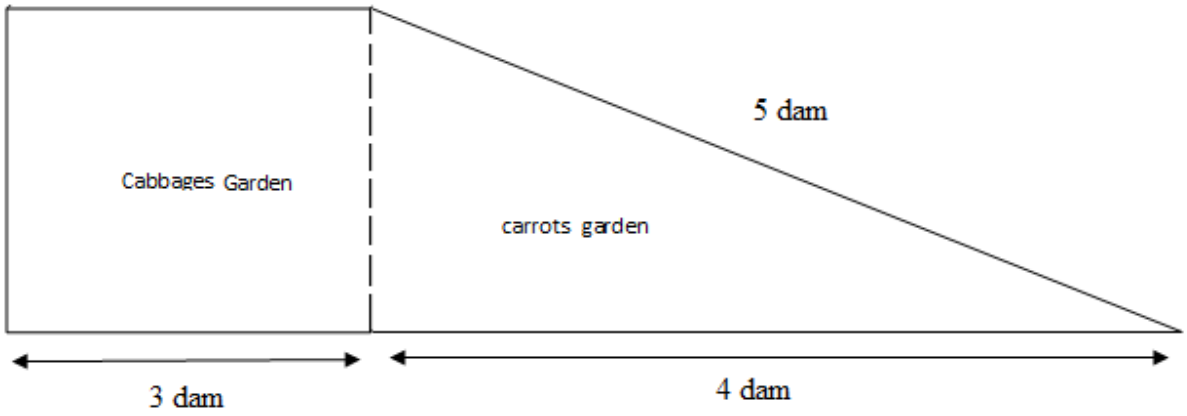
To find the **area** of a shape we divide it into shapes that we know (mostly into parallelograms where Area = base x height and triangles where Area = ½ x base x height

Formative Assessment

1. Ask trainees to work in pairs. They will work together and discuss the processes used as well as the answers so each trainee can discover how well they have understood the lesson.

2. After all trainees have found responses in pairs, ask some volunteers to share their work, including explanations. If trainees do not understand well, provide more examples and go over the concepts of the lesson again during the next class.

A vegetable garden is divided into 2 parts. A squared part of cabbages and a triangular part of carrots as shown below:[12]



a. What is the perimeter of the cabbage garden and that of the carrot garden?
b. What kind of shape do those two sections of a vegetable garden make? Calculate the perimeter of that shape.
c. Calculate the area of the entire garden area.

Answers:

a. Cabbage garden is a square of 3 dam a side, the perimeter of the cabbages garden is 3 dam x 4=12 dam
Carrot garden is a triangle of 4 dam, 5 dam and other side is 3 dam (the side of the square). The perimeter is 3 dam + 4 dam + 5 dam= 12 dam
b. The two sections of the vegetable garden make a trapezium of 7 dam, 5 dam, 3 dam , 3 dam and the perimeter is 7 dam+ 5 dam+ 3 dam + 3 dam=18 dam
c. The area can be found by calculating the area for each section of the garden and adding them together.
Area of a square = s^2. Area of cabbage garden = $(3 \text{ dam})^2 = 9 \text{ dam}^2$.
Area of triangle = ½ bh. Area of carrot garden = ½ (4 dam)(3 dam) = ½ (12 dam^2) = 6 dam^2.
So area of cabbage garden + area of carrot garden = $9 \text{ dam}^2 + 6 \text{ dam}^2 = 15 \text{ dam}^2$.

[12] Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher's Manual and Learner's Book*. Waltham: Education Development Center) Education Development Center.

ⓘ Further Information for the Trainer

Read more at:

https://www.skillsyouneed.com/num/polygons.html

https://www.edplace.com/blog/what-are-regular-and-irregular-shapes

Learning Outcome 2.3: Calculate lengths and areas of similar shapes using their properties (Thales theorem)

Objectives: By the end of the learning outcome, trainees will be able to:

a. Solve problems involving midpoint theorem
b. Calculate lengths of proportional segments in triangles (and trapeziums)
c. Calculate areas of similar shapes using their properties (Thales theorem)

Time Required: 4 hours

Learning Methodologies: pair and small group work, discussions

Materials Needed:

- Standard materials, Trainee Manual, calculators if available and geometric instruments (ruler, T-square, protractor)

Preparation:

- ❑ Prepare additional examples to provide trainees with additional practice should it be needed.

Cross Cutting Issues:

- ✔ **Gender:** trainer will provide equal opportunities to both boys and girls during the lesson as well as balance gender when forming groups
- ✔ **Peace/value education:** trainer promotes team work and respect for others' views and work
- ✔ **Inclusive education:** trainer is sensitive to differences in learning abilities and uses a variety of activities and materials to cater to different learning styles

Prerequisites: Trainees should already be comfortable with and able to:

- Use knowledge of different types of angles (obtuse, acute, right angle, supplementary angles)
- Use knowledge of the sum of angles in a triangle and polygon, as well as other basic properties of triangles, squares, rectangles.

Key Competencies:

Knowledge	Skills	Attitudes
1. Explain the Midpoint Theorem	**1.** Solve problems involving Midpoint Theorem	**1.** Collaborative
2. Explain Thales Theorem and its converse	**2.** Solve problems involving Thales Theorem	**2.** Accurate
3. Identify proportional properties of given figures	**3.** Calculate lengths of proportional sides of different figures	**3.** Focused on Detail

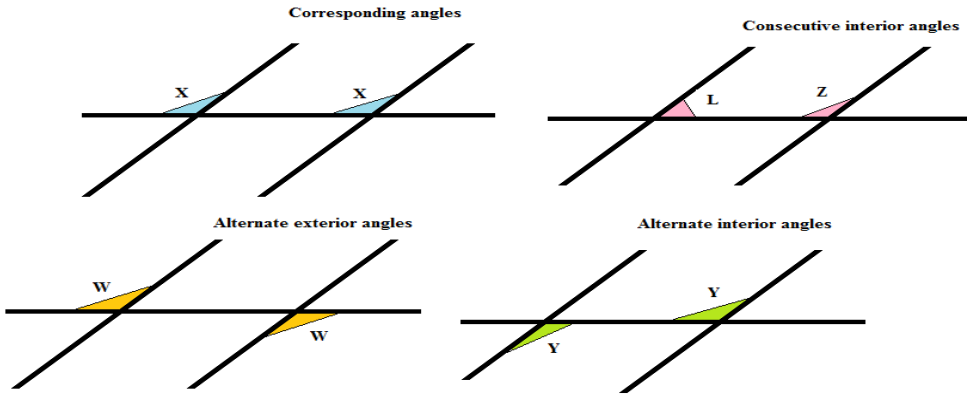
Steps:

Getting Started: What do we know and where are we going?

1. As a warm-up exercise, ask trainees to explain and give examples of the following angles as they relate to a pair of parallel lines cut by a transversal. Trainees can come to the board to draw as they explain their thinking in words:

Topic 2.3 Task 1
Describe and draw an example of each of the following angles:
a) corresponding angles
b) consecutive interior angles
c) alternate exterior angles
d) alternate interior angles

Answer:



Topic 2.3 Task 2
Think about the pairs of angles that are formed when two parallel lines are cut by a transversal. These were named and described in Task 1.
Explain which of the pairs of angles are equal and which are supplementary.

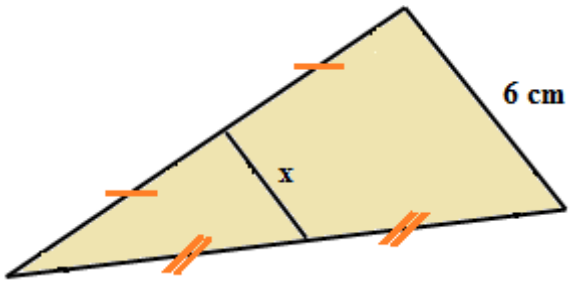
Answer:
The pair of corresponding angles is equal.
The pair of alternate interior angles is equal.
The pair of alternate exterior angles is equal.
Two consecutive interior angles are supplementary.

2. Facilitate trainees to use the local language so they can help one another deepen the meaning for tricky concepts. As trainees explain and draw, they should also explain exactly WHY they know that the angles are equal.

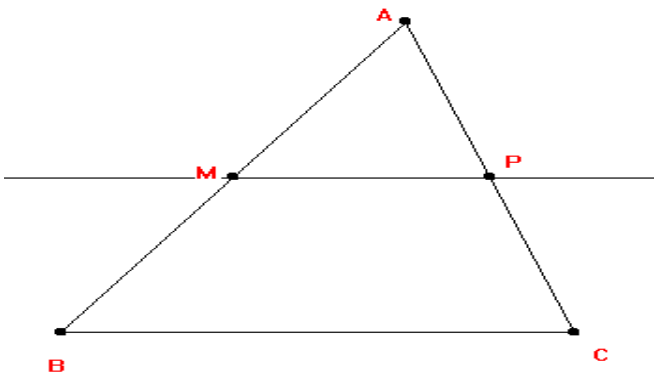
3. Explain that as when we work with triangles and other polygons, we often form parallel lines so we can use the properties of these angles to figure out other angles.

4. Tell trainees that an important theorem related to parallel lines that we will be using during this Learning Outcome is called the **Midpoint Theorem.** Ask trainees to read silently the theorem at the beginning of Task 3.

Topic 2.3 Task 3
Midpoint Theorem:
The segment joining the mid-points of two sides of a triangle is parallel to the third side and that segment is half the length of the third side.
This is illustrated in the following diagram. Segment X bisects the two sides, and thus is parallel to the base side (which measures 6 cm) and is half as long as the base side of the bigger triangle.
Segment X = ½ x 6 cm = 3 cm.



Observe the triangle drawn below while keeping Midpoint Theorem in mind.



If the line through M and P bisects the two sides it intersects, and if Segment BC is 12 m long, tell two facts that we know about segment MP.

Answer:

From the Midpoint Theorem, we know that MP is parallel to BC and we know that MP is half as long as BC so given that BC = 12m we know that MP = 6 m

Problem Solving Activity

1. Pair up trainees and ask them to attempt problem solving activity in Trainee Manual.

2. While trainees work, guide the pairs by identifying errors and providing additional details. Allow trainees to discuss and explain their thinking as well as the methods that they use.

3. After all trainees have completed the activity, let a volunteer present their thinking, methods, and solution to the class.

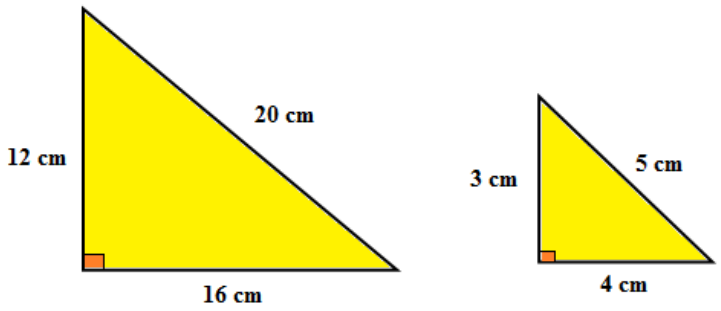
Topic 2.3 Task 4
Carefully observe the two triangles given below.

a. Using a ruler and a protractor, redraw the two right triangles in your notebook.

b. State the pattern that you see when you compare the measures of their sides.

c. Use a protractor to measure the angles of the two triangles.

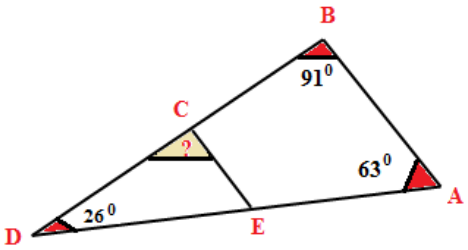
d. What do you notice about the measures of the angles of these two triangles?



Answer:

a. Drawn triangles.
b. Comparing the measures of the sides of the two triangles, observe the small triangle is ¼ of the bigger one. The pattern is shown by 12 cm = 4 x 3cm, 20 cm = 4 x 5 cm, 16 cm = 4 x 4 cm.
c. Measure the angles.
d. Both triangles have a right angle and the other angles have the same measure.
e. The angles of the two triangles are all the same but the lengths of the sides are different.

Topic 2.3 Task 5
The figure below shows a large triangle with vertices A, B and D, and a small triangle with vertices C, D, and E. Point C is the mid-point of BD and point E is the mid-point of AD.



The two triangles in the figure above are similar triangles.

Observe the figure carefully and complete the following sentences.
Knowing that Angle D corresponds to angle D in both triangles, which angle corresponds to:

- **a.** Angle E?
- **b.** Angle C?
- **c.** What is the angle measure for the marked angle at C?
- **d.** If segment DE measures 8 cm, what is the measure of segment EA?
- **e.** If segment DB measures 15 cm, find the length of segment DC.

Answer:

- **a.** Angle E corresponds to angle A and both are equal to 630
- **b.** Angle C corresponds to angle B and both are equal to 910
- **c.** Angle C measures 91 degrees
- **d.** DE=EA= 8cm
- **e.** DC is half of DB, so DC = ½ (15 cm) = 7.5 cm

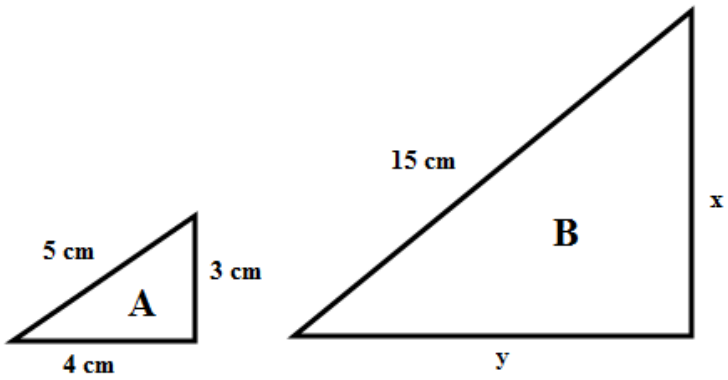
Guided Practice Activity

1. Ask trainees to read **2.3 Key Facts** silently. Provide examples for the class for each of the points. Respond to any questions.

2. Draw the following triangles on the board. Tell trainees that the two triangles are similar. Ask them to look for the "scale factor" and once they have found the scale factor then calculate the lengths for sides x and y.

Topic 2.3 Task 6
Observe the two similar right triangles.

- **a.** Find the scale factor and then the value of X and Y.
- **b.** The perimeter and area of each triangle.



The scale factor is 15:5 = 3

Answer:

a. The scale factor is 3 cm.
5 cm x 3 = 15 cm, 4 cm x 3 = 12 cm, 3 cm x 3 = 9 cm therefore, x = 9 cm and y = 12 cm

b. For triangle A: perimeter is 5+3+4 = 12 cm, the area is ½ (b x h) = ½ x 4 x 3 = 6 cm^2
For triangle B: perimeter is 15+9+12 = 36 cm and the area is ½ (b x h) = ½ x 9 x 12 = 54 cm^2

3. Give trainees time to think and to ask questions before moving on to the next example. **NOTE:** it is important to note that because area is a SQUARED quantity, we must square the 'scale factor' before we multiply it with the smaller area to get the bigger area.

So, the scale factor is 3 in the triangles above. 3^2=9. The area of the small triangle is 6 cm^2. The area of the large triangle is 9 x 6 cm^2= 54 cm^2.

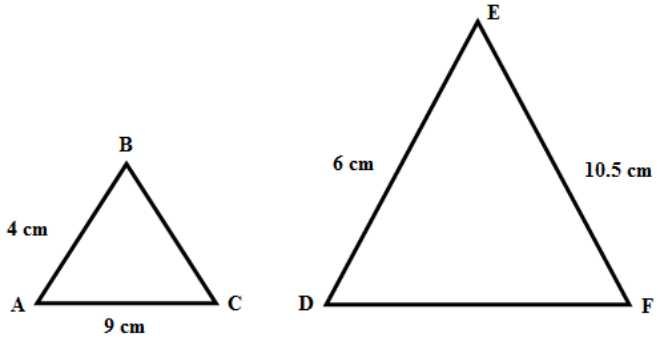
4. Keeping in mind that we must square the scaling factor to find similar areas, ask trainees to attempt Task 7. Once all have attempted it, trainer should explain the thinking and write/draw answer on the board.

Topic 2.3 Task 7

Triangles ABC and DEF are similar.

a. Work out the lengths of DF and BC and give the perimeter of both triangles.

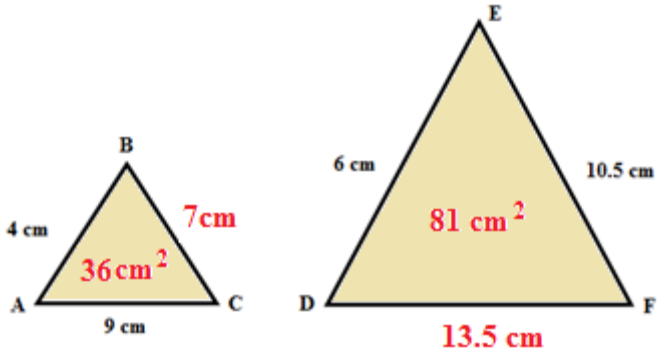
b. If the area of the small triangle is 36 cm^2, what is the area of the bigger triangle?



Answer:

a. Compare the lengths of sides of the two triangles to find that the scale factor is 6 / 4 which is simplified to 3/2= 1.5
Thus DF = 1.5 x 9 cm = 13.5 cm and BC = 10.5 cm/1.5 = 7 cm
Perimeter of the small triangle is 4+7+9=20 cm
Perimeter for the big triangle is 6+10.5+13.5=30 cm

b. For AREA the scale factor should be squared. Therefore, the AREA scale factor is $(3/2)^2$ = 9/4=2.25. Or $(1.5)^2$ = 2.25.
The area of the big triangle is area for the small triangle x squared scale factor so we will get:
36 cm^2 x 2.25 = 81 cm^2 as the area for the bigger triangle.

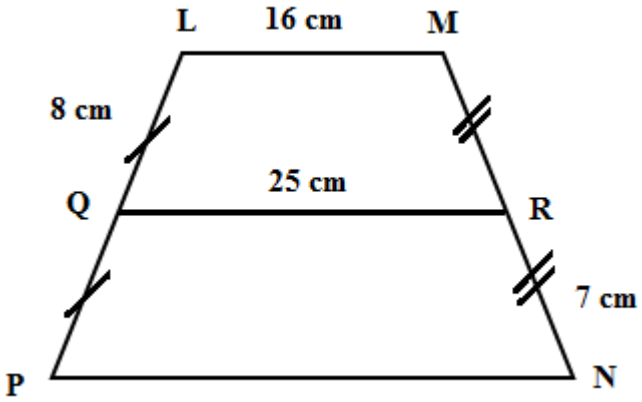


Application Activity

1. Trainees should now work independently on the tasks to consolidate their knowledge and skills. Provide support when needed.

Topic 2.3 Task 8

Given that the length of the mid-segment QR = ½ (LM+PN), find the perimeter of trapezium LMNP.



Answer:

QP=8 because it is equal to QL. MR=7 because it is equal to RN

We are given QR = ½ (LM+PN) Fill in what we know: 25= ½ (16+PN)

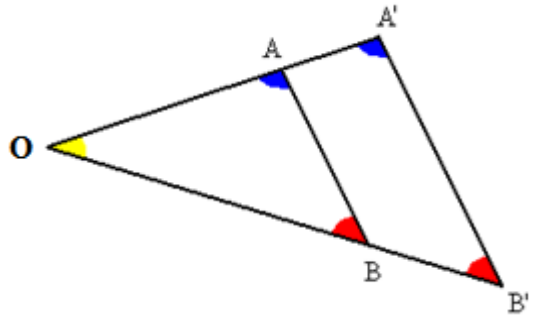
Simplify to find that 50=16 + PN thus PN=34

To calculate the perimeter of trapezium LMNP, sum all sides: 34cm+16cm+16cm+14cm = 80cm

Points to Remember

- The midpoint theorem states that the segment joining two sides of a triangle at the midpoints of those sides is parallel to the third side and is half the length of the third side.
- Two triangles are similar when they have equal angles and proportional sides.
- Thales Theorem: When two triangles have a common angle and they have parallel opposite sides, they are in **Thales position**. Then they are similar triangles and have proportional sides.

- In this example, the two triangles share angle O, the parallel opposite sides are AB and A'B' so the sides of triangle OAB are proportional to the sides of triangle OA'B'.



Formative Assessment

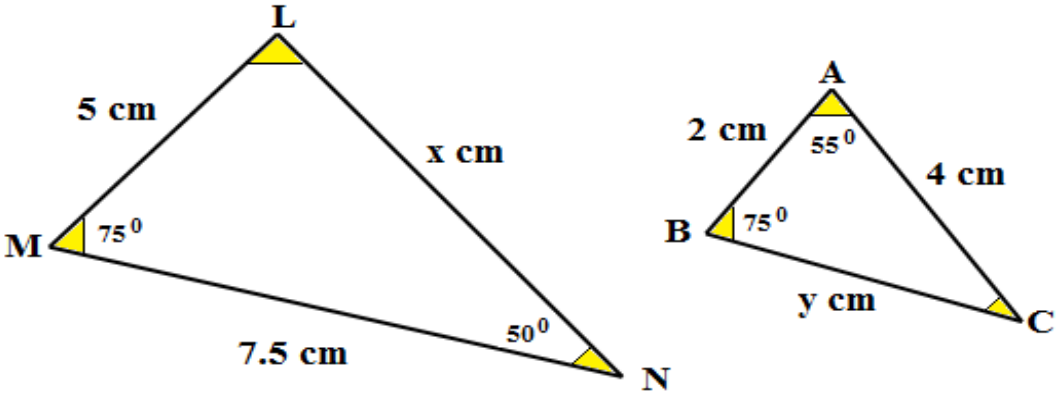
1. Give to trainees the following task from the Trainee Manual. Explain that the formative assessment is to help them check their own understanding of the concepts introduced above.

The triangle ABC is similar to triangle LMN. Find the:

a. values of x and y

b. perimeter of the 2 triangles

c. missing angle in each of the triangles.

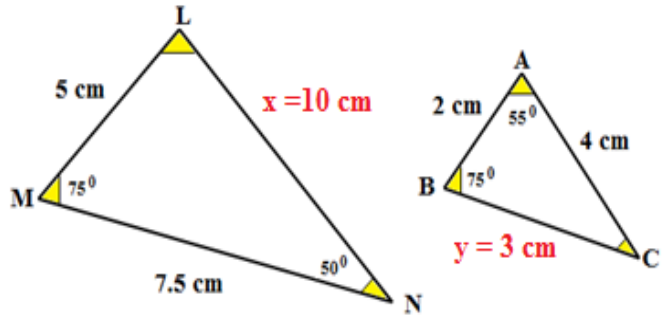


Answer:

a. The scale factor for the two similar triangles is 5/2.
To find x, multiply the length of the corresponding side of the smaller triangle by 5/2:
4 cm x 5/ 2 = 10 cm so x= 10 cm.
To find y, divide the length of the corresponding side of the bigger triangle by 5/2:
7.5 cm / (5/ 2) = 3 cm. So y = 3cm.

b. The perimeter of the bigger triangle is 5+10+7.5=22.5cm
The perimeter of the smaller triangle is 2+4+3 = 9 cm

c. Angle L is equal to angle A so the measure of angle A=55^0.
Angle C is equal to angle N so the measure of angle C=50^0.



2. When trainees have completed the given activities ask volunteers to share their responses.

3. After trainees have self-corrected their work, ask them to circle or draw ☹, 😐, or ☺ at the top of their page to indicate how well they feel they have understood the lesson. If there are many ☹, go over the concepts of the lesson again during the next class.

ⓘ Further Information for the Trainer

- https://flashman.neocities.org/mdptthrm.html
- https://www.siyavula.com/read/maths/grade-10/euclidean-geometry-part-1/07-euclidean-geometry-03

Learning Outcome 2.4: Construct mathematical arguments using the angle properties of parallel lines

Objectives: By the end of the learning outcome, trainees will be able to:

a. Draw two parallel lines and a transversal, identify all angles that are equal, vertically opposite, corresponding, alternate, supplementary angles and memorize these terms

b. Solve missing angle problems, giving reasons for each step in the process

Time Required: 4 hours

Learning Methodologies: individual, pair, and group work, discussion and presentation

Materials Needed:

- Standard materials, Trainee Manual, calculators if available, and geometric instruments like ruler, T-square, protractor

Preparation:

- ❑ Prepare different parallel lines and intersecting line on manila papers or illustrate different types of angles in different colours.

Cross Cutting Issues:

- ✔ **Gender:** trainer will provide equal opportunities to both boys and girls during the lesson as well as balance gender when forming groups
- ✔ **Peace/value education:** trainer promotes team work and respect for others' views and work
- ✔ **Inclusive education:** trainer is sensitive to differences in learning abilities and uses a variety of activities and materials to cater to different learning styles

Prerequisites:

- Knowledge on different types of angles (obtuse, acute, right angle, supplementary angles…) and how to measure them using a protractor
- Knowledge of sides and angle properties of quadrilaterals and parallel lines

Key Competencies:

Knowledge	Skills	Attitudes
1. Explain how to draw parallel and transversal lines	**1.** Draw parallel lines and transversals	**1.** Persistent
2. Describe properties of angles and of parallel lines	**2.** Analyse, compare and construct mathematical arguments related to different angles	**2.** Accurate
3. Provide logical arguments using properties of angles, shapes, and parallel lines	**3.** Solve missing angle problems including providing reasons for each step	**3.** Cooperative

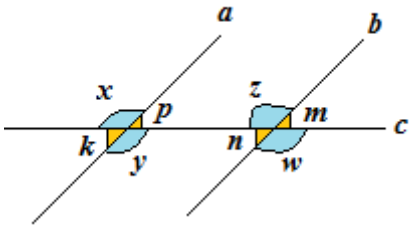
Steps:

Getting Started: What do we know and where are we going?

1. As a warm up exercise, ask trainees to re-draw the figure into their notebooks. They will then use a protractor to measure (and label) all the angles before answering the associated questions.

Topic 2.4 Task 1

Copy the following figure into your notebooks. Then use a protractor to measure each of the angles. Record the angle measures on your drawing.



a. Based on the dimensions of the measured angles, classify all angles as either acute or obtuse.
b. Find four pairs of opposite angles.
c. Find eight pairs of supplementary angles.

Answer:

a. The angles p and k, n and m are acute (less than 90^0), (they are also opposite and equal in the pairs given).

Angles x and y, z and w are obtuse (more than 90^0) (in the given pairs, these angles are also opposite angles and equal.)

b. Pairs of opposite angles are the following: x and y, k and p, z and w, and n and m.

c. Pairs of supplementary angles are the following: k and x, x and p, p and y and y and k and also n and z, z and m, m and w, and w and n.

Problem Solving Activity

1. Divide trainees into groups of four. Within each group, ask trainees to split into pairs. Each pair will work together on one of the two tasks (A or B) once the task is complete each pair will teach the other pair how to solve the other task. As groups are working, trainer should guide students with hints and relevant information on an individual and pair level.

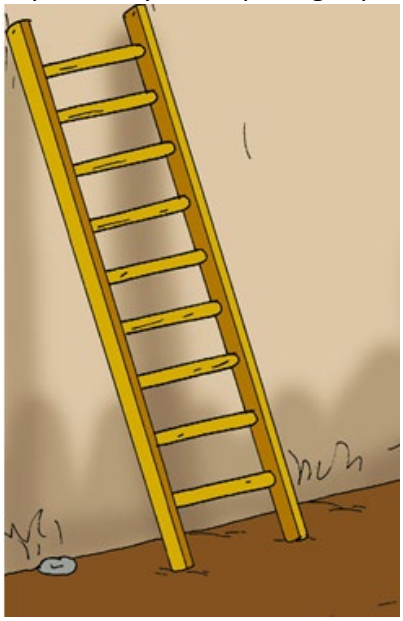
2. When groups have completed both tasks, choose two trainees to present A and B to the class.

Topic 2.4 Task 2

There are two tasks below. Each pair will work on one task then teach the other pair in your group how to solve it. Review the pictures and the given information carefully. Respond to the question asked (find the missing angles)—being sure to provide reasons for each step.

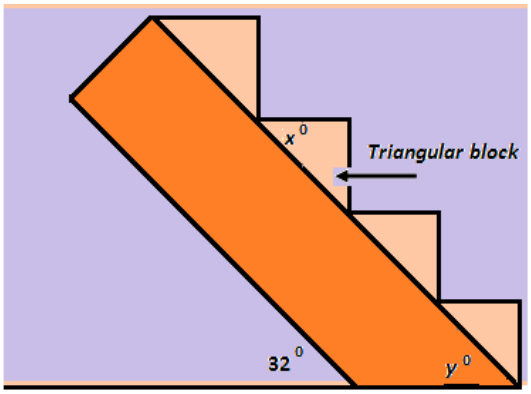
A.

Observe the picture below carefully. Each rung of the latter is parallel to the rung directly above it. Explain why the top rung is parallel to the bottom rung.



B.

One way to build stairs is to attach triangular blocks to an angled support, as shown in the picture. The sides of the angled support are parallel. If the support makes a 32° angle with the floor, what must angle x ^{0} be so that the top of the step will be parallel to the floor? Explain your reasoning.



Answer:

A:

By the definition of parallel, the bottom rung is the same distance from the rung above it on each side of the ladder. That is the distance on one side to the next rung is the same distance as on the other side of the rung. This is true for the next rung up too. And the next rung up too. And so forth. Given this, the distance from the bottom distance on one side of the ladder will be the same distance as on the other side of the ladder—which is to say that, if extended onto infinity the top and bottom rung will never touch—thus they are parallel.

B:

The sides of the angled support are parallel thus angles 32^0 and Y^0 are corresponding angles on the same side of the parallel lines so we know they are equal. Thus, angle $y=32^0$. Next, extend the horizontal line of the triangular block above angle x to see that the line and the floor are parallel lines which are intersected by the angled support. X and Y are on opposite sides of the angled support, and are inside the parallel lines so they are opposite interior angles, thus equal. Given this we know angle X has the same measures as angle Y and thus both measure $32^{0.}$

Guided Practice Activity

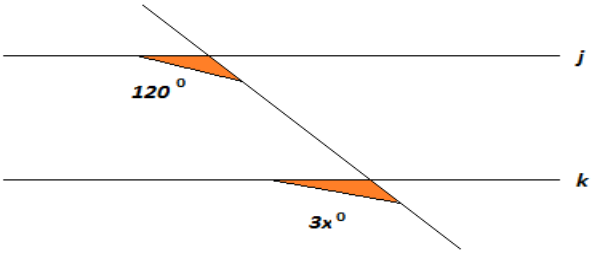
1. Ask trainees to open their manuals and read **2.4 Key Facts** silently.

2. The following problems build on the key facts. Draw the given figures on the board and brainstorm ideas from trainees on how to proceed with each task. List what we know, name the angles, and identify what we are trying to find. Then work the problems for the class – model the thinking, give the reason used at each step, and solve the question.

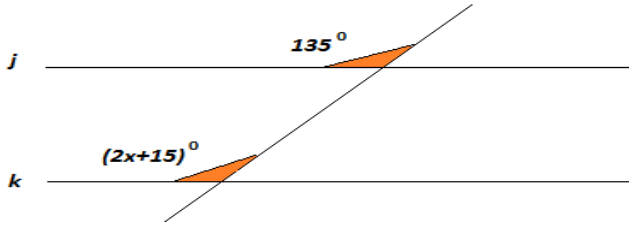
Topic 2.4 Task 3

Observe the angles made by the parallel and transversal lines. Using the information given, identify all the angles and their measures by finding the value for x.

a.



b.



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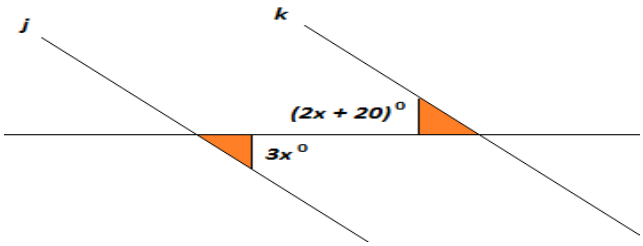
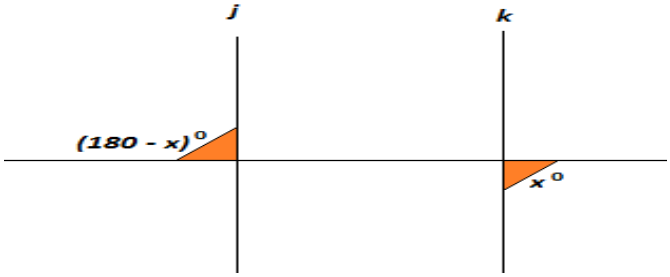
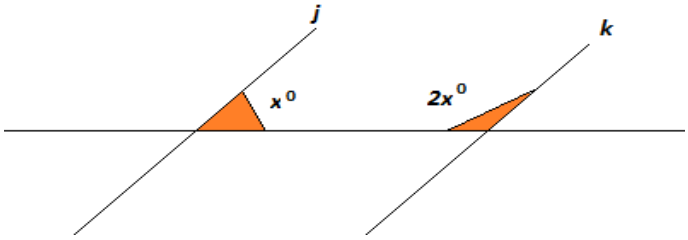


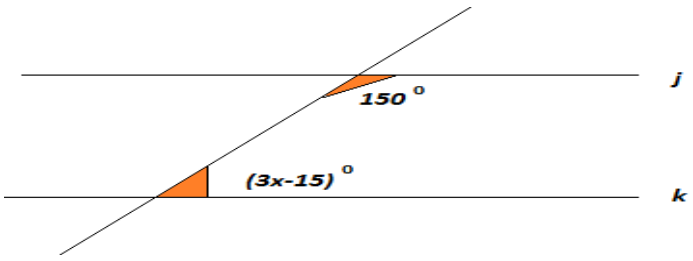
d.



e.



f.



Answer:

- **a.** $120^0 = 3x$ (corresponding angles are equal, solving the equation we find $x = 40^0$)
- **b.** $135^0 = (2x + 15)^0$ (corresponding angles are equal, solving the equation we find $x = 60^0$)
- **c.** $(2x + 20)^0 = 3x^0$ (alternate interior angles are equal, solving the equation we find $x = 20^0$)
- **d.** $(180 - x +)^0 = x^0$ (alternate exterior angles are equal, solving the equation we find $x = 90^0$)
- **e.** $2x^0 + x^0 = 180^0$ (consecutive interior angles are supplementary angles, thus $x = 60^0$)
- **f.** $150^0 + (3x-15)^0 = 180^0$ (consecutive interior angles are supplementary angles, solving the equation we find $x = 15^0$)

3. Upon completion of the exercises above tell the trainees that the problems have been based on the initial fact that the lines are parallel—but that these facts work in two directions:

- If we know the lines are parallel then we can find the related angles

  BUT ALSO

- If we know related angles are equal (or supplementary) then we know the lines are parallel

  Tell trainees that this is called the **conditional statement and its converse**. They work together.

Application Activity

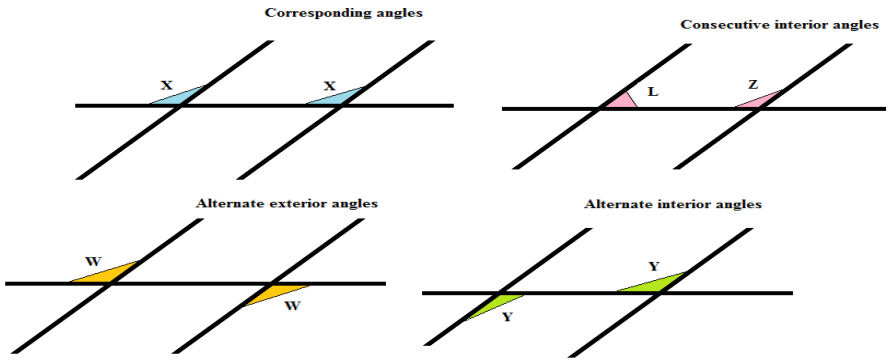
1. Trainees will work on the following problems independently. Encourage trainees to draw and label the parallel lines, then find the solutions.

Topic 2.4 Task 4

Draw two parallel lines and a transversal line and name all formed angles as follow (use different diagrams):

- **a.** Name the corresponding angles by the letter X.
- **b.** Name the alternate interior angles by the letter Y.
- **c.** Name the alternate exterior angles by the letter W.
- **d.** Name the consecutive interior angles by the letter L and Z.

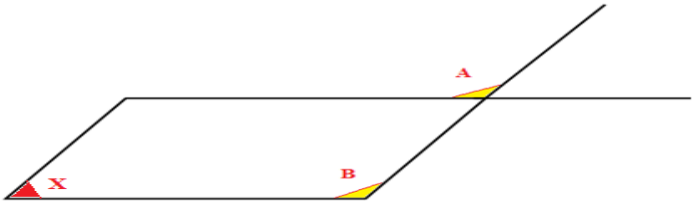
Answer:



Topic 2.4 Task 5

Observe the following figure. Angle X measures 70^0.

a. How many angles are show in the drawn figure?
b. Find the measure of angles A and B.
c. Identify 2 other angles which have the same measure as angle B. Give reasons to explain.



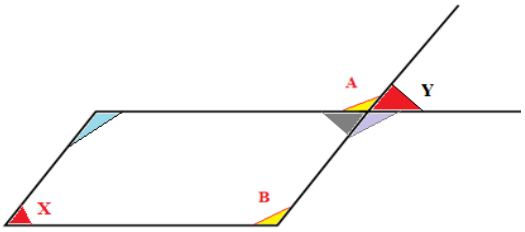
Answer:

a. There are 7 angles in the figure given

b. Angles A and B are on the same side of the two parallel sides of the figure. They are corresponding angles so they are equal.
Label the angle between A and B as angle Y.
Angles A and Y are supplementary because they lie on a line, the sum of angles A and Y is 180^0.
As the angles, B and A are equal, then angles X and Y are equal too.
Angle X is equal to Y and is also equal to the opposite angle to Y.

So, given x = 70^0:
A+Y=180^0
A+X=180^0 (because Y=X)
A+70=180^0 (substitute 70 for X)
A=110^0 and given A+B, B=110^0

c. Angle B is equal to A and to the opposite angle to A.
Angle B is equal to the angle opposite B inside the given parallelogram.



Points to Remember

If two parallel lines are cut by a transversal, then:

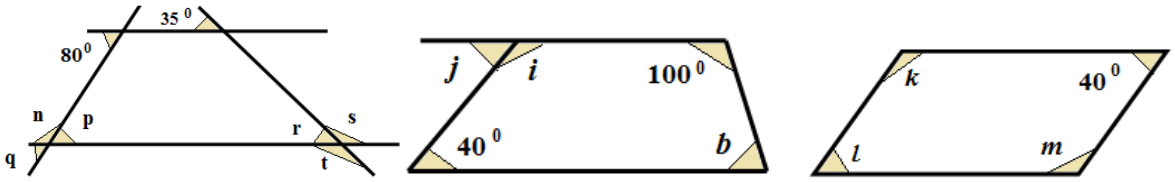
- ✓ The pairs of corresponding angles are congruent/ equal.
- ✓ The pairs of alternate interior angles are congruent/ equal.
- ✓ The pairs of alternate exterior angles are congruent / equal.
- ✓ The pairs of consecutive interior angles are supplementary.

Formative Assessment

1. Ask trainees to work out the given activities. When all trainees have finished, ask some volunteers to share their responses on the board. Ensure reasons and processes are given.

Find the measure of all angles marked with letters in the figures below, given that the two sides of the figure are parallel and other lines are transversal:

a) **b)** **c)**



Answer:

a)

Angles q and 80^0 are on the same side of the parallel sides of the figure. They are corresponding angles and equal angles ($q = 80^0$)

q and p are opposite angles (both acute angles and equal to 80^0). $q = p = 80^0$

$n + p = 180^0$, then $n = 100^0$

Angles r and 35^0 are on the same side of the parallel sides of the figure. They are corresponding angles and equal angles ($r = 35^0$)

r and s are supplementary angles $r + s = 180^0$, then $s = 145^0$

s and t are opposite angles (both obtuse angles and equal to 145^0). $q = p = 145^0$

b)

Angles j and 40^0 are alternate interior angles and equal angles ($j = 40^0$)

j and i are supplementary angles and their sum is 180^0). $J + i = 180^0$, $40^0 + i = 180^0$ ($i = 140^0$)

The angles b and 100^0 are consecutive interior angles and their sum is 180^0. The angle b is 80^0

c)

The angles m and 40^0 are consecutive interior angles and their sum is 180^0. The angle m is 140^0

The angles m and k are alternate interior angles and equal. The angles $m = k = 140^0$

The angles l and 40^0 are opposite angles and equal angles ($l = 40^0$).

2. Ask trainees to draw ☹, 😐, or ☺ to indicate how well they feel they have understood the topic. If there are many ☹, go over the concepts of the topic again during the next class.

Further Information for the Trainer

For more information, consult the following link:
https://www.math-only-math.com/adjacent-angles.html

Learning Outcome 2.5: Calculate the surface area and volume of common geometrical solids using formulas

Objectives: By the end of the learning outcome, trainees will be able to:

a. Determine the number of faces (f), number of vertices (v), and number of edges (e) for a variety of solid figures

b. Investigate the relationship between the surface area of cuboids, prisms, pyramids, and cylinders

c. Calculate the surface area and volume for cube, prism, pyramid, cylinder, cone, and sphere, using given formulae

Time Required: 5 hours

Learning Methodologies: individual, pair, and group work, discussion, presentation

Materials Needed: Empty boxes for each group of trainees and for trainer demonstration, scissors or knife to cut open box, rulers, and calculators if available

Preparation:

- ❑ Prepare different solids out of paper or wood-or find real life examples.
- ❑ Gather objects which are examples of common solid figures (cartons, boxes, tins, balls, oranges, as well as cones and pyramids made from paper).

Cross Cutting Issues:

- ✔ **Gender:** trainer will provide equal opportunities to both boys and girls during the lesson as well as balance gender when forming groups
- ✔ **Peace/value education:** trainer promotes team work and respect for others' views and work
- ✔ **Inclusive education:** trainer is sensitive to differences in learning abilities and uses a variety of activities and materials to cater to different learning styles

Prerequisites: Trainees should already be comfortable with and able to:

- Use addition, subtraction, multiplication, and division correctly
- Calculate area of geometrical shapes like triangle, square, rectangle, circle

Key Competencies:

Knowledge	Skills	Attitudes
1. Describe types of solids: prism, pyramid, cylinder, cone, sphere	**1.** Use formulae to find volumes for different figures	**1.** Collaborative
2. Identify components of solids: Faces, Vertices, Edges	**2.** Represent figures using their Net and use the net to calculate surface areas	**2.** Accurate

3. Describe surface area and volume	**3.** Determine the number of faces (f), number of vertices (v), and number of edges (e) for a variety of solids	**3.** Creative

Steps:

Getting Started: What do we know and where are we going?

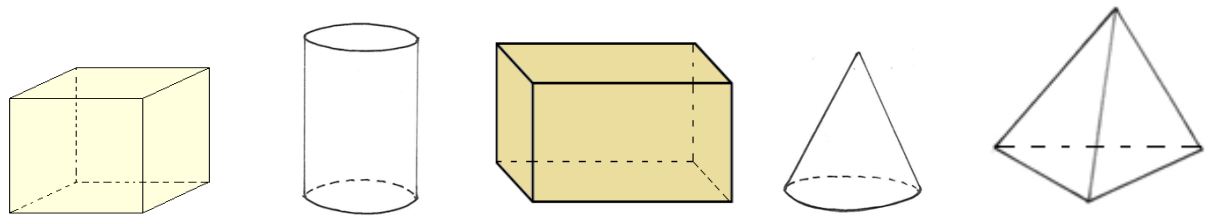
1. Ask trainees to observe the pictures in their manual (or real objects shaped like the pictures) to solve the warm up tasks. Allow trainees to discuss in small groups as they solve the tasks. Let trainees present answers. Correct and guide to ensure all trainees have correct responses.

Topic 2.5 Task 1

Observe the figures below. For each figure find the following:

a) the number faces b) the number of vertices c) the number of edges d) name the figure

a) b) c) d) e)



Answer:

a) **Cube** (all sides are square) Number of sides is 6 all sides are square Number of edges is 12 Number of vertices is 8	b) **Cylinder** Number of sides is 3 (2 circles and 1 rectangle) Sides are 2 circles and 1 rectangle Number of edges is 2 Number of vertices is 0: cylinder has no vertices (no corners)
c) **Rectangular Prism** (all sides are rectangles) Number of sides is 6 Sides are rectangle and square Number of edges is 12 Number of vertices is 8	d) **Cone** Number of sides is 2 (the circle and the slanted side) Shape of side has no name, shape of base is a circle Number of edges 0: cone has no edges Number of vertices 0: cone has no vertices (corners)
e) **Pyramid** (this figure has a triangle base, but some pyramids have a square as a base) Number of sides is 4 (all are triangles) Number of edges is 6 Number of vertices is 4	

Problem Solving Activity

1. Trainer will group trainees into small groups (3 or 4) and give each group an empty box to work with as well as a scissors and ruler. Remind trainees that cm is the unit for length and square centimetres (cm^2) are the unit for surface area, they will use both as they do this investigation.

Upon completion of the investigation, ask each group to present their model to the class.

Topic 2.5 Task 2: Group Investigation
You have been given an example of a solid figure.

- **a.** Count the number of faces, edges and vertices on your solid.
- **b.** Carefully measure each edge of the solid and use your measurements to calculate the area of each face.
- **c.** What do you notice about the relationship between the areas of the faces on your solid?
- **d.** If you wanted to cover your solid with clean paper, what is the total area of the paper you will need to cover the solid completely? This is called SURFACE AREA—that is the area of the entire surface of the solid figure.

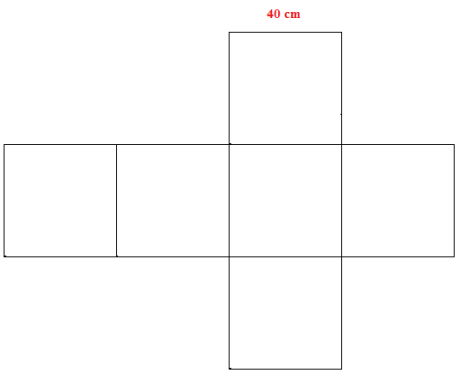
Answer:

- **a.** Answers will vary depending on the solids provided to each group.
- **b.** Answers will vary depending on the solids provided to each group.
- **c.** Trainees will notice that parallel (opposite) sides have equal area.
- **d.** Again, answers will vary. Trainer should ensure that the trainees have added ALL faces.

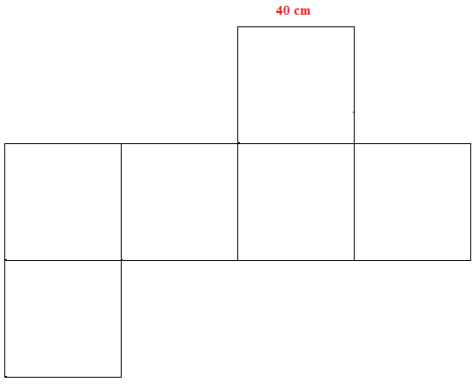
Guided Practice Activity[13]

1. Demonstrate, using a model box, to make a net.

FIRST count the number of faces, edges and vertices of the box and identify it either as a cube or rectangular prism.
THEN cut open the box and lay it out flat on the table. After opening, the solid may look like the following figures:



or



Explain that the opened shape of the box is called a **net** which is a flat way to think about the 3-D shape.

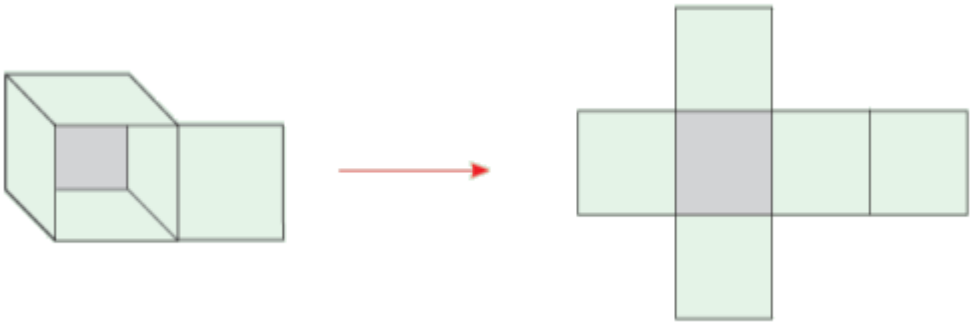
[13] Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher's Manual and Learner's Book*. Waltham: Education Development Center) Education Development Center.

We can cut down a box in different ways, so there are different shapes possible for nets of the same figure per the diagram above, but the number of faces will always remain the same.
How many faces are in each net? (same as in the 3-d shape—in this case 6)
Fold the net up to re-form the solid. Now sketch the net onto the blackboard to
Discuss and take any questions from trainees

Topic 2.5 Task 3
Observe the box given to your group during the investigation. Count the number of faces.
Cut open your box to make a net, confirm that the number of faces has remained the same.
Sketch the net into your copy book.

Re-fold the box to make it into a 3-D figure again, note different groups may have different shaped nets even though their boxes may be similar—this is because of different cuts.

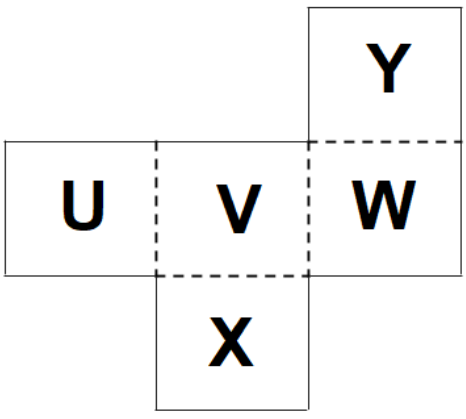
Answer:



2. Trainer will explain to trainees that we have now see how to calculate Surface Area—which is the area of the outside of a solid figure—and an important characteristic of solid figures. Explain that we use flat diagrams—called NETS—which are pictures of how the solids would look if they were cut open. To determine if the NET is correct, we can **imagine** re-folding it to make the original figure again. Ask trainees to work in pairs on the next task doing just that.

Topic 2.5 Task 4
A piece of paper is cut out and libelled as shown in the diagram.

a. If it is folded along the dotted lines what kind of solid will it make?
b. If the box is placed on a table so that the top of the box is upward, then the label at the bottom of the box is ____.



Answer:

a. It will make an open box.
b. Squares U, W, X and Y will form the four sides of the open box with the base of square V.

3. Ask trainees to observe their solid figures take up space. Explain that their NET is 2 dimensional, but their solid is 3 dimensional—or 3-D. Because it is 3-D, a solid takes up space. The amount of space a solid takes up is called VOLUME which is an important characteristic of a 3-d solid. To find volume we use formulas that are related to each different type of solid.

4. Ask a trainee to read aloud **2.5 Key Facts** from their manual. If trainees have questions, use the models and other real-life examples to explain (such as a tin of food, an orange, etc.).

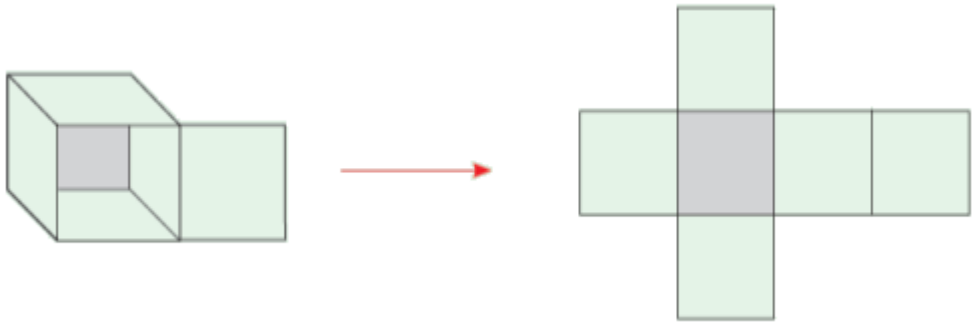
5. Ask groups to quickly review Task 4 and calculate the volume of their model solid. Let each group present their finding to the class. Remind class that while Surface area is a squared measure, volume is a cubic measure.

Topic 2.5 Task 5
With your group, determine the kind of solid you have as your example then use your measurements in the formula given in **2.5 Key Facts** to determine the volume of your solid.

Answer: Answers will vary depending on the box used by each group—trainer should verify calculations and units used for each group to ensure accuracy.

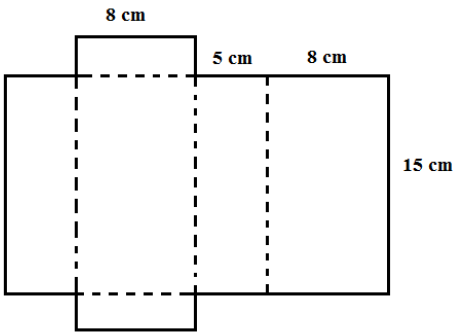
6. Ask trainees to attempt both parts of Task 5 individually. After 5 minutes trainer should write answer on the board, explaining each step carefully to aid trainee understanding and processes.

Topic 2.5 Task 6
Figure 1:
The figure drawn below is a cube. Each edge measures 40 cm.



a. Using properties of a square calculate the surface area.
b. Using the correct formula, calculate the volume of the given cube.

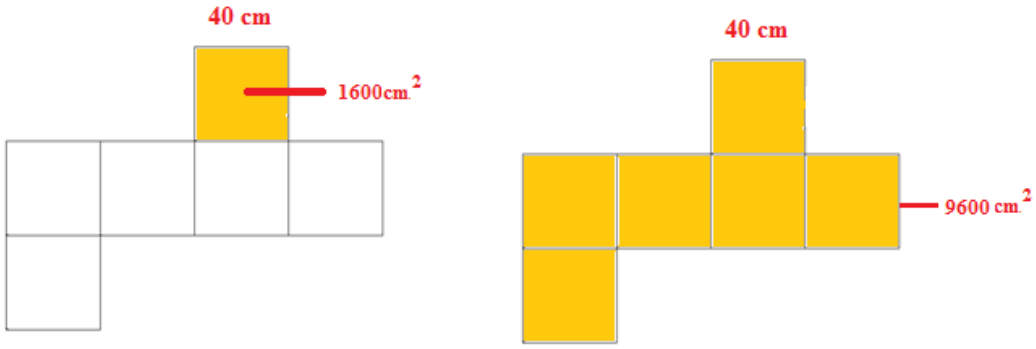
Figure 2:



a. Using properties of a square calculate the surface area.
b. Using the correct formula, calculate the volume of the given cube.

Answer:
Figure 1

a. Surface area of the cube is calculated by finding out the surface are of one square. We know all sides are equal, so we multiply the area of one square by 6 because the cube has 6 faces.
Each square has edge length of 40 cm, thus the area of each face is 40 cm x 40 cm = 1600 cm^2.
The surface area of all 6 squares in the net is 1600 cm2 x 6 = 9600 cm^2.
The surface area of the cube net is 9600 cm^2.



b. The volume of a cube is found by $V=E^3$. In this case E=40 thus V = $(40\ cm)^3$ = 40 cm x 40 cm x 40 cm = 64,000 cm^3.

Figure 2:

a. To calculate Surface Area, let us consider the faces in pairs that are opposite one another. We can observe that these pairs are equal. As such we get:
5 cm x 8 cm = 40 cm^2. This we multiply by 2 to get: 40 cm^2 x 2 = 80 cm^2
5 cm x 15 cm = 75 cm^2. This we multiply by 2 to get: 75 cm^2 x 2 = 150 cm^2
15 cm x 8 cm = 120 cm^2. This we multiply by 2 to get: 120 cm2 cm^2 x 2 = 240 cm^2
The total for 6 faces is 80 + 150 + 240 = 470 cm^2.

b. Volume for a rectangular prism is found using the formula V = L x W x H
Thus, we have V = 5 cm x 15 cm x 8 cm = 600 cm^3.

Topic 2.5 Task 7
Calculate Surface Area and Volume for the solids described below:

a. a sphere with radius 5 m
b. a cone with height of 10 m and radius of 2 m (only find volume)
c. a square-based pyramid with square edges of 5 cm, height of 4 cm, and triangular faces of area 10 cm^2

Answer:

a. Surface Area= 4 x 3.14 x $radius^2$ = 4 x 3.14 x 5^2 = 4 x 3.14 x 25 = 314 cm2
Volume= 4/3 x 3.14 x $radius^3$ = 4/3 x 3.14 x 5^3 = 4/3 x 3.14 x 125 = 523.33 cm3
b. Volume = 1/3 x 3.14 x $radius^2$ x height = 1/3 x 3.14 x 2^2 x 10 m = 41.87 m2
c. Surface Area= square base + 4 triangles = (5cm x 5 cm) + 4 x 10 cm^2 = 65 cm2
Volume = 1/3 x area of base x height = 1/3 x 25 cm2 x 4 cm = 33 cm3

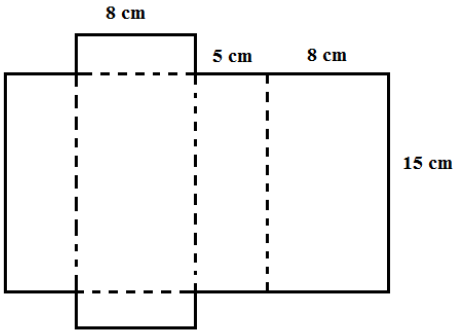
Application Activity[14]

Topic 2.5 Task 8

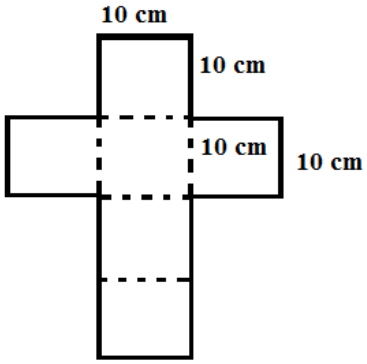
Draw the following nets on Manila paper. Cut the nets out. Fold them along the dotted lines.

a. Which of the nets will make a cube?
b. If the net does not make a cube, explain reasons why you think it does not.

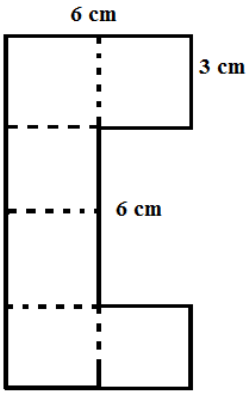
i)



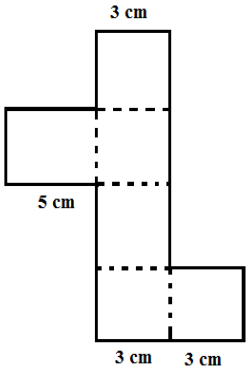
ii)



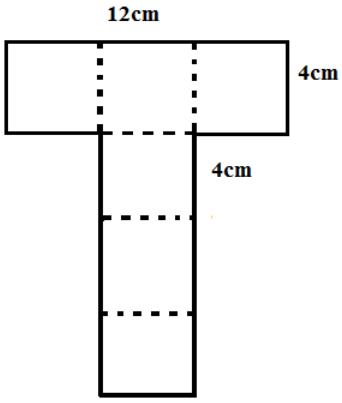
iii)



iv)



v)



[14] Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher's Manual and Learner's Book*. Waltham: Education Development Center) Education Development Center.

Answer:

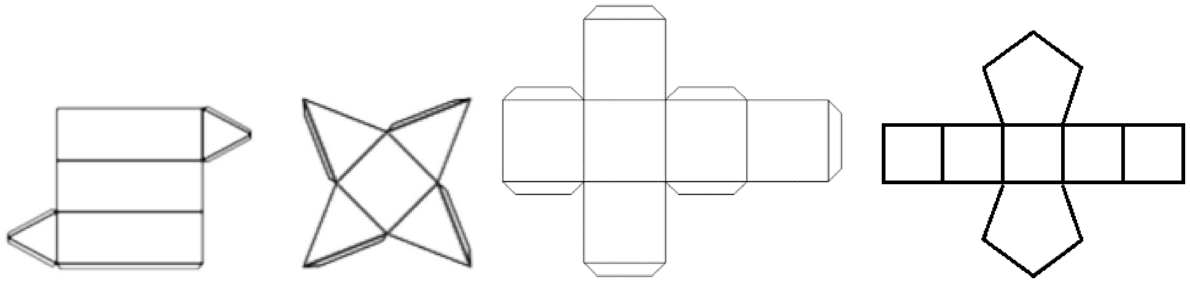
i) All edges of this net are not equal (8 cm, 5 cm and 15 cm), and not squares therefore the net cannot make a cube.	ii) All edges of the nets are equal to 10 cm thus squares. Therefore, folding the net we can get a cube.
iii) All edges of the nets are equal to 3 cm thus squares, but two of the faces are on the same side of the net—they overlap. Folding the net we cannot get a well-covered cube: two faces coincide/ match or correspond.	iv) All edges of this net are not equal (5 cm and 3 cm). They are not all squares therefore the net cannot make a cube.
v) All edges are equal to 4 cm so they are squares. Folding the net we can get a cube.	

Topic 2.5 Task 9

For each figure below, answer the following:

- **a.** List the 2-dimensional shapes you see in the net.
- **b.** What is the name of the solid given by the net in the figure?

i) ii) iii) iv)



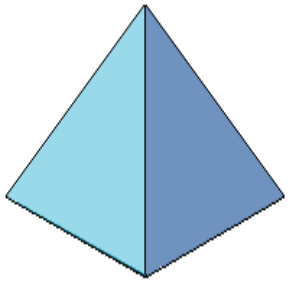
Answer:

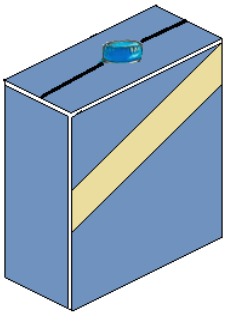
i) Triangular prism with 2 triangles as base, 3 rectangles form the other surfaces	ii) Square-based pyramid: a square forms the base, and 4 triangles form the other surfaces
iii) Cube with all 6 square faces	iv) Pentagon prism with 2 pentagons as bases and 5 rectangles as the other surface

Topic 2.5 Task 10

Draw the nets of these three-dimensional shapes.

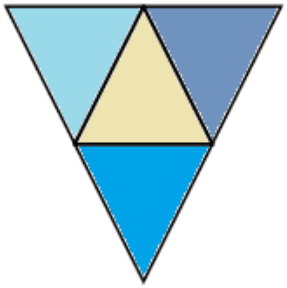
a) b) c)

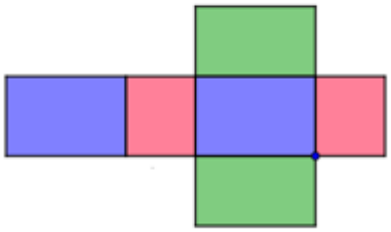


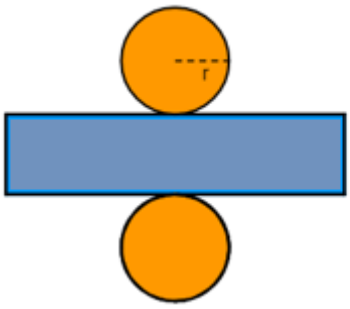


Answer:

a) b) c)







Topic 2.5 Task 11

Find the surface area and the volume of a cylinder of radius 3cm and height 4cm.

Answer:

Surface Area:

S= top and bottom circles + curved side which is a rectangle so $S=2\pi r^2 + 2\pi rh$

$S = (2 \times 3.14 \times 3^2) + (2 \times 3.14 \times 3 \times 4) = 56.52 + 75.36 = 131.88\ cm^2$

Volume:

Volume is the area of the base multiplied by the height so $V = (\pi r^{2)} \times h$

$V = 3.14 \times 3^2 \times 4 = 113.04\ cm^3$

Points to Remember

- Solids in geometry are three-dimensional shapes. Common examples are cubes, rectangular prisms, cylinders, spheres, cones, and pyramids.
- To find surface area we use a net of the solid which is a 2-dimensional shape that can be folded to form to become our 3-D solid.
- To find the volume of a solid we use the given formulae.

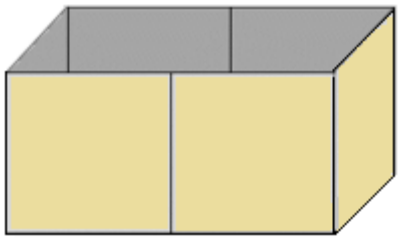
Formative Assessment

1. Ask trainees to work out the given activities to assess their level of understanding of the topics in this learning outcome. When all trainees have finished working independently ask volunteers to share their responses on the board. Focus on process used as well as the answer. Trainer will correct and guide as the trainees show their work to ensure full understanding.

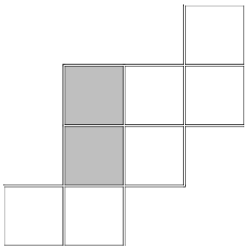
Formative Assessment[15]

1.

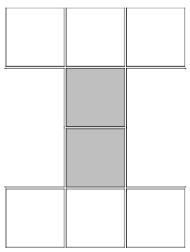
Here is an open box. Imagine folding each of the given nets. Which will fold together to make the box drawn below? (The grey squares show the base.)



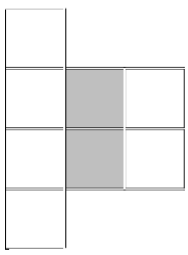
A



B



C



Answer:

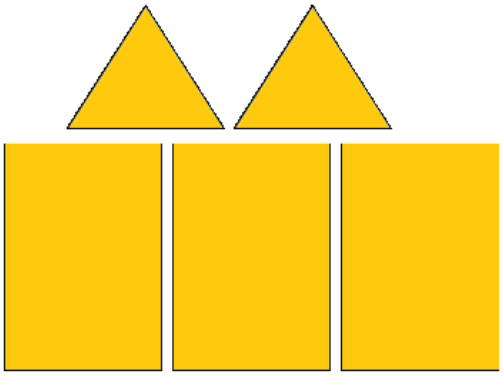
The nets B and C can be folded to make the open box. When the net A is folded, the formed box is open in two places, one opening is on top and the other on the left side.

2.

Aloys is making a 3D object using just the shapes below.

a. Which object is he making?

b. Once constructed (note that this is NOT a NET), give the number of faces, edges and vertices.



.

Answer:

a. This will be a Triangular Prism – but this picture must be rearranged to become a net.

b. There will be 5 faces, 9 edges, and 6 vertices when the figure is constructed.

3.

Observe the net below carefully.

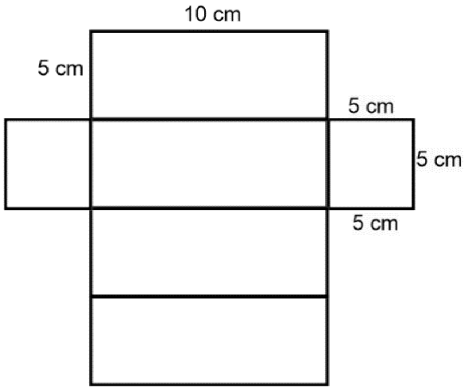
a. Accurately draw this net.

b. Find the area of each face then find the total surface area for this figure.

[15] Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher's Manual and Learner's Book*. Waltham: Education Development Center) Education Development Center.

c. Cut out and fold this net to create a rectangular prism.

d. Calculate the volume of this rectangular prism.



Answer:

a. Answers should be drawn in trainee notebooks.

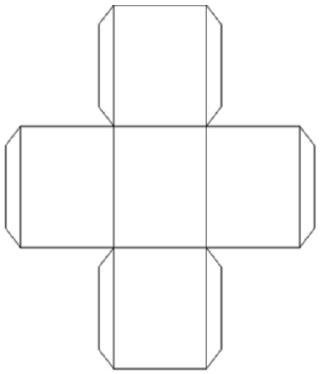
b. Square area 5 x 5 = 25 cm^2, rectangle area is 5 x 10=50 cm^2. Total Surface Area is given by S= 4 x (5 cm x 10 cm) + 2 x (5 cm x 5 cm) = 200 cm^2 + 50 cm^2 = 250 cm^2

c. Active task—no answer.

d. Volume is base x depth. V = (5 x 5) x 10 = 250 cm^3.

4.

Alice thinks that this net will fold to create a cube. Do you agree with Alice? Explain your answer and if the solid is not a cube then give the name of the solid that is formed.



Answer:

The presented net has 5 faces. To fold it, one can get an open box with only 5 faces. Furthermore, we can observe that the faces are not squares—thus the open box has the form of a rectangular prism, not a cube.

2. Ask trainees to circle or draw ☹, 😐, or 😤 to indicate how well they feel they have understood the lesson. They should also list particular areas of difficulty. If there are a lot of ☹, go over the concepts of this lesson again during the next class.

ⓘ Further Information for the Trainer

✓ Solid figures are three-dimensional objects, meaning they have length, width, and height (or depth). Solid figures are identified by the numbers of faces, edges, and vertices, as well as the

shape of the faces. Solids take up space which means they have volume, which is determined by using a formula.

- ✓ The flat surfaces of a solid figure are its faces. The edge of a solid figure is the line segment where two faces meet. A vertex (plural: vertices) is the corner that is formed where the ends of the line segments of two or more edges meet.
- ✓ A sphere is a solid figure that has no faces, edges, or vertices. This is because it is completely round; it has no flat sides or corners.
- ✓ A **net** is a 2-dimensional shape that can be folded to form a 3-dimensional solid. Alternatively, a net is a pattern made when the surface of a three-dimensional figure is laid out flat showing each face of the figure. A solid may have different nets, depending on what edges are cut apart and what edges remain attached.
- ✓ To determine whether a net is correctly related to a solid:
  - Make sure that the solid and the net have the same number of faces.
  - Ensure that shapes of faces on the solid match shapes of the corresponding faces in the net.
  - Visualize folding the net to form the solid and make sure that all sides fit together properly.

https://www.ck12.org/geometry/faces-edges-and-vertices-of-solids/lesson/Classification-of-Solid-Figures-MSM7/
https://www.onlinemathlearning.com/geometry-nets.html
https://byjus.com/surface-area-of-a-prism-formula/

Summative Assessment—Unit 2

1. List the Learning Outcomes for this Learning Unit on the blackboard. Trainees have been trained on these topics. Ask trainees if there are particular topics that they are still finding difficult. If so, Trainer should provide a couple examples (or re-work problems from the sections that trainees identify as particularly difficult) Trainer works those examples on the board to clarify the content for class.

2. Explain that trainees will each take the test below to check understanding of this Learning Unit. Trainees will work independently. When complete, trainees give test to trainer and trainer will correct the test after all have finished. Corrected tests should be returned to each trainee and trainees use the corrected test to complete the self-assessment below the test.

Summative Assessment:
For each of the following multiple-choice questions, find the correct answer then write a few sentences, or show calculations to explain why you believe your answer is correct.

1.
What is the name of a three-dimensional figure with six square faces?

a) pyramid b) cube c) solid d) edge e) sphere

2.
I am a three-dimensional figure with five faces. My base is a rectangle; my other faces are triangles. I have five vertices and eight edges. What am I?

a) rectangular prism b) rectangular pyramid c) triangular prism d) triangular pyramid

3.
How many vertices does a rectangular prism have?
a) 14 b) 12 c) 8 d) 6 e) 4

4.
Which of the following has a base that is a circle?
a) Cube b) Sphere c) Prism d) Cylinder

5.
I am a three-dimensional figure with four faces. All of my faces, including the base are triangles. What am I?

a) rectangular prism b) triangular prism c) triangular pyramid d) rectangular pyramid

6.
How many edges are there on a rectangular prism?

a) 6 b) 8 c) 10 d) 12

7.

Alice is going to build a table for her family room. It is in the shape of a triangular prism. What shapes will she need to build the table?

a) 5 triangles b) 3 triangles and 2 rectangles c) 2 triangles and 3 rectangles d) 5 rectangles

8.

The photo below shows a portion of the X international Airport. What is the relation between angle 3 and angle 6?

a) Alternate Interior angles b) Alternate Exterior angles
c) Corresponding angles d) Supplementary Angles

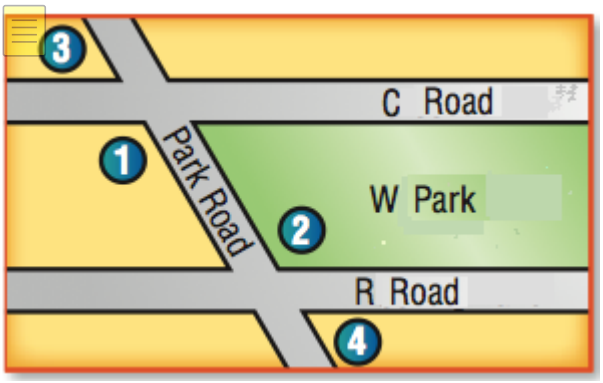
9.

The lines that separate parking spaces are parallel. The measure of angle 1 is 60 degrees. What is the measure of the angle that is supplementary to angle 2?

a) 30 degrees b) 60 degrees c) 120 degrees d) 180 degrees

10.

R Road and C Road are parallel streets that intersect Park Road. If angle 1 is 118 degrees, then what is angle 4?



a) 59 degrees b) 62 degrees c) 118 degrees d) 180 degrees

11.

In a football game, one of the successful ball passes made angles as shown by the picture. Find the value of X.

a) 58 degrees b) 64 degrees c) 116 degrees d) 122 degrees

Answers:

(explanations will vary)

Question1: b; Question 2: d; Question 3: c; Question 4: d ; Question 5: c ; Question 6: d ; Question 7: c ; Question 8: b ; Question 9: c ; Question 10: b ; Question 11: b

Self-Reflection

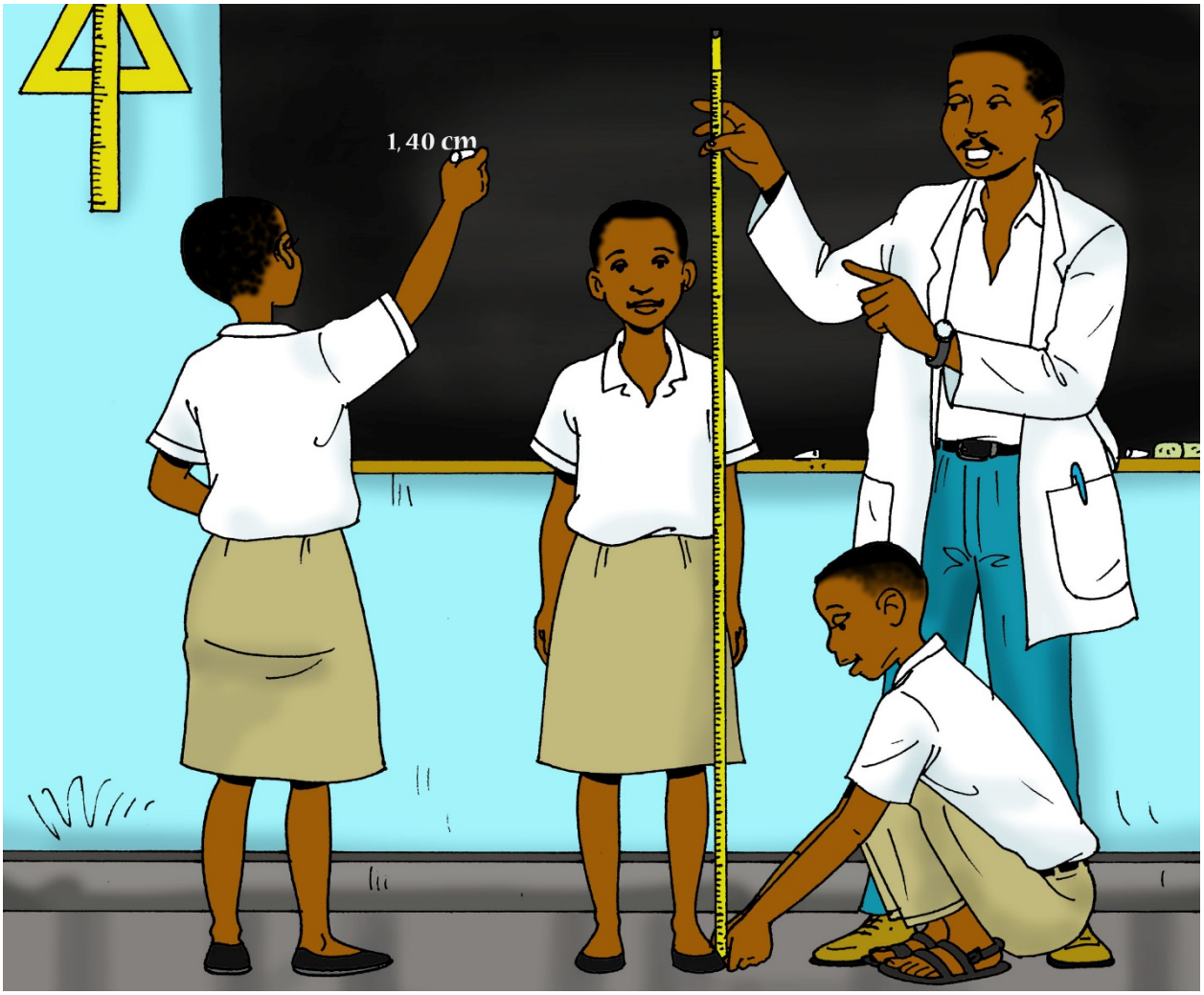
1. Take few minutes and ask trainees to re-take the self-assessment they took at the beginning of the Learning Unit 2. Trainees will fill in the table with an X to mark where they feel they have understood the topics on each line now they have completed the unit of study.

2. Explain that this self-assessment is to help trainees identify areas of strength and areas for improvement as well as actions they can take to improve their learning for the next unit. After completing the self-assessment, ask trainees to complete the table below to focus their intentions for the next unit of study and to guide review of this unit.

Areas of strength	Areas for improvement	Actions to be taken to improve
1. **2.**	**1.** **2.**	**1.** **2.**

3. Discuss individual results with each trainee privately. Identify the areas that are giving many trainees difficulties and plan to provide additional mini-lessons for extra support those topics that trainees found difficult. (For example, trainers can use class time before you begin the next learning outcome to go through commonly identified difficult concepts).

Learning Unit 3: Apply statistical concepts and calculations



Learning Outcomes

By the end of the Learning Unit, trainees will be able to:

3.1 Record and organize statistical data for analysis

3.2 Compute the measures of central tendency (Mean, Mode, and Median) using statistical data sets based on life situations

3.3 Present and communicate statistical data and analysis of results using mathematical representations (graphs, charts and tables)

Learning Unit 3: Self-Assessment

1. Ask trainees to look at the illustration at the beginning of the unit in their manual. Choose a trainee to describe the illustration in detail. After brainstorming, share the main topics and learning outcomes.

2. As trainees now know, the purpose of a self-assessment is to become familiar with the topics in the unit and to think about what they already know (or do not know). At the end of the unit, they will do the same self-reflection again, review their progress, and think of actions to improve their learning overall. The self-assessment is not a test!

3. Ask trainees to fill out the self-assessment found at the beginning of Unit 3 in their manual. This should take 2 or 3 minutes.

Learning Outcome 3.1: Record and organize statistical data for analysis

Objectives: By the end of the learning outcome, trainees will be able to:

a. Collect and record data

b. Organize data into tables with data both grouped and ungrouped.

Time Required: 5 hours

Learning Methodologies: group and pair work, individual work, presentations

Materials Needed:

- Standard materials, Trainee Manuals, calculators if available, flip chart paper

Preparation:

- ❑ Prepare empty data tables on flip chart.
- ❑ Fix tape measure on the wall so trainee can measure themselves.

Cross Cutting Issues:

- ✔ **Gender:** trainer will provide equal opportunities to both boys and girls during the lesson as well as balance gender when forming groups
- ✔ **Peace/value education:** trainer promotes team work and respect for others' views and work
- ✔ **Inclusive education:** trainer is sensitive to differences in learning abilities and uses a variety of activities and materials to cater to different learning styles

Prerequisites: Trainees should already be comfortable with and able to:

- Use addition, subtraction, multiplication, and division

Key Competencies:

Knowledge	Skills	Attitudes
1. Describe qualitative, quantitative, discrete and continuous data	**1.** Differentiate types of data	**1.** Persistent
2. Understand how to use frequency distribution tables for grouped and ungrouped data sets	**2.** Collect and organize data	**2.** Innovative
	3. Group data into classes, calculate frequencies, and make frequency distribution tables	**3.** Accurate

Steps:

Getting Started: What do we know and where are we going?

1. As trainees enter the classroom, each trainee should write their age, and the number of siblings, and favourite animal in the chart drawn onto the blackboard like the one below:

Age	Number of siblings	Favourite animal

2. When trainees are all in class, ask a volunteer to describe what they see in the chart.
Possible answers: lots of numbers, some numbers repeated and some different, two sets of numbers, big and smaller numbers, many different animals, words, numbers, etc.

3. Ask trainees if they can draw any conclusions from these three data sets. Explain that we organize numbers and words to analyse and draw conclusions. This example explains why statistics can help us. Introduce learning outcome and objectives.

Problem Solving Activity

1. Ask trainees to look at Task 1 data in their manuals. These data are similar to the data gathered from this class (still be on the blackboard) but are organized. Work in pairs to complete Task 1. After 5 minutes ask trainees to share their observations and thinking. Emphasize vocabulary words highlighted in the answers below.

Topic 3.1 Task 1

The following data was gathered from one class of primary students.

Age	Tallies	Number of pupils
11 years	𝍸	5
12 years	𝍸 𝍸 𝍸 ////	19
13 years	𝍸 ////	9
14 years	///	3
15 years	////	4
	Total:	40

a. Explain why some tallies have cross marks while others are just vertical lines.
b. Describe a relationship between the column with tallies and the column with numbers.
c. What is the most common age for students in this class?
d. What is the age difference between the oldest students and the youngest students?
e. Consider the students in the 12-year-old group: are all the students in that age group EXACTLY the same age?

Answer:

a. Tallies are simple marks used to count: we count up to four with vertical tallies. Then we use a cross tally to make the grouping 5. This makes tallies easy to read and work with.
b. Tallies are used to organize as we count data points into groups. When collection is complete, the number of tallies is listed. The number of tallies in a group is the frequency.
c. The most common age for students in this class is 12 years old. That row has the most tallies —and the frequency is the highest for that group. The most common data point is the mode.
d. The oldest students are 15. The youngest are 11. The difference is 15 – 11 = 4 years. The difference between the biggest and the smallest data points is called the "**range**".
e. NO. This kind of data is called "**continuous data**" which means that there are lots of possible values for the data—like numbers on a number line. Students grouped into the 12-year-old category are NOT the same age. Some will be 12 years and 3 months; others will be 12 years and 5 or 6 months. There may be a rare case when two students are born on the same day, but one may be hours older. We group data to make them easier to organize and analyse. BUT, Number of siblings is NOT continuous data because the numbers can ONLY be whole numbers. Number of siblings is called "**discreet data**" and everyone that has 2 siblings has exactly 2 siblings—it is different from age data which is continuous data.

Guided Practice Activity

1. Ask the trainees to look back at the data we gathered at the beginning of the class. Notice that Age is **'continuous data'** although we have simplified it into yearly groups. Number of siblings, on the other hand, is **'discreet data'** because there are only specific numbers—no decimals or fractions are possible. Nonetheless, both data sets use numbers so we call these data sets **'quantitative'** which means related to quantities—or numbers. The data set for favourite colour does not use numbers—rather is a grouping of words. This sort of data is called **'qualitative'** which means that data are words (rather than number values).

2. Divide the class into three groups: one will work with the age data set, one will work with the sibling data set, and one will work with the animal data set. Give each group a piece of flip chart paper. They will make a large table to organize their data (shown below). Each group will share their work with the class, showing their table, giving a summary of what they did and learned.

Topic 3.1 Task 2
In groups, organize the data from the blackboard into a table like the table below. When data is organized, write down anything interesting you see about trainees in this class.

Groups will show their table and present in front of the class the following:

- **a.** Explain the processes used to complete the table
- **b.** Share two interesting facts learned from the organized data

Sample data table:

Age (or # of siblings) (or favourite animal)	**Tallies**	**Number of trainees**
Add rows below if needed per data set		

3. As the groups are working the trainer walks around to ensure all groups understand the task, and trainee is actively engaged. After 10 minutes, each group presents their table and findings.

Answer:
Actual answers will depend on numbers from the trainees in your class but here is an example:

Age	**Tallies**	**Number of trainees**
15	/	1
16	////	4
17	~~HHT~~	5
18	////	4
19	~~HHT~~ /	6
20	////	4
21	~~HHT~~	5
22	///	3
23		0
24	/	1
	Total	33

- **a.** The data are gathered into the correct row on the table using tallies as you work methodically through the original set of raw (un-organized) data.

b. (Answers will vary.) Possible answers may include: 19 is the most common age for trainees. The range of ages in this class is 9 years (24 – 15 = 9). There is a total of 33 trainees...

(Note: this answer is provided as an example—based on the data above—class data will differ.)

Sample answer for group 2:

# of siblings	Tallies	Number of trainees
0	/	1
1	////	4
2	𝍸 ////	9
3	𝍸 𝍸 /	11
4	𝍸 //	7
5	/	1
6		0
7		0
	Total	33

a. same as above

b. (Again, answers will vary depending on class data) for the above set: the most common number of siblings is 3. The range of number of siblings is 5 years (5 – 0 = 5).etc.

(This answer is provided only as an example.)

Sample answer for group 3:

Favourite animal	Tallies	Number of trainees
pig	///	3
goat	////	4
cat	𝍸 /	6
lion	𝍸 𝍸	10
cow	𝍸 //	7
sheep	//	2
chicken	/	1
dog		0
	Total	33

a. same as above

b. Lion is the most common favourite animal for trainees. Because this is QUALITATIVE data it is impossible to find the range of the data. There are 33 trainees in the class in total. (Again—these answers are provided only as an example.)

4. Respond to any questions posed, ensure that the information presented is accurate, and conclusions drawn are correct based on the data given. When groups have completed their presentations, ask a volunteer to read aloud **3.1 Key Facts** from the Trainee Manual.

5. Respond to any questions regarding the vocabulary, and provide additional examples to clarify concepts if needed.

6. Explain to trainees that we will work together on the next task which has a set of raw data—that is, a set of data that has not yet been organized. Trainees will organize the data into a

table and then answer questions regarding that data. Ask one trainee to come to the blackboard to be the "writer" for the class. Other trainees take turns giving directions to the 'writer' as they solve Task 3. Pay close attention to ensure correct processes, answers, and use of vocabulary.

Topic 3.1 Task 3

A teacher recorded the age of her pupils.
Make an organized table. Use tallies to put the data points where they belong inside the table. Then answer the questions below the data set to help you analyse this data set.

14	11	12	12	13
13	14	15	11	12
13	12	12	15	12
12	13	15	12	13
14	12	12	12	11
12	13	15	12	12
13	13	12	12	11
12	12	13	11	12

a. How many students are there in this class?
b. How many of the pupils in this class are 12 years old?
c. How old are the oldest pupils in that class?
d. How old is the youngest pupil in this class?
e. What is the age range for pupils in this class?
f. What is the most common age for pupils in this class?
g. What is the statistics term for the most common age? The data point with the highest frequency?

Answer:

Age	**Tallies**	**Number of pupils**
11 years	𝍸	5
12 years	𝍸 𝍸 𝍸 ////	19
13 years	𝍸 ////	9
14 years	///	3
15 years	////	4
	Total:	40

a. There are 40 pupils in the class
b. There are 19 pupils who are 12 years old in this class.
c. The oldest pupil is 15 years old.
d. The youngest pupil is 11 years old.
e. The age range in this class is 15-11= 4 years
f. The most common age for pupils in this class is 12 years old
g. The most common age (12 years old) is the mode of this data set.

Application Activity

1. Trainees work in pairs to help each other and ensure that all have understood clearly. Explain that pairs will measure each other in cm using a measuring tape fixed to the wall. Place the 0 cm end on the floor and measure upward to 200 cm. If an actual tape is not available, trainer can make an informal tape on a folded piece of flipchart. Trainees should measure to the nearest cm and write their height on the board, as we did for their ages, etc. so the entire class's data points (one from each trainee) are visible for all. Complete Task 4 in pairs.

Topic 3.1 Task 4

Work with a partner to organize the data points (trainee heights in cm) from the board into a table. Then answer the following questions about your collected and organized data:

- **a.** What class (or group) has the most students?
- **b.** What vocabulary word is used to describe the class with the highest frequency?
- **c.** What is the range of the heights of trainees in this class?
- **d.** Our data is now grouped 'classes.' Guess what mid-class value is, and give an example.

Answer: (this will depend on the data gathered by the class!):

A possible table will look something like the following:

Height (actual answers will depend on data gathered in class – this is just an example)	**Tallies**	**Number of trainees (Frequency: number of trainees in each height group)**
146-150 cm	////	4
151-155 cm	///	3
156-160 cm	~~////~~ ////	9
161-165 cm	~~////~~ ~~////~~ /	11
166-170 cm	~~////~~ ~~////~~	10
171-175 cm		0
176-180 cm	////	4
	Total:	41

NOTE: the following responses are based on data in the example table above—the class data will differ. The responses below are examples to help trainer understand how to guide trainees.

a. The height group 161 cm - 165 cm is the most common. There are 11 trainees in that group—that is, the frequency of that group is 11 which is higher than other groups.

b. The group with the highest frequency (the most trainees) is the group 161-165 cm. This is called the mode (or in this case, the modal class).

c. Range will depend on the data collected: find the tallest and shortest heights and find the difference. With grouped data, we know that the range is approximately 180 - 146 = 34 cm,

using the class boundaries, not actual data points. Each group has a range of 4 cm (e.g. 150 – 146 = 4 cm).

d. Mid-class value is the value that is in the exact centre of a class—For example: let us look at the first class which includes the values: 146, 147, 148, 149 and 150. There are 5 values and the exact middle value is 148 so we can call 148 our Mid-Class Value. This value is used in the next Learning Outcome, so this is a little 'flash forward' for learning.

2. Ensure all trainees understand how to work with data sets, tally the values, find frequencies, and do some initial analysis (find mode or modal class and range as well as class frequencies).

3. Trainees will work individually on Tasks 5 and 6. Trainees will have approximately 10 minutes to complete the two tasks—then discuss the answers as a class. Walk around the class to ensure trainees have understood these foundational concepts. Provide support as needed.

Topic 3.1 Task 5

A trainee did a survey of the families in her community. She asked each family how many children were living in their household. The following table shows the results:

Number of children	Tally marks	Number of families
1	𝍸	
2	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ////	
3	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 /	
4	𝍸 𝍸 𝍸 //	
5	𝍸 𝍸 ///	
	Total	

a. Complete the table, the third column, which gives the frequency (number of families) for each data point (number of children in the household).
b. How many families have five children living in their homes?
c. Which number of children is the most common in this community??
d. Which is the least likely number of children for a family in this community to have?
e. How many families were surveyed? That is, what is the total frequency?

Answer:

a.

Number of children	Tally marks	Number of families
1	𝍸	5
2	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ////	39
3	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 /	31
4	𝍸 𝍸 𝍸 //	17
5	𝍸 𝍸 ///	13
	Total:	105

b. 13 families have 5 children in their households
c. The most common number of children per family is 2 children.
d. A family in this community is least likely to have one child.

e. The total number of families surveyed is 105. That is, the total frequency is 105.

Topic 3.1 Task 6

Kamali sells soft drinks in his wholesale shop. The table below shows the types of drinks and the number he sold at his shop in one week.

Type of drink	A	B	C	D	E	F
Number of bottles sold	11000	9636	10800	12024	8842	9000

a. Which type of drink is the most likely to be sold? (**Answer:** soda A is the most likely because it has the highest frequency.)
b. Which type of drink had the least likely to be sold? (**Answer:** soda E is the least likely to be sold because it has the lowest frequency).
c. What was the total number of bottles sold that week? (**Answer:** 61 302 bottles were sold which gives us the total frequency for numbers of bottles sold.)

Points to Remember

- **There are two main kinds of data: Qualitative data (not numbers) and Quantitative data (numbers—and can either be discrete data** (only counting numbers) or **continuous data** (which may have decimals or fractions included).
- Data, which is not yet organized, processed, or ordered in any way, is called **raw data**.
- **Frequency** is the number of data points in each group, in a sample, or in the population.
- **Mode is** the most common data point (the data point with the highest frequency).
- **Range** is the difference between the highest data value and the lowest data value.
- The **Mid-Class Value** is the value of the term in the exact centre of any given class.

Formative Assessment

1. Ask trainees to work independently on the following tasks. Explain that this formative assessment is an opportunity for each trainee to self-assess their own understanding so they should work carefully and if any parts are not clear, ask the trainer after work time is complete.

Formative Assessment

1.

The mass of 50 tomatoes (measured to the nearest g) were noted and shown in the table.

86	101	114	118	87	92	93	116	105
102	92	93	101	111	96	117	100	106
118	101	107	96	101	102	104	92	99
107	98	105	113	100	103	108	92	109
95	100	103	110	113	99	106	116	101
105	86	88	108	92				

a. Construct a frequency distribution table, using equal class intervals of width 5 g, starting with the class 85 – 89 g, 90 – 94 g, …, 115 – 119 g.

b. Find the frequency for each group, and cite the modal class and the range of the data.

Answer:

a. Using classes defined above we get the following table which lists frequencies for each class:

Mass	**Tallies**	**Frequency**
85 – 89	////	4
90 – 94	𝍸 /	6
95 – 99	𝍸 //	7
100 – 104	𝍸 𝍸 ///	13
105 – 109	𝍸 𝍸	10
110 - 114	𝍸	5
115 - 119	𝍸	5
		50

b. The modal class (the class with the highest frequency) is 100 – 104 g, which tells us that the most common weight for a tomato is between 100 and 104 grams.
The approximate range (using the class boundaries) is 119 – 85 = 34 g, but if we use actual data points, we find the smallest (86g) and largest (118g) data points then find the difference to get the ACTUAL range: 118 – 86 = 32 g

2.

The table below shows the number of days in which different temperatures were recorded in 60 days. Temperatures were rounded to the nearest whole number.

Temperature	**Tally marks**	**Number of days**
22^0 C	𝍸 𝍸 𝍸	15
24^0 C	𝍸 𝍸 //	
25^0 C	𝍸 𝍸	
26^0 C		9
28^0 C		13
	Total:	

a. Copy and complete the table.

b. How many days had a temperature of 26^0 C?

c. Which temperature was recorded on the most number of days?

d. Which temperature was recorded on the least number of days?

e. Is this data qualitative or quantitative? Continuous or Discreet?

Answer:

a.

Temperature	Tally marks	Number of days
22^0 C	𝍸 𝍸 𝍸	15
24^0 C	𝍸 𝍸 //	12
25^0 C	𝍸 𝍸	10
26^0 C	𝍸 ////	9
28^0 C	𝍸 𝍸 ///	13
	Total:	59

b. The days which have 26^0 C are 10.

c. The temperature recorded on most number of days is 22^0 C

d. The temperature recorded on least number of days is 26^0C.

e. This data is quantitative because it is numbers. It is Continuous because you could have a decimal temperature (25.4 degrees which we have rounded to 24 degrees)

Learning Outcome 3.2: Determine the measures of central tendency (Mean, Mode and Median) using statistical data sets based on life situations

Objectives: By the end of the learning outcome, trainees will be able to:

a. Define the mode, mean and median

b. Determine mean, mode and median for a given data set

Time Required: 5 hours

Learning Methodologies: brainstorming; pair, group, and individual work

Materials Needed:

- Standard materials, trainee workbook, calculators if available

Preparation:

❑ Prepare sample sets of data for trainees to analyse, write them on flip charts as additional examples should trainees need more practice.

Cross Cutting Issues:

✔ **Gender:** trainer will provide equal opportunities to both boys and girls during the lesson as well as balance gender when forming groups

✔ **Peace/value education:** trainer promotes team work and respect for others' views and work

✔ **Inclusive education:** trainer is sensitive to differences in learning abilities and uses a variety of activities and materials to cater to different learning styles

Prerequisites: Trainees should already be comfortable with and able to:

- Use addition, subtraction, multiplication, and division

Key Competencies:

Knowledge	Skills	Attitudes
1. Explain how to make a frequency table, and how it can be used	**1.** Make frequency table of both grouped and ungrouped data	**1.** Creative
2. Explain how to calculate mean, mode, and median	**2.** Calculate the measures of central tendency for any given data set.	**2.** Accurate
	3. Discuss the meaning of the different measures of central tendency for real life situations	**3.** Focused on Detail

Steps:

Getting Started: What do we know and where are we going?

1. Remind trainees that they have worked with averages before in mathematics—and very likely in their own lives too.

Topic 3.2 Task 1
Brainstorm some times in real life when averages are used. List out in your notebook.

In class, trainees call out ideas, trainer writes ideas on the board.

Possible answers may include: for class grades we use average test scores, average temperatures during hot season, average mm of rainfall per year in different places in the country, average goals per season for football stars in Africa cup teams, average cost of a moto taxi to school, average time it takes to walk to school, etc. …

2. Ask if trainees remember how to calculate averages, ask volunteer to explain to the class:
To calculate an average, we use both addition and division:
First, add up all the data values, then divide that total by the number of values.

Example: Given 5 quiz scores: 15, 12, 17, 19, and 18, to find the average, first add:
15 + 12 + 17 + 19 + 18 = 81 then divide 81 by 5 (because there were 5 scores).
81/5 = 16.2 so the average score for the 5 quizzes is 16.2.

3. Ask trainees to read Task 2 carefully and work on this independently.

Topic 3.2 Task 2
Find the averages of the following three sets of numbers:

a. Find the average temperature for the week. The 7 daily temperatures were:
21, 25, 24, 20, 19, 20, 21

b. Find the average number of bananas on a hand of bananas sold at Simba Supermarket. 8 banana hands that were examined, and we counted the following bananas: 9, 13, 14, 11, 12, 13

c. In a nearby village 5 families were raising goats. The families had the following numbers of goats each: 6, 7, 8, 9 and 11. On average, a family owns how many goats?

Answer:

a. 21 + 25 + 24 + 20 + 19 + 20 + 21 = 150. Then 150/7 = 21.4 so the average weekly temperature is 21.4.

b. 9 + 13 + 14 + 11 + 12 + 13 = 72 then 72/6 = 12 the average number of bananas in a hand is 12.

c. 6 + 7 + 8 + 9 + 11 = 41 then 41/5 = 8.2 goats. NOTE: it is not possible to have a decimal (part) of a goat it must be either 8 goats or 9 goats. It is impossible to have 8.2 goats so we round the answer to the closest whole number—thus the average is 8 goats per family.

4. Write the following words from Learning Outcome 3.1 on the blackboard. Ask trainees to write a definition for each one and ask a volunteer to explain each one:

Frequency **Range** **Mode (and Modal Class)**

5. Write the following words on the board too. Explain that we will learn about these terms during the lesson but they should write possible definitions for each. They can check their answers as the terms come up in the lesson.

Mid-class value **Mean** **Median**

Problem Solving Activity

1. Ask students to open their manuals to Task 3. They will work in pairs to complete the task. Give trainees about 5 minutes to complete the task. While trainees work, walk around the class to give hints and guidance if trainees are finding the task difficult.

Topic 3.2 Task 3

Below are marks scored in a test:

30 50 80 45 90 65 75 30 80 85 10 65 50
50 15 85 50 65 75 10 90 55 50 100 85

a. Re-write the scores, listed in order from the smallest to the biggest. Repeat any scores that occur more than once. The total number of scores in the new list will equals the number of scores shown in the original list above.

b. Find the score that is in the exact centre of the ordered scores and underline it on your list.

c. Label the score that is in the exact centre of the list as MEDIAN SCORE then, on the list, count how many scores are below the median AND how many scores are above. Are they equal?

d. Which score appears most often?

e. What is the correct term for the term that occurs most often?

f. Find the average (which, in statistics, we call this number the MEAN).

Answer:

a. The ordered list should be the following:
10, 10, 15, 30, 30, 45, 50, 50, 50, 50, 50, 55, 65, 65, 65, 75, 75, 80, 80, 85, 85, 85, 90, 90 ,100
Note that there are 25 values just as there are in the original list. Some values are repeated.

b. 10, 10, 15, 30, 30, 45, 50, 50, 50, 50, 50, 55, 65, 65, 65, 75, 75, 80, 80, 85, 85, 85, 90, 90 , 100

c. **65 is the median** for this data set—which means that half the values are lower and half the values are higher than this particular value. In this case there are 12 values lower and 12 values higher than 65. The number of values lower than the median should always be equal to the number of values higher than the median. Yes, 12 = 12.

d. The score that appears most often is 50—that value appears on our list 5 times.

e. The data point that appears most often is called the **MODE**.

f. To find the average, add all data values then divide by the number of data points in the set.
10 + 10 + 15 + 30 + 30 + 45 + 50 + 50 + 50 + 50 + 50 + 55 + 65 + 65 + 65 + 75 + 75 + 80 + 80 + 85 + 85 + 85 + 90 + 90 + 100 = 1485 and we have 25 data points so we divide that total by 25 to get:
1485/25 = 59.4
This tells us that the average score is 59.4
In statistics we use the word **MEAN** to describe the average of all the data point values.

2. When all the trainees have completed Task 3, discuss methods for putting data in increasing order. Read (carefully and slowly) the ORDERED list of data points so trainees can check if their lists are correct. Explain how to count the total number of data points and divide that by two to find the exact centre of the list and identify the MEDIAN. The number appearing most often is the MODE, and the average, or MEAN is calculated by adding values then dividing by frequency.

3. If trainees have questions, provide additional examples to illustrate and clarify the concepts.

Guided Practice Activity

1. Ask a volunteer to read **3.2 Key Facts** aloud. If any trainees have questions, see if another trainee can respond and explain—if not, explain concept to the class.

2. Work Task 4 on board, together with class, to review the measures of central tendency.

Topic 3.2 Task 4
The following table shows the marks obtained by students of form 3 in a math test.

Mark	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39
No. of students	5	8	12	18	14	11	6	3

a. Compare this data set to the data set of test scores in Task 3.
b. Find the range, mean, median and mode of the data presented in the table above.

Answer:

a. Answers will vary. Brainstorm, as a class, how the two sets are similar then how the two sets are different. Possible answers may include:

Similar: both are sets of test scores, both have lots of students' data, both have a variety of different scores, both are made-up, etc.

Different: the first set has a max of 100 points, this set has a max of 39 points, the range of the first set is 90 points while the range of this set is 39 points, the first had an overall frequency of 25 students this has a total frequency of 77 students, the first group was raw data, this is already organized into groups, etc.

Note: one important difference is that with raw data we know exact scores, but once the data has been grouped into classes we cannot be sure of exact data points any longer—we will be considering APPROXIMATE data values, but we can still work with the set and understand some interesting things about the set as a whole. To do this we will still determine the measures of central tendency – as we did during the first Learning Outcome when we found the Modal Class rather than the exact Mode. We also did this with the Range, an approximate range is the difference between the highest possible value (39) and the lowest possible value (0) so in this case the Range is 39 – 0 = 39.

b. **To find the MEDIAN of this data set:** total frequency of this data set is 77 (from adding frequencies of each of the classes). The median value is the data point in the exact centre of the data set so we take 77/2 = 38.5—thus the central data point, is the 39^{th} data point. We start in lowest classes and add up frequencies to find the class which holds the 39^{th} data point:

5 + 8 = 13 (not a high enough frequency to have the median yet so median is not in the first two classes

13+ 12 = 25 still not containing the median--the 39^{th} term. The median is not in this 3^{rd} class.

25+ 18 = 43. YES! Because 43 is higher than 39 we know that the 39^{th} value falls before the end of this class, so the median class is the 4^{th} class, which is 15-19. The median class is [15-19].

To find a more exact value for median class, use the mid-class value or (15+19)/2=17

Finding the MEAN of grouped data takes more calculations. Work carefully and organize well!

Again, we have the following table, so we re-organize the table into a new format:

The given table is the following:

Mark	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39
No. of students	5	8	12	18	14	11	6	3

Copy the two rows above into the first two **columns** on the black board per below. Using data from the first two rows, explain how to calculate the mid-class value and fill the 3^{rd} column.

With values for F and X, the final column can be filled. Do calculations with trainees.

Mark	Frequency (F)	Mid class value (X)	F x X
0-4	5	(0+4)/2 = 2	5 x 2 = 10
5-9	8	(5+9)/2 =7	8 x 7 = 56

10-14	12	(10+14)/2 = 12	12 x 12 = 144
15-19	18	(15+19)/2 = 17	18 x 17 = 306
20-24	14	(20+24)/2 = 22	14 x 22 = 308
25-29	11	(25+29)/2 = 27	11 x 27 = 297
30-34	6	(30+34)/2 = 32	6 x 32 = 192
35-39	3	(35+39)/2 = 37	3 x 37 = 111
Total:	77	Total:	1424

Note: we assume all the members of a class in our table take on the value of the mid-class value. The last column gives the total value per the number of members in that class. That is, each person in a class gets assigned the mid-class value as their data point. This is not exactly accurate, but it is a good approximation without more detailed information.

Add the last column (F x X for each class) to get the total value then divide by the total frequency to find the mean. The average – or mean -- score is 1424/77 = 18.5.

In summary, using grouped data, we found the measures of central tendency for this set of data as follows: Modal class is [15-19] and an approximate mode is thus the mid-class value thus 17. Median data point is the 77/2 = 38.5 (thus look for the 39^{th}) spot. The class is [15-19], or 17
The Mean is 18.5 which is found by making an organized table like the one above.

3. Ask trainees to pair up to complete Task 5 so they can practice the methods worked together during Task 4. Trainer should circulate among the groups to offer support and encouragement to trainees and to respond to any questions posed.

Topic 3.2 Task 5
A different group of students in form 3 took the same math test as was taken in Task 4. Their results were slightly different and are shown in the table below.

Mark	Frequency F	Mid class value, X	F x X
4-8	2	(4 + 8) /2 = 6	2 x 6 = 12
9-13	4	(9 + 13)/2= 11	4 x 11 =…
14-18	7	(14+18)/2 = …	…
19-23	14	…	…
24-28	8	…	…
29-33	5	…	…

a. Complete the table.
b. Identify the modal class and give and approximate value for the mode.
c. Find the range of the data and the median class and an estimate for the median value.
d. Use your completed table to calculate the mean of this class.
e. Consider the measures of central tendency that you have calculated here in Task 2and those that were calculated for Task 1 and draw a conclusion about which class has stronger students based on your analysis.

Answer:
a.

Mark	Frequency F	Mid class value X	F x X
4-8	2	6	12
9-13	4	11	44
14-18	7	16	112
19-23	14	21	294
24-28	8	26	208
29-33	5	31	155
Total:	40	Total:	855

b. The class modal is [19 – 23], which has the highest frequency (thus most data points) the approximate value for the mode is the mid-class value: approximate mode = 21

c. The range is 33-4 = 29, so to find median we take 29 / 2 = 14.5 so our median will be the 15^{th} value of the ordered data.
To find this we take the cumulative frequency by class: 2 + 4 = 6: median is not in the first two classes, 6 + 7 = 13 so median is not in the third class, 13 + 14 = 27 so the 15^{th} data value will fall into the 4^{th} class or in [19-23]. And estimated for median is the mid-class value so (19+23)/2 = 21

d. Mean is calculated by total F x X divided by the total frequency so 855/40 = 21.3

e. Although the Task 4 group had a few higher test scores than the Task 5 group of students (where no values were above 33), the Task 4 group of students also had lower scores than the Task 5 group. The overall class average was higher in the Task 5 class which tells us that overall, more students did better in the Task 5 class. We see Task 5 class also has a higher modal class than the Task 4 class—again the students in Task 5 are slightly stronger in math than those students in Task 4 class.

Application Activity

1. Trainees will now work on a similar problem, Task 6, independently. Walk around the class to help struggling trainees and to provide additional support as needed.

2. After approximately 10 minutes, trainees pair up and compare answers. If answers are different, trainees help each other find if there is a fault and to correct the work together. Trainees should have approximately 5 minutes to help and correct their peers.

3. At the end of the peer work time, a volunteer writes the correct answers on the board. Ensure the given answers are correct. Trainees self-correct their own work in their notebooks.

Topic 3.2 Task 6
The 30 members of a certain music band were asked how many instruments each could play. The results are set out in the frequency distribution table shown below:

Number of instruments, X	1	2	3	4	5	6
Frequency, F	11	10	5	3	1	0

a. Complete the table below:

Number of Instruments played by each student X	Frequency F	F x X
1		
2		
3		
4		
5		
6		
Totals:		

b. Determine the mode for this data set.
c. Identify the median value for this data set.
d. Calculate the mean for this data set.
e. Find the range of this data set.

Answer:

a.

Number of Instruments played by each student	Frequency F	F x X
1	11	11
2	10	20
3	5	15
4	3	12
5	1	5
6	0	0
Totals:	30	63

b. Mode is 1. Eleven students play 1 instrument (this data point has the highest frequency).
c. There are 30 data points so our median is the 30/2=15th data point. Once data points are ordered we can count up or look at cumulative frequency and see that the 15th data point is 2.
d. To calculate the mean, we take 63/30 = 2.1 but since you cannot play a partial instrument the decimal does not make sense so we round it to simply 2. The mean of this data set is 2.
e. The range of this data set is 5-1=4, note that we do not use 6 when we calculate our range because there are no students that can play 6 instruments.

Points to Remember

There are three main statistical measures, which attempt to locate a typical value of a data set. These are:

1. **Mean**—or average value of the data points
2. **Mode**—or the value that appears most often (also a modal class: highest frequency)

3. **Median**—the data value that has the same number of data values lower and higher

These values are called the **measures of central tendency** of a set of data.

When we are only given grouped data, we can only estimate these values and we rely on the mid-class value as the representative value for all data points in that class.

Formative Assessment

1. The Formative Assessment is designed to help trainees check their own knowledge. They will have 15 minutes to independently work on this assessment.

2. After 15 minutes, ask trainees to pair up and check their answers against their partner's answers. If they are not the same, work out which is correct by explaining methods and checking arithmetic.

3. Ask a volunteer to explain both methods and answers. Confirm the responses are correct.

Formative Assessment

1.

Find the median and the mean for the data sets below:

a. 20, 30, 16, 28, 42

b. 60, 50, 75, 56, 66, 80

Answer:

a. We arrange data in order of magnitude, smallest to largest: 16, 20, 28, 30, 42

There are 5 data values so the 3rd value has two values lower and two values higher so that is our median. In this case the value is 28.

To find the mode we add the values and divide by the frequency so we get:
(16 + 20 + 28 + 30 + 42)/5 = 136/5 = 27.2 so the mean of this data set is 27.2

b. We arrange the data: 50, 56, 60, 66, 75, 80

There are 6 data values so we take the 3rd and the 4th and find the average of those two:
(60 + 66)/2 = 63 so our median value is 63.
To find the mode we add the values and divide by the frequency so we get:
(50 + 56 + 60 + 66 + 75 + 80)/5 = 387 /6 = 64.5 so the mean of this data set is 64.5

2.

The mass of 31 boys is measured to the nearest kg and noted in the following table:

Mass (kg)	60 - 64	65 - 69	70 - 74	75 - 79	80 - 84	85 - 89
Frequency	2	4	10	9	5	1

Use the data provided in the table to answer the following questions:

a. Use the given data to complete the following table:

Mass	Frequency F	Mid class value X	F x X
60 - 64	2	(60 + 64)/2= 62	2 x 62 = 124
65 - 69	4	(65 + 69)/2=67	4 x 67 = …
70 - 74	10	(70 + 74)/2 = …	…
75 - 79	9	…	…
80 - 84	…	…	…
85 - 89	…	…	…
Total	…	Total	…

b. Find the modal class.
c. Calculate the range of this data set.
d. Identify the approximate median value.
e. Calculate the mean.

Answer:

a. Use the given data to complete the following table:

Mass	Frequency F	Mid class value X	F x X
60-64	2	(60 + 64)/2 = 62	2 x 62 = 124
65 - 69	4	(65 + 69)/2 = 67	4 x 67 = 268
70 - 74	10	(70 + 74)/2 = 72	10 x 72=720
75 - 79	9	(75 + 79)/2 = 77	9 x 77 = 693
80 - 84	5	(80 + 84)/2 = 82	5 x 82 = 410
85 - 89	1	(85 + 89)/2 = 87	1 x 87 = 87
Total	31	Total	2302

b. The modal class is the class with the highest frequency [70-74] with a frequency of 10.
c. The range of this data set is the difference between highest and lowest values: 89 -60 = 29
d. The approximate median value is found by finding the middle data point. 31/2=15.5 thus the 16th data point which is in the 3rd class [70-74], an approximate value, the mid-class value, is 72.
e. The mean is: 2302/31 = 74.3 kg. The decimal is okay because mass is continuous.

Learning Outcome 3.3: Present and communicate statistical data and analysis using mathematical representations (graphs, charts, and tables)

Objectives: By the end of the learning outcome, trainees will be able to:

a. Represent collected data in raw, tallied, or grouped formats on a graph, chart, or table

b. Analyse data that is presented in graph, chart, or table formats

Time Required: 5 hours

Learning Methodologies: pair, small group, and individual work, presentations

Materials Needed:

- Standard materials, trainee workbooks, calculators if available

Preparation:

❑ Prepare various data sets that are relevant to trainees' lives as additional examples should trainees need more practice

Cross Cutting Issues:

✔ **Gender:** trainer will provide equal opportunities to both boys and girls during the lesson as well as balance gender when forming groups

✔ **Peace/value education:** trainer promotes team work and respect for others' views and work

✔ **Inclusive education:** trainer is sensitive to differences in learning abilities and uses a variety of activities and materials to cater to different learning styles

Prerequisites: Trainees should already be comfortable with and able to:

- Use addition, subtraction, multiplication, and division
- Understand and calculate mathematical averages

Key Competencies:

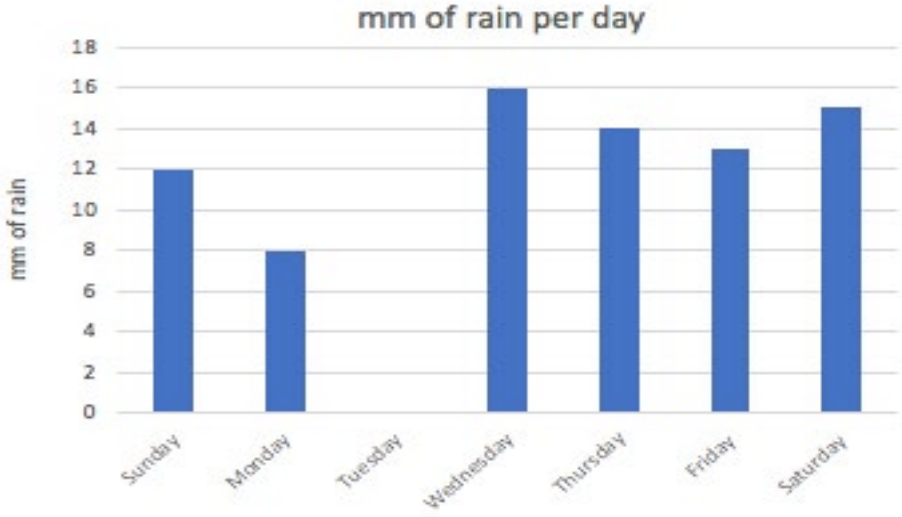
Knowledge	Skills	Attitudes
1. Differentiate types of statistical data	**1.** Collect and represent data	**1.** Team work
2. Explain the concept of mean, mode, median for ungrouped data	**2.** Calculate frequency, range, and mean (or average)	**2.** Persistence
3. Read and analyse data presented in charts, tables, and graphs	**3.** Create different types of statistical data presentation (polygons, bar chart, pie chart, tables)	**3.** Accuracy

Steps:

Getting Started: What do we know and where are we going?

1. Display the following bar chart on the blackboard. Ask trainees to observe it carefully. After 1 minute, brainstorm observations and ideas about what it represents. Write trainee observations and ideas on the board.

Topic 3.3 Task 1



a. Observe the above bar chart carefully. Brainstorm. Write down your observations and ideas.
b. Which day had the most rain? The least?
c. Calculate the mean rainfall during the week using information displayed on the chart.

Possible Answers:

a. This is a bar chart, it shows a rainy season week—because it rains nearly every day, some days rain more than others, on one day there was no rain at all, the next week data will probably be different because rains vary, etc.
Note: the bar chart is a visual display of data giving a quick idea about information being presented. BUT we can also use charts to calculate the measures of central tendency.

b. Wednesday had the most rain; Tuesday had the least rain.

c. Note there are 7 data points (one for each day of the week). All data points are "mm of rain."
Give the trainees about 2 minutes to calculate the mean individually then ask for a trainee to come to the board and share their working and answer with the class.
Data points are: 12, 8, 0, 16, 14, 13, 15
Add the data values and then divide by the number of data points:
12 + 8 + 0 + 16 + 14 + 13 + 15 = 78 then 78/7 = **11.1 mm of rain is the average (mean) during this week**.

2. Introduce the learning outcome, objectives, and the knowledge, skills and attitudes the trainees will gain. This outcome builds on knowledge and skills gained in the first 2 outcomes.

Problem Solving Activity

1. Trainees work in groups of 3. They have approximately 5 minutes to respond to the task—then the class will review the answers together. Groups must ensure that all members participate and understand the concepts clearly.

Topic 3.3 Task 2
A class of L2 students has recorded their ages. The ages all fall between 15 and 22 years old. The number of students at each age is recorded in the table below:

Age	Number of trainees
15	2
16	4
17	5
18	4
19	7
20	3
21	5
22	3

Using the data provided in the table above:

a. Display this data in a bar chart with clear axis labels so we can visualize the situation.

b. Calculate the range of this data.

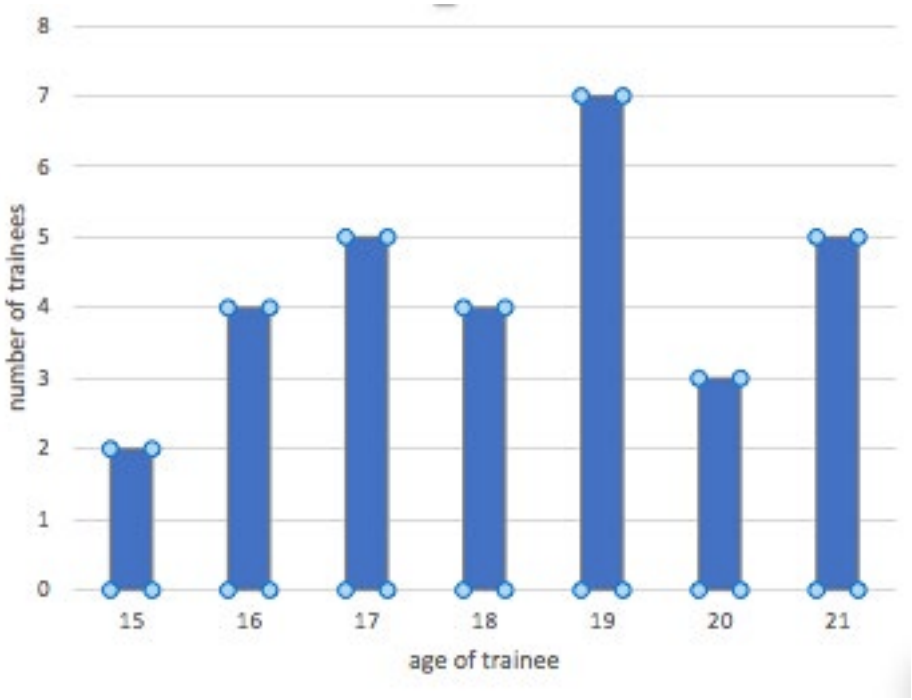
c. Find the median value for this data set.

d. Identify the mode of this data set—and explain how the bar chart make this task easy.

e. Calculate the mean of this data set (remember you will need to take the F x X for each age).

Answer:

a. A bar chart that displays this data is the following:



b. The range of this data is 21-15 = 6 years

c. The total frequency is 2 + 4 + 5 + 4 + 7 + 3 + 5 = 30 so there must be 30 students in this class.

d. The mode of this data set is 19—the value with the highest frequency—the bar chart makes the mode easy to see because it is simply the tallest bar.

e. To calculate the mean of this data set, take the F x X for each age, so add a column to the original table and calculate F x X:

Age (X)	Number of trainees (F)	X x F
15	2	30
16	4	64
17	5	85
18	4	72
19	7	133
20	3	60
21	5	105
Totals:	30	549

Use total frequency (30) and the total sum for the F x X to calculate the mean: 549/30 = 18.3. The mean age in this class is 18.3 years old.

2. Have trainees show their bar graphs representing this data. Using the bar graph which has been drawn on the board representing this data, trainer will explain the horizontal and vertical axes—and show the importance of labels.

-- Ask questions to help trainees become familiar with reading bar graphs:
How many children are 18 years of age? How many students are more than 19? Which bar is the highest? What does that mean? What does the lowest bar mean? Etc.

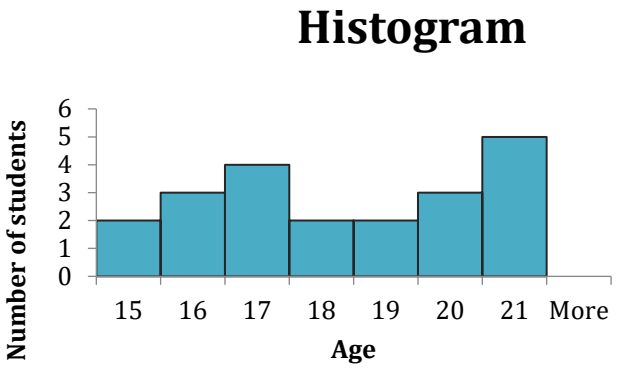
Guided Practice Activity

1. Remind trainees that the focus now is on ways to display data—or analysing data displays (rather than provided as raw data or in a data table). Ask a volunteer to read **3.3 Key Facts** aloud.

2. The class has already seen two different examples of **bar charts**—probably the most common type of data display. Ask if any questions remain about bar charts; respond and give examples.

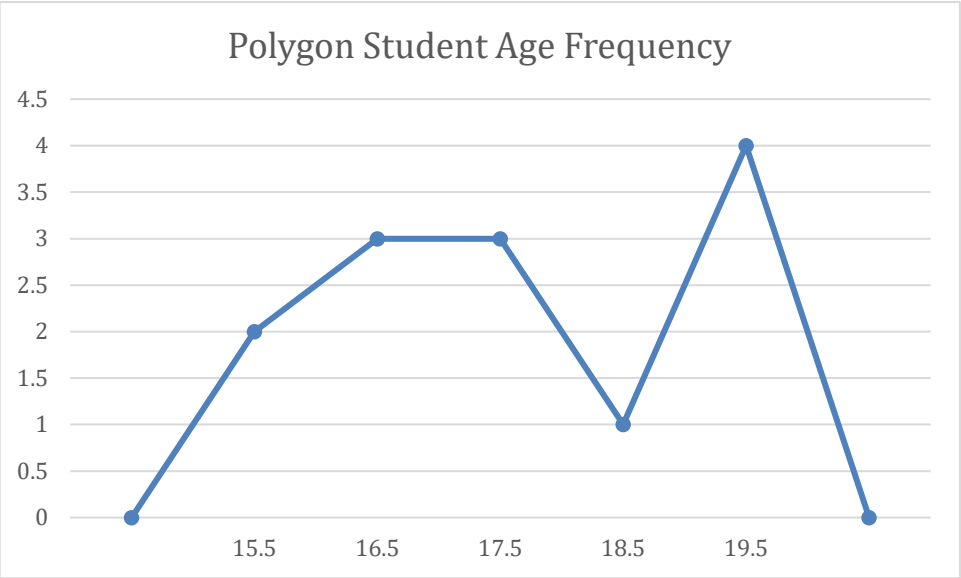
3. Tell trainees that a bar chart and a **histogram** are similar. A histogram is a bar chart for either continuous or grouped data so class boundaries touch – a bar chart is for discreet data. As an example, look at the age example worked during the Problem-Solving section above.

Age data is continuous data (a person can be 18 years and 6 months (that is 18.5) years old so trainees grouped as age of 18 are actually in a class that spans all the ages ranging from 18 to 19 years old. In this case we can use a histogram to display this age data with the x-axis changing slightly so the first bar starts at 15 and ends at 16 (where the second bar starts). The second bar starts at 16 and ends at 17 (where the third bar begins) and so forth. We do not change the y axis which remains frequency.



4. Discuss the histogram: what do they observe? How is it different from the bar chart?
Possible answers: the rectangles representing data values touch one another, in this case the ages that are considered a single age, say 19 which ranges from 19 to 20 years old.

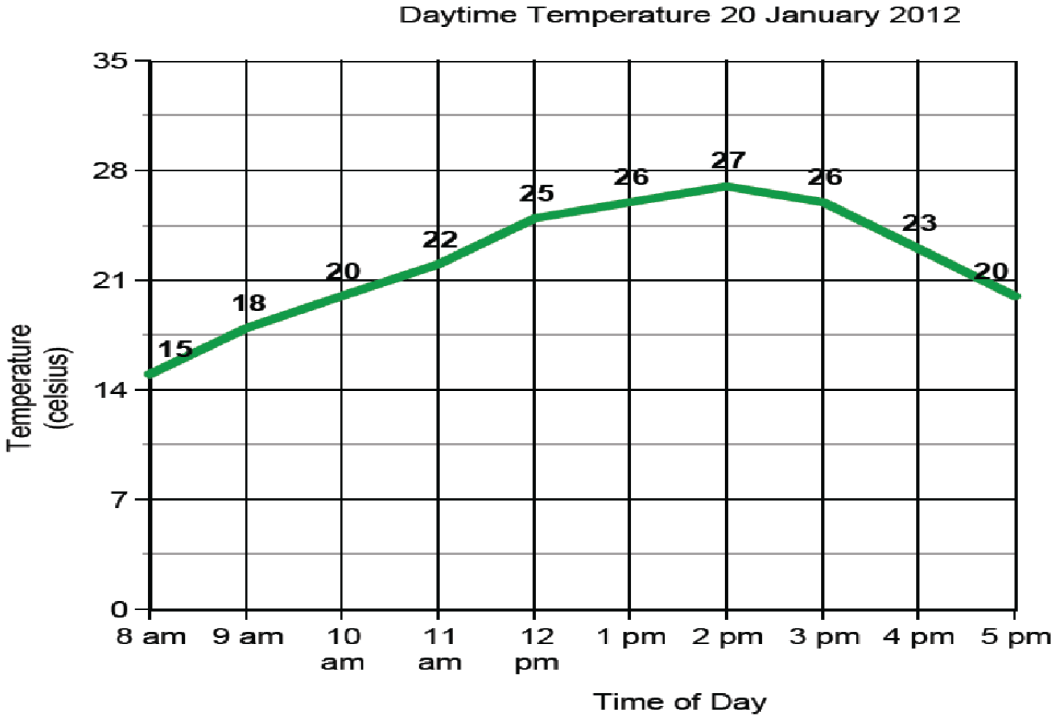
5. Refer to the **key facts**: a FREQUENCY POLYGON is related to a histogram. The midpoint of the top of each rectangle is joined by line segments to form a polygon (a many-sided figure with straight sides). Make a frequency polygon from the histogram above by adding midpoints and joining those midpoints with lines on the board.



6. More common graphs are **line graphs** and **pie charts**. Ask trainees to work in pairs on Task 3 and 4 in their manuals. Trainees have about 5 minutes to complete this task. Walk around to give support as needed and refer trainees back to the key facts.

Topic 3.3 Task 3

The Line graph below shows the temperature during some hours during the day on 20 January 2012. [16]



a. What type of data is presented in the table?
b. What was the temperature at 12 noon?
c. When was it the hottest? The coldest?

[16] Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher's Manual and Learner's Book*. Waltham: Education Development Center) Education Development Center.

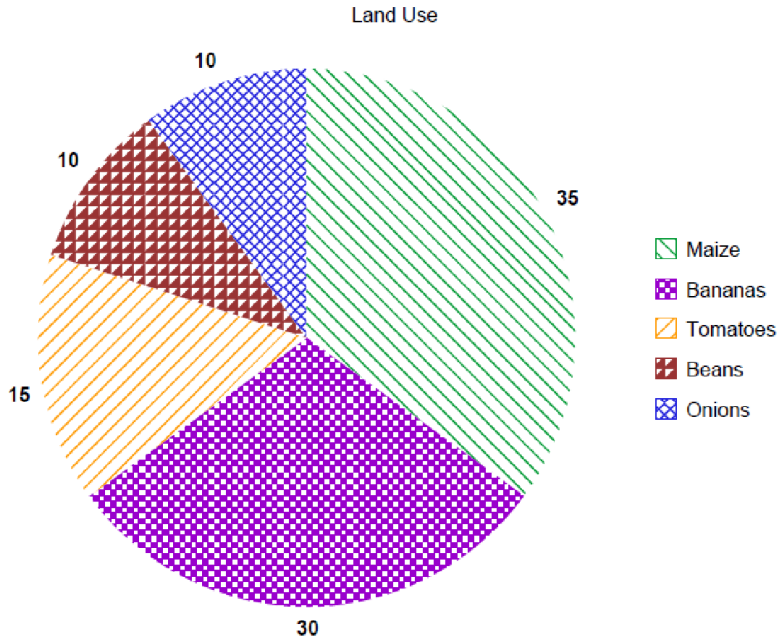
d. What is the range of temperatures that were measured?
e. Find the average temperature during the 10 hours that temperatures were recorded.

Answer:

a. Quantitative data (it is numbers) which is continuous (temperature is measured so you can have decimal values for the data points). NOTE: line graphs are best used for quantitative data.
b. 25 degrees Celsius
c. hottest: 2pm (27 degrees), coldest 8 am (15 degrees)
d. 27 – 15 = 12 so the range is 12 degrees
e. 15+18+20+22+25+26+27+26+23+20)/10 = 222/10 = 22.2 degrees C is the mean temperature

Topic 3.3 Task 4

Below you will see a method of representing data called a pie chart. The entire circle represents all the gathered data, and each portion represents one of the possible categories for data, the size showing the percentage of data that falls into that category. [17]



In this case, the pie chart represents use of agricultural land in a rural village. Crops are divided into 5 possible categories by what is planned: maize, bananas, tomatoes, beans, or onions.

The gathered data is the following:

Crop planted	Maize	Bananas	Tomatoes	Beans	Onions
Number of Hectares	36	30	16	10	10

a. How many hectares were surveyed for land use?
b. What information does this pie chart tell you?
c. What percentage of land is used for growing maize? Beans? Onions? Bananas?

[17] Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher's Manual and Learner's Book*. Waltham: Education Development Center) Education Development Center.

d. Which crop uses the most land?
e. Which crops together make up 50% of the land?

Answer:

a. 102 hectares were surveyed (the total of all the hectares)
b. The pie chart illustrates how much land goes to plant each of the different crops
c. To find the percentages we take the part over the whole and then find the decimal to the hundredth place.

Maize: 36/102=0.352 so we round to 35%
Bananas: 30/102=0.294 so we round to 29%
Tomatoes: 16/102=0.158 so we round to 16%
Beans: 10/102=0.098 so we round to 10%
Onions: 10/102=0.098 so we round to 10%

These percentages tell us how much of the circle each crop represents, so for example the part showing Maize should be 35% of the entire circle.

d. Maize uses the most land (35% of the total land)—the biggest part of the pie
e. If we take bananas, beans and onions together we will cover almost 50% of the land. Similarly, if we take maize and tomatoes together, that will cover a bit over 50% of the land.

7. The final sort of data representation is a **cumulative frequency graph**.
Ask if anyone remembers what cumulative means. Give trainees time to think then share ideas. Answer: cumulative means successively added—that is, the second number is added to the first, then the third is also added. Next the fourth is also added, and so on.

Since cumulative means all added together, then a cumulative frequency graph that means a graph of data values where frequencies are added together. Data values go on the x-axis and the cumulative (added up) frequency goes on the y-axis.

8. Ask trainees to carefully read Task 5 then write their initial thinking in their notebooks. They will work individually. After 3 minutes discuss answers as a class and verify and demonstrate as needed to ensure trainee understanding.

Topic 3.3 Task 5

The following frequency table shows marks earned by students in L2 in a Mathematics exercise:

Mark	1	2	3	4	5	6	7	8	9	10
Number of students	3	4	2	8	6	12	5	2	5	3

Data Value (Mark on exercise)	Frequency (number of students)	Cumulative Frequency
1	3	3
2	4	3+ 4 = 7
3	2	7 + 2 = 9
4	8	9 + 8 = 17
5	6	17 + 6 = 23
6	12	**23 = 12 = 35**
7	5	35 + 5 = 40

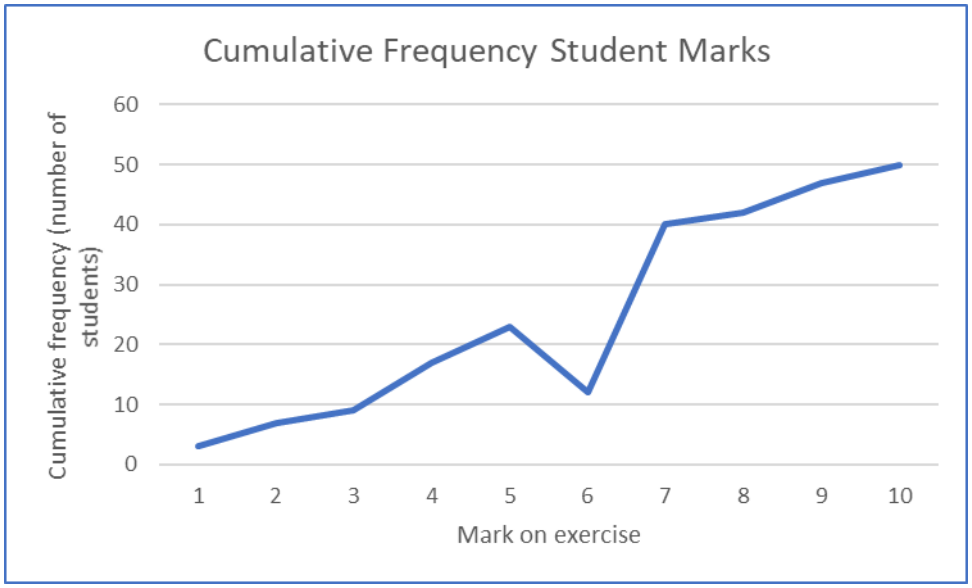
8	2	40 = 2 = 42
9	5	42 + 5 = 47
10	3	47 + 3 = 50
Total:	50	

Complete:

- **a.** Graph using Data Value as the x axis and Cumulative Frequency as the y axis
- **b.** The mode
- **c.** The median mark for this class

Answer:

a.



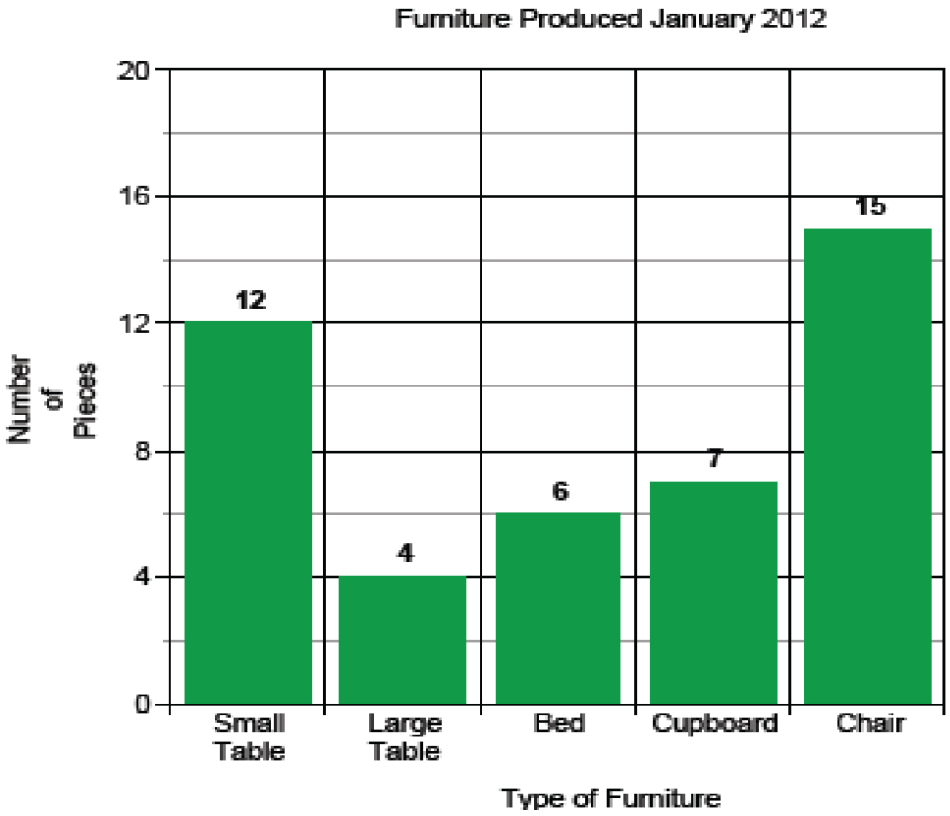
- **b.** The mode is the data value with the highest frequency: 12 students earned a mark of 6 so the data value of 6 is the mode for this data set.
- **c.** The total frequency is 50 so median is the 25th data point. Looking at the cumulative frequency we see that the mark 5 has only reached a cumulative frequency of 23 so we need to go to the next higher data value—with data value of 6, the cumulative frequency goes up to 35 (in bold above) thus the 25th data point falls into that value with a mark of 6. So 6 is our median.

Application Activity

1. Trainees can work individually or in pairs if they choose. Allot 15 minutes to complete Tasks 6, 7 and 8 then discuss the answers together. Walk around the class to address any questions and to guide trainees that are having difficulties.

Topic 3.3 Task 6

The following chart shows how many different types of furniture a factory produced during one month[18]:



a. What type of data is presented in the table?
b. Why would a carpenter want to keep track of the number of pieces of furniture he made in a month?
c. How many beds did the carpenter make? Chairs? Small tables?
d. The carpenter made 7 pieces of which type of furniture?
e. List the types of furniture produced, in order of the most to least number of pieces.

Answer:

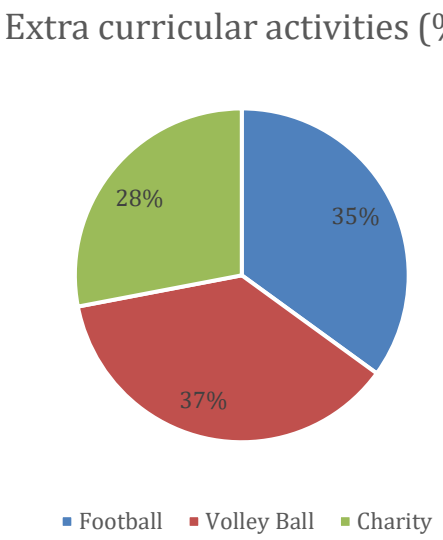
a. The data presented is quantitative data because you can count the number of pieces – How many small tables were produced? (12).
b. This data could help the managers plan for the coming month—how much wood they need, how many hinges, how many labourers, estimate the monthly income and expenses, etc.
c. Beds: 6, Chairs: 15, Small tables: 12.
d. The carpenter made 7 cupboards.
e. Chair, small table, cupboard, bed, and large table

Topic 3.3 Task 7

The following table and pie chart show the number of students involved in extracurricular activities.

[18] Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher's Manual and Learner's Book*. Waltham: Education Development Center) Education Development Center.

No	Extracurricular Activity	Number of Student	Percentage
1	Football	15	35%
2	Volley Ball	16	37%
3	Charity	12	28%



a. What information does the pie chart tell you?
b. Which extracurricular activity is the most popular?
c. How might the administration at a school use this information?

Possible Answers:

a. The pie chart tells the percentage of students involved in different types of extracurricular activities – football, volleyball and charity work.
b. Volleyball is the most popular, with the highest percentage (37%) of students participating. Then football, followed by charity.
c. The administration could use such information in planning and budgeting of extracurricular activities.

Topic 3.3 Task 8

At home, after school today:

a. Identify ways in which people around you use data or statistics to help their lives.
b. Look in books, magazines, or other written materials for tables, graphs, or charts that represent some sort of statistical data.

Points to Remember

- Data can be presented using a variety of visual representations. Those representations can be used to understand data more clearly.
- A **pie chart** is best used for qualitative data (categories rather than numbers)

- A **histogram** is best used for continuous, quantitative data.
- **Bar charts** can display either qualitative or quantitative data.
- A **line graph** plots two data points against one another, for example rainfall per day.

Formative Assessment

1. Ask trainees to look in their manuals at the Formative Assessment. There are two tasks that they will work on independently to check their own knowledge of the content of this Learning Outcome. Explain that this is designed to help trainees check their own knowledge. They will have 15 minutes to work on this assessment independently.

2. After 15 minutes, ask trainees to pair up to check answers with their partner. If answers are not the same try to work out which one is correct by explaining the methods used to convince your partner, help each other check the arithmetic, and review operations used.

3. Choose a volunteers to explain the methods and answers to the class. Trainer will confirm that the responses given are correct.

Formative Assessment

1.

In our village, there is a man called Kagabo who is an exemplary farmer. He has lots of animals and grows different types of crops. He divided his land into four plots as follows:

- 10 ha for bananas plants
- 5 ha for pineapples
- 5 ha for maize and
- 30 ha for animals

a. Determine what percentage of the whole plot of land is used for each different purpose.

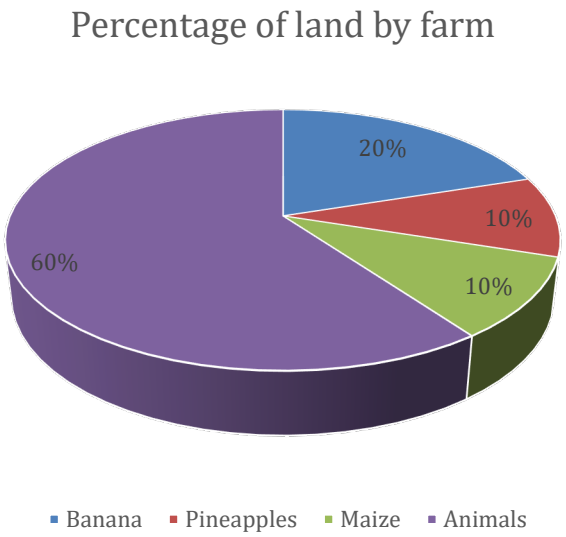
b. Use the percentages to sketch a pie chart for a visual representation of Kagabo's land use.

Answer:

a. The total amount of land is 10 + 5 + 5 + 30 = 50 ha so the percentages of each purpose are the following:

b. Bananas: 10/50 = 0.20 = 20%
Pineapples: 5/50 = 0.10 = 10%
Maize: 5/50 = 0.10 = 10%
Animals: 30/50 = 0.60 = 60%

c.



2.
Kagabo decided to record the milk production of his 12 cows. He found his cows provided the following litres of milk on Monday: 8, 3, 4, 5, 3, 10, 4, 3, 4, 4, 10, 4

a. Organize the data first in ascending order then into a frequency table.

b. What is the total amount of milk produced that day by Kagabo's cows?

c. What is the average number of litres produced by each of Kagabo's cows?

d. What is the most common amount of milk for a cow to provide?

Answer:

a. The ordered data is the following: 3,3,3,4,4,4,4,4,5,8,10,10
If we put this data into a frequency chart, we get the following:

Data Value: Litres X	Frequency: Number of cows F	Total Litres Produced: F x X
3	3	9
4	5	20
5	1	5
8	1	8
10	2	20
total	12 cows	62 litres total

b. The cows produced a total of 62 litres that day.
c. The average number of litres per cow is 62/12=5.2 litres per cow.
d. The most common amount is the mode—and that is 4 litres in this data set.

3.
The table below shows the summary records of milk produced daily for the whole week.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Litres	62	60	55	50	60	65	53

a. Present the data on a graph, reporting on X-Axis the days and on Y-Axis the litres.

b. How many litres were milked on Thursday?

c. How many litres were milked on Friday?

d. Which day had the highest number of litres and how many?

e. Which day had the lowest number of litres and how many?

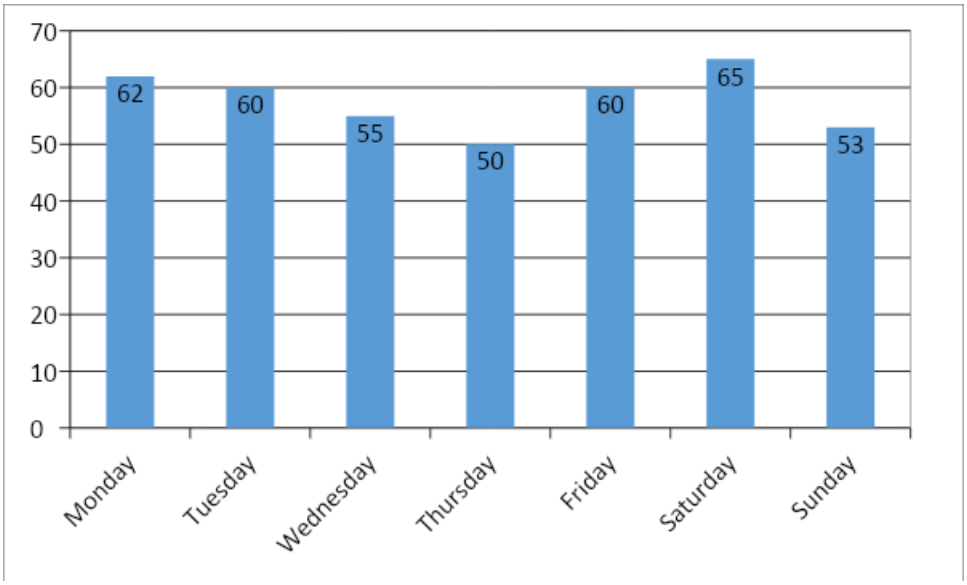
f. What was the total amount of milk collected during that week?

g. Calculate the average production per day.

h. If 1 litre of milk costs 200 Frw, how much money did Kagabo get within that week?

Answer:

a.



b. 50 litres

c. 60 litres

d. Saturday has the highest litres—65 litres were produced that day.

e. Thursday was the lowest day with only 50 litres.

f. 62 + 60 + 55 + 50 + 60 + 65 + 53 = 405 litres were collected during that week.

g. Average production: 405/7 = 58 litres per day

h. 405 litres x 200 Frw = 81,000 Frw is the money that the farmer made that week.

Summative Assessment - Unit 3

1.

The data listed below is the weights (in kg) of bags of maize grains sold by farmers in Huye.

87 89 104 101 99 85 92 109
84 86 98 97 93 100 94 88
91 108 88 98 114 99 103 91
80 96 113 82 87 90 93 100
105 101 106 95 102 96 90 99
447 480 509. 473. 495 470. 472. 487

a. Organize this data by constructing a grouped frequency table with class intervals of 5kg starting with the classes [80-84], [85-89], [90-94], …. and so on.

b. Calculate the range of this data.

c. Determine the modal class for this data.

d. Calculate the mean weight of a sack of maize in Huye town.

Answer:

a.

Weight (in kg)	Number of pieces of luggage (frequency: F)	Mid class value (X)	F x X
80-84	3	(80 + 84)/2 = 82	246
85-89	7	(85 + 89)/2 = 87	609
90-94	8	(90 + 94)/2 = 92	736
95-99	9	(95 + 99)/2 = 97	873
100-104	7	(100 + 104)/2 = 102	714
105-109	4	(105 + 109)/2 =107	428
110-114	2	(110 + 114)/2 =112	224
	Total: 40		Total: 3830

b. 114 (highest weight) – 80 (lowest weight) = 34 kg which is the range.

c. Modal class for this data is [95-99]

d. The total of all weight values is 3830 kg. There are 40 sacks so the mean is 3830/40=95.75 kg.

2.

The table below shows the weights (in kg) of 50 pieces of luggage carried by Onatracom bus from Kigali to Bujumbura.

Weight (in kg) (data value: X)	Number of pieces of luggage (frequency: F)
7-9	2
10-12	8
13-15	14
16-18	19
19-21	7

a. Determine the mid-class value for each of the classes in the table above.

b. Identify the median value for this data set.

c. Use the mid-class value to find the F x X value for each row.

d. Calculate the mean (or average) weight for a piece of luggage on the bus.

Answer:

a. and c.

Weight (in kg) (data value)	Number of pieces of luggage (frequency: F)	Mid class value (X):	F x X
7-9	2	8	16
10-12	8	11	88
13-15	14	14	196
16-18	19	17	323
19-21	7	20	140

b. 50/2 = 25. The 25th data point is in class [16-18]. We assign the mid class value: 17 as median.

c. Sum total of all F x X values is 763. Total frequency is 50. Mean is 763/50 = 15.26 kg.

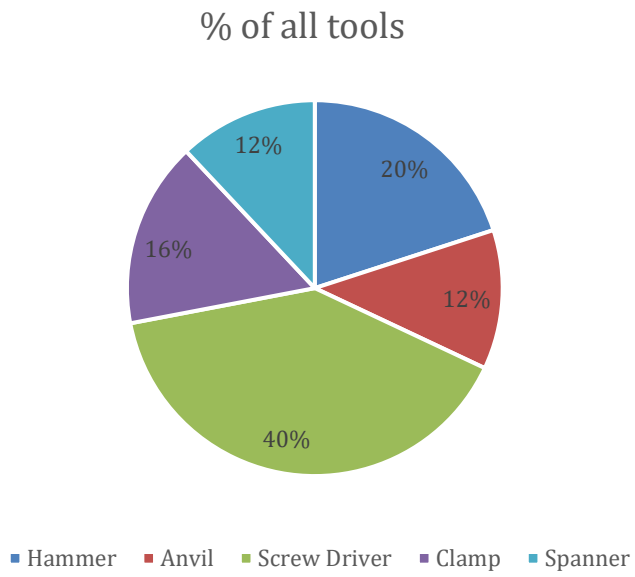
3.

A blacksmith had the following tools: 5 hammers, 3 anvils, 10 screw drivers, 4 clamps, 3 spanners. Record this data in a table and sketch a pie chart to represent this situation.

Answer:

Tool:	**Frequency:**	**% of all tools**
hammer	5	5/25 = 20%
Anvil	3	3/25 = 12%
screw driver	10	10/25 = 40%
Clamp	4	4/25 = 16%
spanner	3	3/25 = 12%
Total:	**25**	**25/25 = 100%**

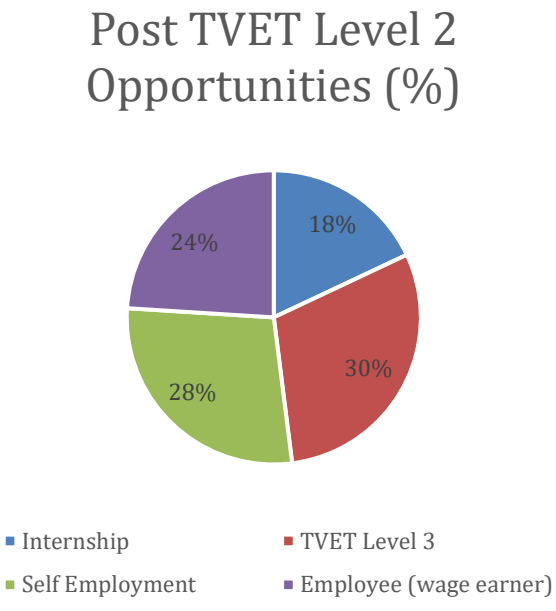
Pie chart:



4.

Use the following data table and pie chart to answer the questions below.

No	Post Level 2 Options	Percentage	Number of Students
1	Internship	18%	9
2	TVET Level 3	30%	15
3	Self Employment	28%	14
4	Employee (wage earner)	24%	12



a. Interpret the data table and pie chart. What type of information does it give you?
b. According to the pie chart, what do most trainees in this class do after their Level 2 TVET programme?
c. What percentage of trainees become self employed or employees (wage earners) after their Level 2 TVET progamme?
d. Assuming each trainee only does one of these post Level 2 activities and they are from the same program, how many trainees are there in the program?

Answers:

a. The tables provide information on what trainees are doing after a Level 2 TVET programme. Some go on to internships, others become self employed, some go on to Level 3 programmes and others become employees.
b. Most trainees in this particular class go on to a Level 3 TVET programme.
c. 28% + 24% = 52% of the trainees.
d. 9 + 15 + 14 +12 = 50 trainees.

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