TVET CERTIFICATE IV in SOLAR ENERGY

# BASICS OF ELECTRICITY

SOLBE401

## Apply basics of electricity

Competence

**Credits: 8**

**Learning hours: 80**

**Sector: Energy**

**Sub-sector: Solar Energy**

**Module Note Issue date:** June, 2020

### Purpose statement

This core module describes the skills, knowledge and attitude required to interpret basic concepts in electricity, analyze characteristics of AC signal, study the behavior of different AC circuit and analyze 3-phase circuit. The learner will be able to apply DC circuits laws and theorems, apply magnetism and electromagnetism, identify and interpret waveform signals. He/she will also be able to identify power in AC circuits and apply power factor improvement techniques.

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# Learning Unit 1 – Interpret basic concepts in electricity

## LO 1.1 – Identify electrical quantities

- Content/Topic 1: Introduction to electricity and its means of generation

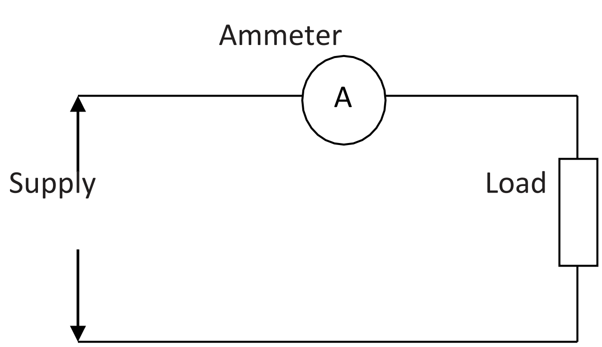
Alternating current (AC): is the current which flow in two directions and is coming from a.c generator.

Direct current (DC): is the current which flow in one direction and is coming from cell, battery, d.c generator etc.

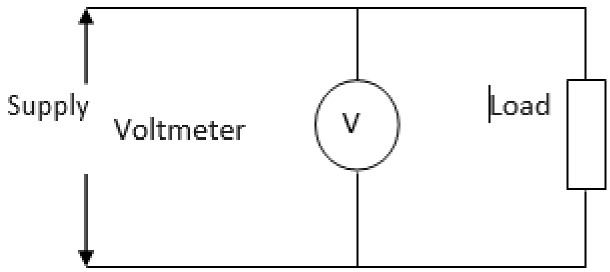
Generally says,the current flows from positive (+) terminals to negative (-) terminal. But electrons flow from negative terminal to positive terminal. The flow of electron is called current. Related to this, we study about some of the electrical terms.

- Content/Topic 2: Different electrical quantities

(a) Electrical current: The continous flow of free electrons constitutes an electric current. The unit of current is Ampere (A) and is measured by Ammeter. Ammeter is connected in series with the circuit. It denoted by the letter ''I''




(b) Voltage: The electrical pressure which is used to move the electrons is called Voltage. It is denoted by the letter ''V''. The unit of voltage is Volt and is measured by Voltmeter. Voltmeter is connected in parallel with the circuit.




(c) Resistance: The property of conduct which opposes the flow of current through it is called resistance. It is denoted by the letter ''R''. The unit of resistance is ohms (Ω) and is measured by ohmmeter.

(d) Electromotive force(e.m.f): In a circuit, a force is used to conduct the electrons from one point to another point is called Electromotive force. The unit of e.m.f is volt.

*e.m.f = P.d +voltage drop.*

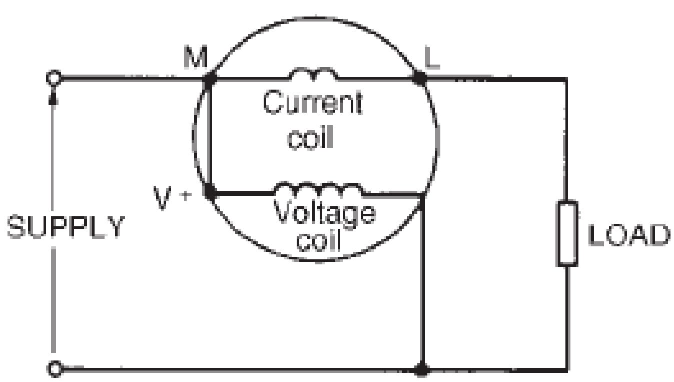
(e) Potential difference: It is represented by, the potenti1al difference between two point in the electrical circuit. Shortly it is called P.D and the unit is volt.

(f) Electrical Power: Power is defined as the product of voltage and current. Unit of power is watts. The energy absorbed by an appliance in one hour is called the energy consumed by the appliance.It unit is watt and denoted by the letter ''P''.

P = VxI Watts

Electric work Q = Pxt Watthour.

One killo watt hour = 1unit



EX.1) A 100 W electric light bulb is connected to a 250 V supply. Determine (a) the current flowing in the bulb, and (b) the resistance of the bulb.

$$\text{Power } P = V \times I, \text{ from which, current } I = \frac{P}{V}$$

$$\text{(a)} \quad \text{Current } I = \frac{100}{250} = \frac{10}{25} = \frac{2}{5} = \mathbf{0.4\ A}$$

$$\text{(b)} \quad \text{Resistance } R = \frac{V}{I} = \frac{250}{0.4} = \frac{2500}{4} = \mathbf{625\ \Omega}$$

EX.2) A current of 5 A flows in the winding of an electric motor, the resistance of the winding being 100Ω. Determine (a) the p.d. across the winding, and (b) the power dissipated by the coil.

$$\text{(a)} \quad \text{Potential difference across winding, } V = IR = 5 \times 100 = \mathbf{500\ V}$$

$$\text{(b)} \quad \text{Power dissipated by coil, } P = I^2R = 5^2 \times 100 = \mathbf{2500\ W\ or\ 2.5\ kW}$$

EX.3) A source e.m.f. of 5 V supplies a current of 3 A for 10 minutes. How much energy is provided in this time?

$$\text{Energy} = VIt = 5 \times 3 \times (10 \times 60) = 9000 \text{ Ws or J} = \mathbf{9\ kJ}$$

(g) Conductance:

- Conductance is reciprocal of resistance whereas resistance of a conductor measures the opposition which offers to the flow of current, hence the conductance measures the inducement, which offers to flow of current.
- Conductance is de degree to which an object conducts electricity.
- Conductance is the ability for electricity to flow a certain path.

Its unit is SIEMENS (S) and denoted by the letter G.

Conductance *G = 1/R S.*

Generally, the materials are classified by its conductance as they are:

1. **A conductor** is a material having a low resistance which allows electric current to flow in it. All metals are conductors and some examples include: copper, aluminum, brass, platinum, silver, gold and carbon.

2. **An insulator** is a material having a high resistance which does not allow electric current to flow in it. Some examples of insulators include: plastic, rubber, glass, porcelain, air, paper, cork, mica, ceramics and certain oils.

3. **Semiconductor:** The material whose conductivity lie in between conductor and Insulator is called semiconductor.

**EX:** Germanium, Silicon.

## LO 1.2 – Apply DC circuits laws and theorems

- Content/Topic 1: Ohm's law

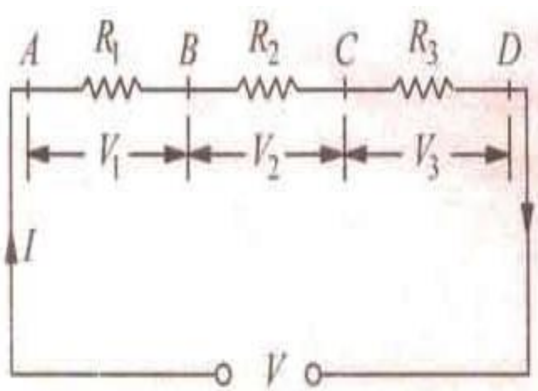
Ohm's law states that the current I flowing in a circuit is directly proportional to the applied voltage V and inversely proportional to the resistance R, provided the temperature remains constant. Thus,

$$I = \frac{V}{R} \text{ or } V = IR \text{ or } R = \frac{V}{I}$$

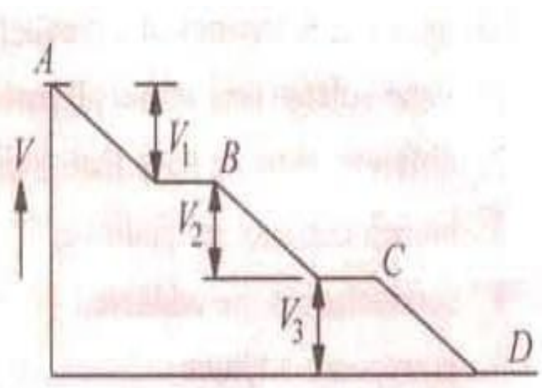
- Content/Topic 2: Series, parallel and mixed connection of resistors

Resistance in Series

When some conductors having resistances *R1*,*R2* and *R3* etc. are joined end-on-end as in Fig.(a) below, they are said to be connected in series. It can be proved that the equivalent resistance or total.
resistance between points *A* and *D* is equal to the sum of the three individual resistances. Being a series circuit, it should be remembered that (i) current is the same through all the three conductors *(ii)* but voltage drop across each is different due to its different resistance and is given by Ohm's Law and *(iii)* sum of the three voltage drops is equal to the voltage applied across the three conductors. There is a progressive fall in potential as we go from point *A* to *D* as shown in Fig.(b) below.




(a)




(b)

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

But $V = IR$

where $R$ is the equivalent resistance of the series combination.

$$IR = IR_1 + IR_2 + IR_3 \text{ or } R = R_1 + R_2 + R_3.$$

Also,

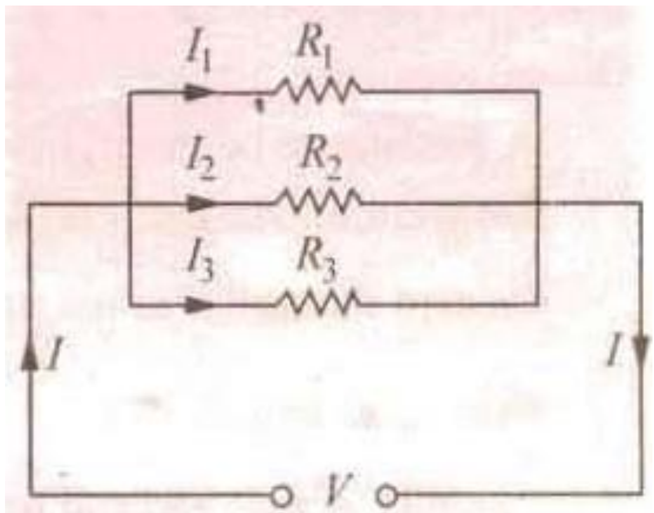
$$\frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}$$

**The main characteristics of a series circuit are:**

1. same current flows through all parts of the circuit.
2. different resistors have their individual voltage drops.
3. voltage drops are additive.
4. applied voltage equals the sum of different voltage drops.
5. resistances are additive.
6. powers are additive.

**Resistances in Parallel**

Three resistances, as joined in Fig. below are said to be connected in parallel. In this case (i) p.d. across all resistances is the same (ii) current in each resistor is different and is given by Ohm's Law and(iii) the total current is the sum of the three separate currents.




$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Now,

$$I = \frac{V}{R} \text{ where } V \text{ is the applied voltage.}$$

$R$ = equivalent resistance of the parallel combination.

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \text{ or } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

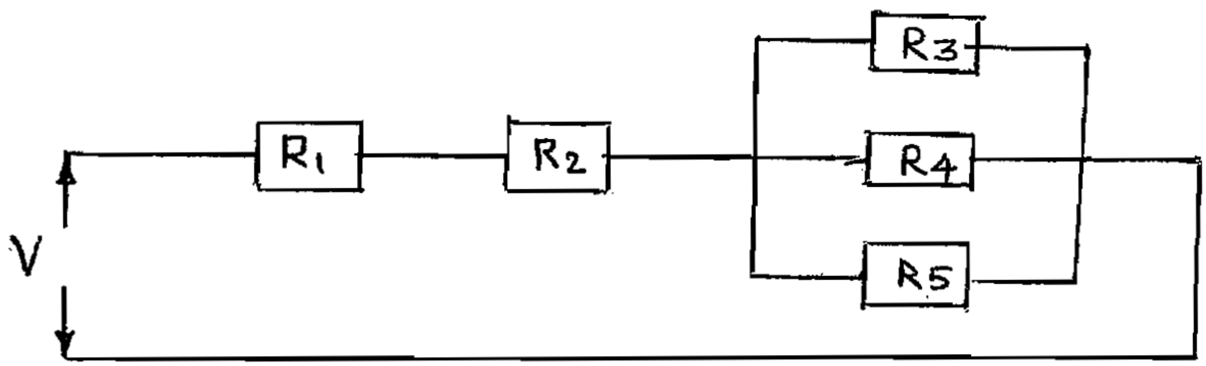
Also,

$$G = G1 + G2 + G3$$

**The main characteristics of a parallel circuit are:**
1. same voltage acts across all parts of the circuit
2. different resistors have their individual current.
3. branch currents are additive.
4. conductances are additive.
5. powers are additive.

**Resistance in series parallel circuit**

In this circuit one and more resistors connected in series with one more resistors connected in parallel. It is a combination of series and parallel circuit.




In the above series parallel circuit, there are five resistors (R1, R2, R3, R4, and R5) placed in it among them R1, R2 are connected in series and R3,R4,R5 are connected in parallel. The parallel resistors are connected in series with R1 and R2.
Hence the total resistance (R) of the circuit is:

$$R = R_1 + R_2 + \frac{R_3 \times R_4 \times R_5}{R_4 R_5 + R_5 R_3 + R_3 R_4}$$

EX.1. What is the resistance of a coil which draws a current of (a) 50 mA and (b) 200 μA from a 120 V supply?

Solution

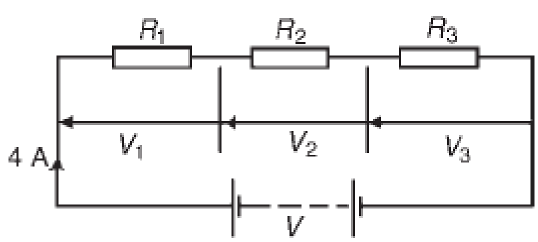
$$\text{(a)}\quad \text{Resistance } R = \frac{V}{I} = \frac{120}{50 \times 10^{-3}}$$

$$= \frac{120}{0.05} = \frac{12\,000}{5} = \mathbf{2\,400\ \Omega \text{ or } 2.4\ k\Omega}$$

$$\text{(b)}\quad \text{Resistance } R = \frac{120}{200 \times 10^{-6}} = \frac{120}{0.0002}$$

$$= \frac{1200\,000}{2} = \mathbf{600\,000\ \Omega \text{ or } 600\ k\Omega \text{ or } 0.6\ M\Omega}$$

EX.2. For the circuit shown in Figure below, determine (a) the battery voltage V, (b) the total resistance of the circuit, and (c) the values of resistance of resistors R1, R2 and R3, given that the p.d.'s across R1, R2 and R3 are 5 V, 2 V and 6 V respectively.



**Solution**

(a) Battery voltage $V = V_1 + V_2 + V_3$

$$= 5 + 2 + 6 = \mathbf{13\ V}$$

(b) Total circuit resistance $R = \frac{V}{I} = \frac{13}{4} = \mathbf{3.25\ \Omega}$

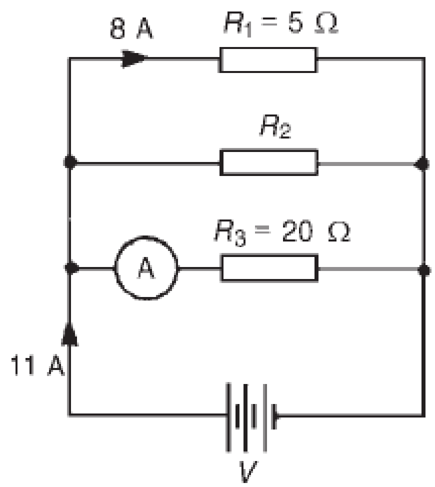
(c) Resistance $R_1 = \frac{V_1}{I} = \frac{5}{4} = \mathbf{1.25\ \Omega}$

Resistance $R_2 = \frac{V_2}{I} = \frac{2}{4} = \mathbf{0.5\ \Omega}$

Resistance $R_3 = \frac{V_3}{I} = \frac{6}{4} = \mathbf{1.5\ \Omega}$

(Check: $R_1 + R_2 + R_3 = 1.25 + 0.5 + 1.5 = 3.25\ \Omega = R$)

EX.3. For the circuit shown in Figure below, determine (a) the reading on the ammeter, and (b) the value of resistor R2




Solution

P.d. across $R_1$ is the same as the supply voltage $V$.

Hence supply voltage, $V = 8 \times 5 = 40$ V

(a) Reading on ammeter, $I = \frac{V}{R_3} = \frac{40}{20} = \mathbf{2\ A}$

(b) Current flowing through $R_2 = 11 - 8 - 2 = 1$ A

Hence, $R_2 = \frac{V}{I_2} = \frac{40}{1} = \mathbf{40\ \Omega}$

- <u>Content/Topic 3: Kirchhoff's laws</u>

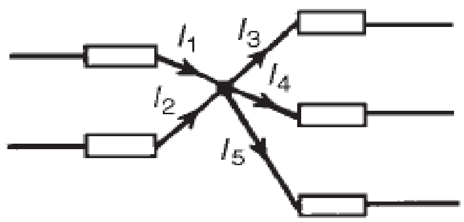
Kirchhoff's laws

Kirchhoff's laws state:

**(a) Current Law:** At any junction in an electric circuit the total current flowing towards that junction is equal to the total current flowing away from the junction, i.e. $\Sigma I=0$

Thus, referring to Figure below

$I_1 + I_2 = I_3 + I_4 + I_5$ or $I_1 + I_2 - I_3 - I_4 - I_5 = 0$

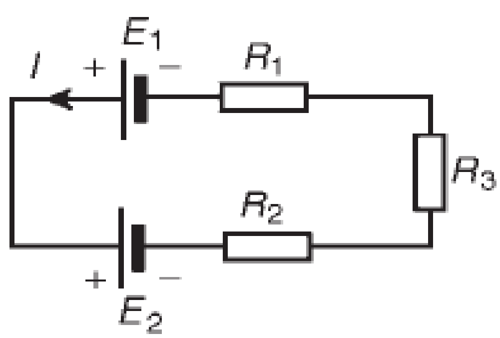



**b) Voltage Law:** *In any closed loop in a network, the algebraic sum of the voltage drops (i.e. products of current and resistance) taken around the loop is equal to the resultant e.m.f. acting in that loop*.

Thus, referring to Figure below: $E_1 - E_2 = IR_1 + IR_2 + IR_3$

(Note that if current flows away from the positive terminal of a source, that source is considered by convention to be positive.

Thus moving anticlockwise around the loop of Figure below, $E_1$ is positive and $E_2$ is negative.)




- <u>Content/Topic 4: DC circuits analysis theorems</u>

Introduction

The laws which determine the currents and voltage drops in d.c. networks are: (a) Ohm's law, (b) the laws for resistors in series and in parallel, and (c) Kirchhoff's laws.

In addition, there are a number of circuit theorems which have been developed for solving problems in electrical networks. These include:

(ii) Thevenin's theorem
(ii) Norton's theorem
(iii) The maximum power transfer theorem

**Thévenin's theorem**

It provides a mathematical technique for replacing a given network, as viewed from two output terminals, by *a single voltage source with a series resistance.* It makes the solution of complicated networks (particularly, electronic networks) quite quick and easy. The application of this extremely useful theorem will be explained with the help of the following simple example.

Suppose, it is required to find current flowing through load resistance $R_L$ as shown in Fig. (a) below.

We will proceed as under:
*1.* Remove $R_L$ from the circuit terminals *A* and *B* and redraw the circuit as shown in Fig. *(b) below.* Obviously, the terminals have become open-circuited.
2. Calculate the open-circuit voltage $V_{OC}$ which appears across terminals *A* and *B* when they are open *i.e.* when $R_L$ is removed.
As seen, $V_{OC}$ = drop across $R_2 = IR_2$ where *I* is the circuit current when *A* and *B* are open.

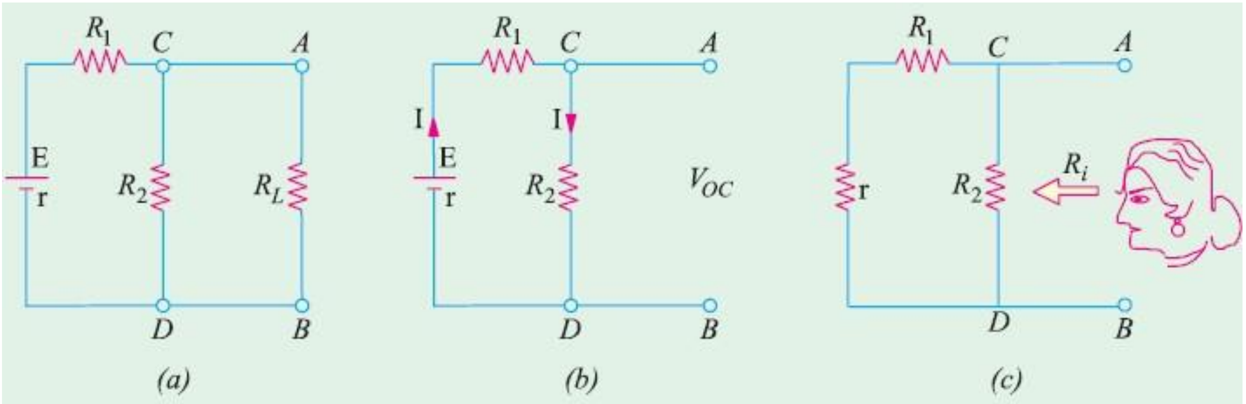
$$I = \frac{E}{R_1 + R_2 + r} \qquad \therefore V_{oc} = IR_2 = \frac{ER_2}{R_1 + R_2 + r}$$

*[r is the internal resistance of battery]*
*Voc* is also called 'Thevenin voltage' *Vth.*

3. Now, imagine the battery to be removed from the circuit, leaving its internal resistance *r* behind and redraw the circuit, as shown in *Fig. (c) below.* When viewed *inwards* from terminals *A* and *B,* the circuit consists of two parallel paths: one containing $R_2$ and the other containing $(R_1 + r)$. The equivalent resistance of the network, as viewed from these terminals is given as:

$$R = R_2 || (R_1 + r) = \frac{R_2(R_1 + r)}{R_2 + (R_1 + r)}$$

This resistance is also called, *Thevenin resistance $R_{th}$ (though, it is also sometimes written as $R_i$ or $R_o$).

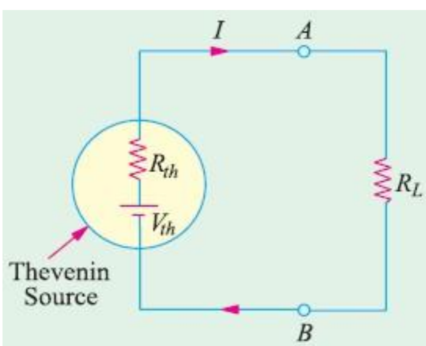



Consequently, as viewed from terminals *A* and *B,* the whole network (excluding *$R_1$)* can be reduced to a single source (called Thevenin's source) whose e.m.f. equals $V_\propto$ (or $V_{th}$) and whose internal resistance equals $R_{th}$ (or $R_i$) as shown in Fig. below.

4. $R_L$ is now connected back across terminals *A* and *B* Source

from where it was temporarily removed earlier. Current flowing through $R_L$ is given by

$$I = \frac{V_{th}}{R_{th} + R_L}$$




It is clear from above that any network of resistors and voltage sources (and current sources as well) when viewed from any points *A* and *B* in the network, can be replaced by a single voltage source and a single resistance in series with the voltage source.

After this replacement of the network by a single voltage source with a series resistance has been accomplished, it is easy to find current in any load resistance joined across terminals *A* and *B.*

This theorem is valid even for those linear networks which have a nonlinear load.

Hence, Thevenin's theorem, as applied to d.c. circuits, may be stated as under :

*The current flowing through a load resistance $R_L$ connected across any two terminals A and B of* *a linear, active bilateral network is given by $V_{OC}$ ||($R_i$+$R_L$) where $V_{OC}$ is the open-circuit voltage (i.e. voltage across the two terminals when $R_L$ is removed) and $R_i$ is the internal resistance of the network as viewed back into the open-circuited network from terminals A and B with all voltage sources replaced by their internal resistance (if any) and current sources by infinite resistance.*

How to Thevenize a Given Circuit?

1. Temporarily remove the resistance (called load resistance $R_L$ whose current is required.

*2.* Find the open-circuit voltage $V_{OC}$ which appears across the two terminals from where resistance has been removed. It is also called Thevenin voltage $V_{th}$.

*3.* Compute the resistance of the whose network as looked into from these two terminals after all voltage sources have been removed leaving behind their internal resistances (if any) and current sources have been replaced by open-circuit *i.e.* infinite resistance. It is also called Thevenin resistance $R_{th}$

*4.* Replace the entire network by a single Thevenin source, whose voltage is $V_{th}$ or $V_{OC}$ and whose internal resistance is $R_{th}$ or $R_i$.

5. Connect $R_L$ back to its terminals from where it was previously removed.

6. Finally, calculate the current flowing through $R_L$ by using the equation.

$$I = \frac{V_{th}}{(R_{th} + R_L)} \quad or \quad I = \frac{V_{oc}}{(R_i + R_L)}$$

**Norton's theorem**

This theorem is an alternative to the The venin's theorem. In fact, it is the dual of The venin's theorem. Whereas The venin's theorem reduces a two-terminal active network of linear resistances and generators to an equivalent constant-voltage source and series resistance, Norton's theorem replaces the network by an equivalent constant-current source and a parallel resistance.

**This theorem may be stated as follows:**

*(i)* Any two-terminal active network containing voltage sources and resistance when viewed from its output terminals is equivalent to a constant current source and a parallel resistance. The constant current is equal to the current which would flow in a short-circuit placed across the terminals and parallel resistance is the resistance of the network when viewed from these open- circuited terminals after all voltage and current sources have been removed and replaced by their internal resistances.

**Explanation**
As seen from *Fig. (a) below* a short is placed across the terminals *A* and *B* of the network *with all its energy sources present.* The short-circuit current $I_{SC}$ gives the value of constant-current source.
For finding $R_i$ all sources have been removed as shown in *Fig. (b) below.* The resistance of the network when looked into from terminals *A* and *B* gives $R_i$ The Norton's equivalent circuit is shown in *Fig. (c) below.* It consists of an ideal constant-current source of infinite internal resistance having a resistance of $R_i$ connected in parallel with it.

*(ii)* Another useful generalized form of this theorem is as follows:
*The voltage between any two points in a network is equal to $I_{SC}.R_i$ where $I_{SC}$ is the short-circuit current between the two points and $R_i$ is the resistance of the network as viewed from these points with all voltage sources being replaced by their internal resistances (if any) and current sources replaced by open-circuits.*

Suppose, it is required to find the voltage across resistance $R_3$ and hence current through if *[Fig. (d) below].* If short-circuit is placed between *A* and *B,* then current in it due to battery of e.m.f. $E_1$ is
$E_1/R_1$ and due to the other battery is $E_2/R_2$

$$I_{sc} = \frac{E_1}{R_1} + \frac{E_2}{R_2} = E_1G_1 + E_2G_2$$

where $G_1$ and $G_2$ are branch conductances.

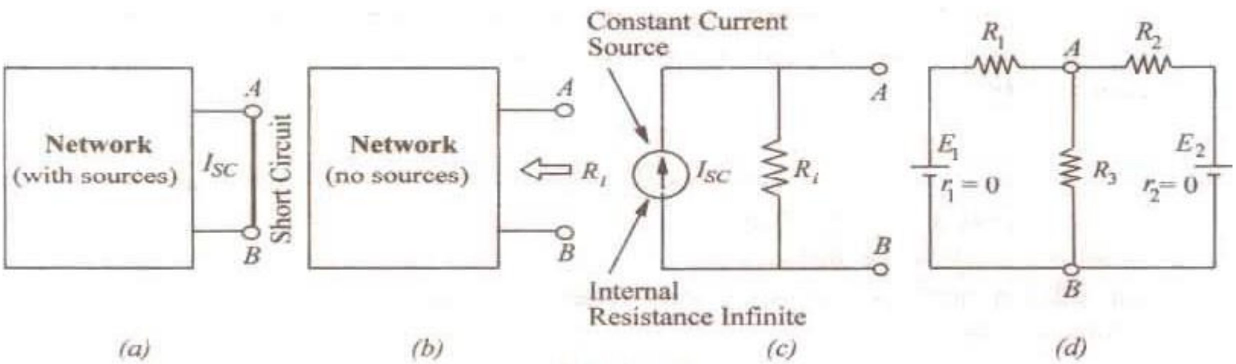
Now, the internal resistance of the network as viewed from *A* and *B* simply consists of three resistances *R1, R2* and *R3* connected in parallel between *A* and *B.* Please note that here load resistance
*R3 has not been removed.* In the first method given above, it has to be removed.

$$\frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = G_1 + G_2 + G_3$$

$$R_i = \frac{1}{G_1 + G_2 + G_3}$$

$$V_{AB} = I_{sc}.R_i = \frac{E_1G_1 + E_2G_2}{G_1 + G_2 + G_3}$$

Current through *R2* is *I3* = *VAB*




**How To Nortonize a Given Circuit?**

This procedure is based on the first statement of the theorem given above.
1. Remove the resistance (if any) across the two given terminals and put a short-circuit across them.

2. Compute the short-circuit current *Isc*

3. Remove all voltage sources but retain their internal resistances, if any. Similarly, remove all current sources and replace them by open- circuits *i.e.* by infinite resistance.

4. Next, find the resistance *Ri* (also called *RN)* of the network as looked into from the given terminals. It is exactly the same as *Rth*

5. The current source *(Isc)* joined in parallel across *Ri* between the two terminals gives Norton's equivalent circuit.

**Thévenin and Norton equivalent networks**

The Thévenin and Norton networks shown in Figures below are equivalent to each other. The resistance 'looking-in' at terminals AB is the same in each of the networks, i.e. *$R_i$.*
If terminals AB in Figure (a) below are short-circuited, the short-circuit current is given by $V_{oc}/R_i$.
If terminals AB in Figure (b) below are short-circuited, the short-circuit current is $I_{SC}$. For the circuit shown in Figure (a) below to be equivalent to the circuit in Figure (b) below the same short-circuit current must flow.
Thus,

$$I_{SC} = V_{oc}/R_i.$$

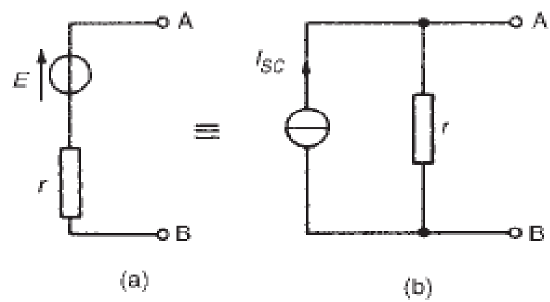
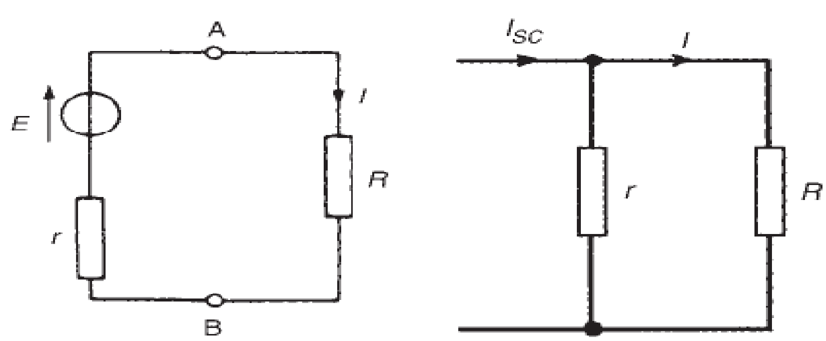



Figure (a) below shows a source of e.m.f. $V_{oc}$ in series with a resistance $R_i$ feeding a load resistance $R_L$.
From Figure (b) below it can be seen that, when viewed from the load, the source appears as a source of current $I_{SC}$ which is divided between $R_i$ and $R_L$ connected in parallel.
Thus the two representations shown below are equivalent.

$$I = \frac{V_{oc}}{R_i + R_L} = \frac{V_{oc}/R_i}{(R_i + R_L)/R_i} = (\frac{R_i}{R_i + R_L})\frac{V_{oc}}{R_i}$$

$$\boldsymbol{I = (\frac{R_i}{R_i + R_L})I_{sc}}$$



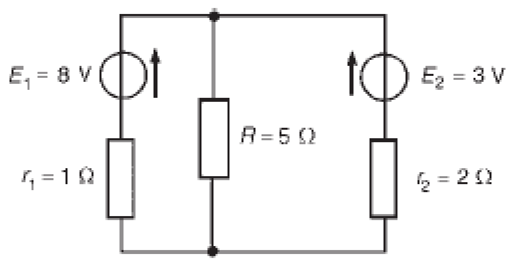

(a) (b)

**The superposition theorem**

The superposition theorem states:
*'In any network made up of linear resistances and containing more than one source of e.m.f., the resultant current flowing in any branch is the algebraic sum of the currents that would flow in that branch if each source was considered separately, all other sources being replaced at that time by their respective internal resistances.'*

Using the superposition theorem

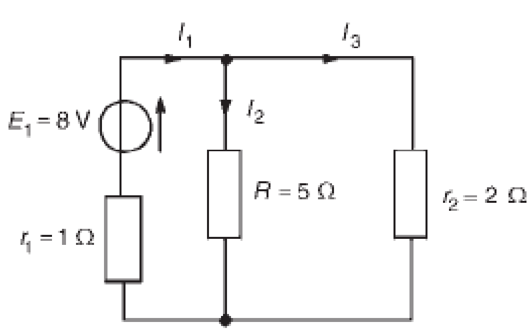
The superposition theorem, may be applied to both d.c. and a.c. networks. A d.c. network is shown in Figure below and will serve to demonstrate the principle of application of the superposition theorem.
To find the current flowing in each branch of the circuit, the following six-step procedure can be adopted:




(i) Redraw the original network with one of the sources, say $E_2$, removed and replaced by $r_2$ only, as shown in Figure below.
(ii) Label the current in each branch and its direction as shown in Figure below, and then determine its value. The choice of current direction for $I_1$ depends on the source polarity which, by convention, is taken as flowing from the positive terminal as shown.

R in parallel with $r_2$ gives an equivalent resistance of

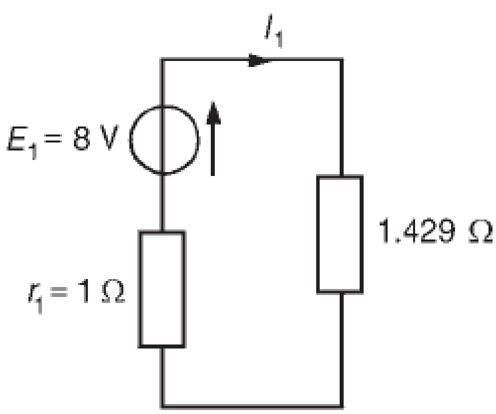
$$(5 \times 2)/(5 + 2) = 10/7 = 1.429\ \Omega$$



$$\text{current } I_1 = \frac{E_1}{(r_1 + 1.429)} = \frac{8}{2.429} = 3.294 \text{ A}$$

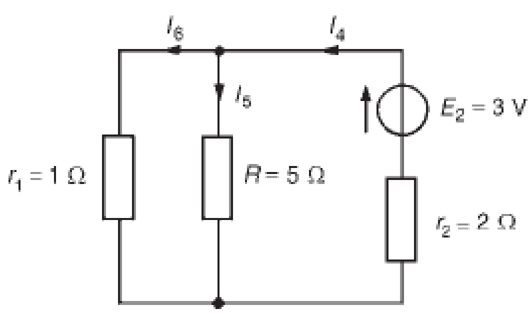
$$\text{current } I_2 = \left(\frac{r_2}{R + r_2}\right)(I_1) = \left(\frac{2}{5+2}\right)(3.294) = 0.941 \text{ A}$$

$$\text{and current } I_3 = \left(\frac{5}{5+2}\right)(3.294) = 2.353 \text{ A}$$




(iii) Redraw the original network with source E1 removed and replaced by r1 only, as shown in Figure below.
(iv) Label the currents in each branch and their directions as shown in Figure below, and determine their values.

R and r1 in parallel gives an equivalent resistance of



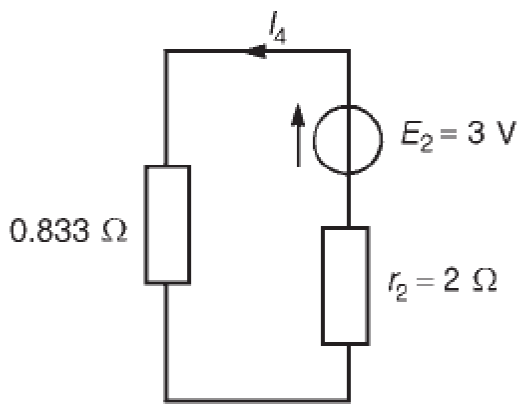

$(5 \times 1)/(5 + 1) = 5/6\ \Omega$ or $0.833\ \Omega$,

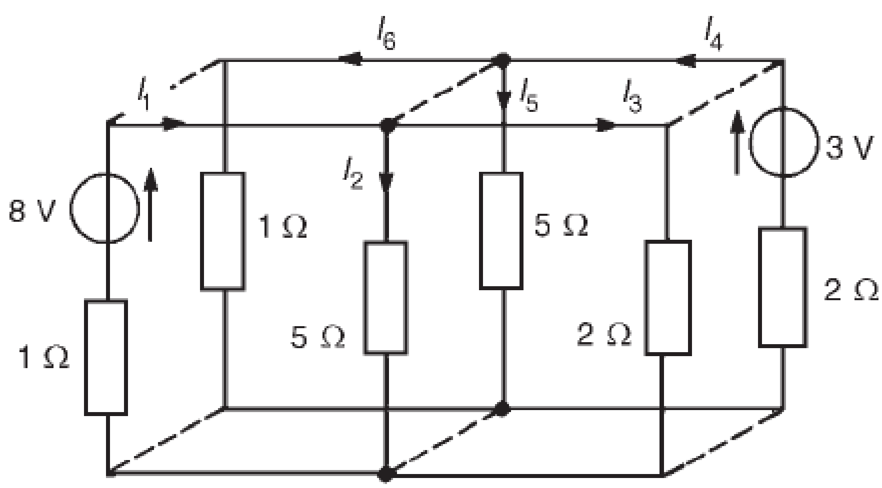
$$\text{current } I_4 = \frac{E_2}{r_2 + 0.833} = \frac{3}{2.833} = 1.059 \text{ A}$$

From Figure 32.4,

$$\text{current } I_5 = \left(\frac{1}{1+5}\right)(1.059) = 0.177 \text{ A}$$

$$\text{and current } I_6 = \left(\frac{5}{1+5}\right)(1.059) = 0.8825 \text{ A}$$




(vi) Determine the algebraic sum of the currents flowing in each branch. (Note that in an a.c. circuit it is the phasor sum of the currents that is required.)

From Figure below, the resultant current flowing through the 8 V source is given by

I1- I6 = 3.294- 0.8825 = 2.41 A (discharging, i.e., flowing from the positive terminal of the source).

The resultant current flowing in the 3 V source is given by

$I_3 - I_4 = 2.353 - 1.059 = 1.29$ A (charging, i.e., flowing into the positive terminal of the source).

The resultant current flowing in the 5Ω resistance is given by

$I_2 + I_5 = 0.941 + 0.177 = 1.12$ A

The values of current are the same as those obtained by using Kirchhoff's laws.

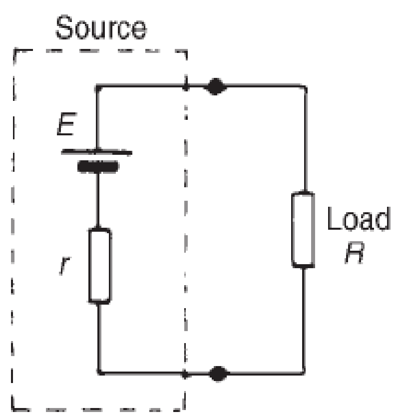
The following problems demonstrate further the use of the superposition theorem in analysing a.c. as well as d.c. networks. The theorem is straightforward to apply, but is lengthy. Th´evenin's and Norton's theorems (described in Chapter 33) produce results more quickly.

**Maximum power transfer theorem**

The maximum power transfer theorem states:

*'The power transferred from a supply source to a load is at its maximum when the resistance of the load is equal to the internal resistance of the source.'*

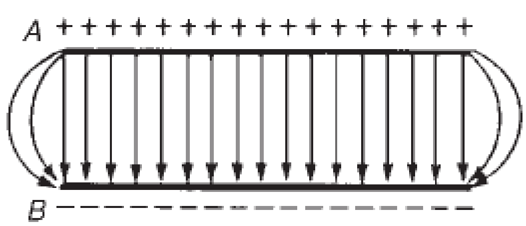
Hence, in Figure below, when *R* = *r* the power transferred from the source to the load is a maximum.



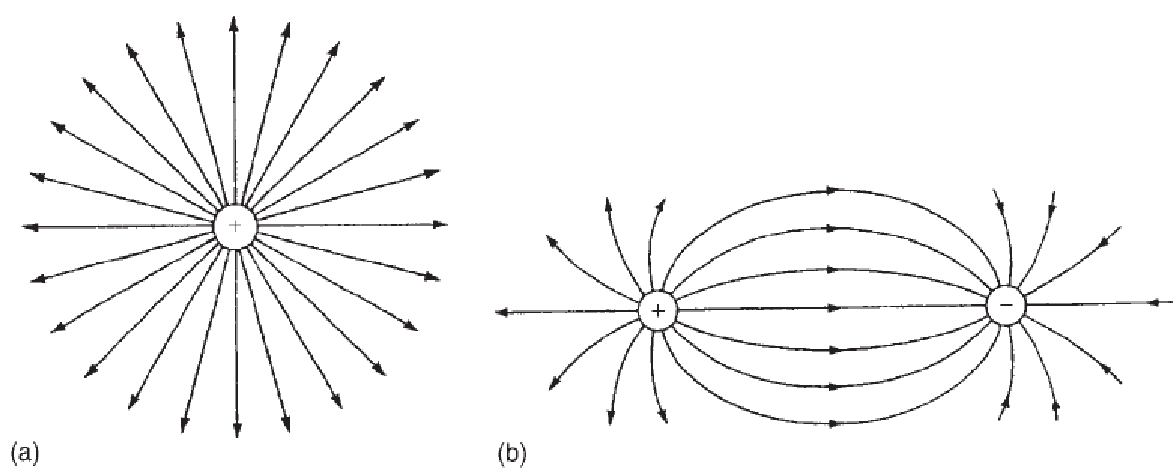
- Content/Topic 5: Capacitors and capacitance

**Electrostatic field**

Figure below represents two parallel metal plates, A and B, charged to different potentials. If an electron that has a negative charge is placed between the plates, a force will act on the electron tending to push it away from the negative plate B towards the positive plate, A. Similarly, a positive charge would be acted on by a force tending to move it toward the negative plate. Any region such as that shown between the plates in Figure below, in which an electric charge experiences a force, is called an electrostatic field. The direction of the field is defined as that of the force acting on a positive charge placed in the field. In Figure below, the direction of the force is from the positive plate to the negative plate.




Such a field may be represented in magnitude and direction by lines of electric force drawn between the charged surfaces. The closeness of the lines is an indication of the field strength. Whenever a p.d. is established between two points, an electric field will always exist. Figure (a) below shows a typical field pattern for an isolated point charge, and Figure (b) below shows the field pattern for adjacent charges of opposite polarity. Electric lines of force (often called electric flux lines) are continuous and start and finish on point charges. Also, the lines cannot cross each other. When a charged body is placed close to an uncharged body, an induced charge of opposite sign appears on the surface of the uncharged body. This is because lines of force from the charged body terminate on its surface.




*(a) Isolated point charge (b) adjacent charges of opposite polarity*

The concept of field lines or lines of force is used to illustrate the properties of an electric field.

However, it should be remembered that they are only aids to the imagination. The force of attraction or repulsion between two electrically charged bodies is proportional to the magnitude of their charges and inversely proportional to the square of the distance separating them,

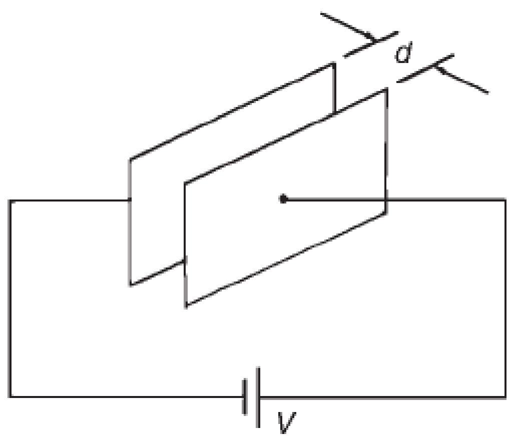
i.e. force where constant in air

$$\propto \frac{q_1 q_2}{d^2} \text{ or } \boxed{\text{force} = k\frac{q_1 q_2}{d^2}} \quad k \approx 9 \times 10^9$$

This is known as Coulomb's law.

**Electric field strength**

Figure below shows two parallel conducting plates separated from each other by air. They are connected to opposite terminals of a battery of voltage V volts.
There is therefore an electric field in the space between the plates. If the plates are close together, the electric lines of force will be straight and parallel and equally spaced, except near the edge where fringing will occur. Over the area in which there is negligible fringing,




$$\boxed{\text{Electric field strength, } E = \frac{V}{d} \text{ volts/metre}}$$

where *d* is the distance between the plates. Electric field strength is also called potential gradient.

## Capacitance

Static electric fields arise from electric charges, electric field lines beginning and ending on electric charges. Thus the presence of the field indicates the presence of equal positive and negative electric charges on the two plates of previous Figure. Let the charge be +Q coulombs on one plate and -Q coulombs on the other. The property of this pair of plates which determines how much charge corresponds to a given p.d. between the plates is called their capacitance:

$$\text{capacitance } C = \frac{Q}{V}$$

The unit of capacitance is the farad F (or more usually $\mu F = 10^{-6} F$ or $pF = 10^{-12} F$), which is defined as the capacitance when a pd of one volt appears across the plates when charged with one coulomb.

## Capacitors

Every system of electrical conductors possesses capacitance. For example, there is capacitance between the conductors of overhead transmission lines and also between the wires of a telephone cable. In these examples the capacitance is undesirable but has to be accepted, minimized or compensated for. There are other situations where capacitance is a desirable property.

Devices specially constructed to possess capacitance are called capacitors (or condensers, as they used to be called). In its simplest form a capacitor consists of two plates which are separated by an insulating material known as a dielectric. A capacitor has the ability to store a quantity of static electricity.

The symbols for a fixed capacitor and a variable capacitor used in electrical circuit diagrams are shown in Figures below.

Fixed capacitor

Variable capacitor

The charge *Q* stored in a capacitor is given by:

$$Q = I \times t \text{ coulombs,}$$

where I is the current in amperes and t the time in seconds.

**Practical types of capacitor**

1. Variable air capacitors.
2. Mica capacitors.
3. Paper capacitors.
4. Ceramic capacitors.
5. Plastic capacitors.
6. Titanium oxide capacitors
7. Electrolytic capacitors.

Electric flux density

Unit flux is defined as emanating from a positive charge of 1 coulomb. Thus electric flux ϕ is measured in coulombs, and for a charge of Q coulombs, the flux ϕ = Q coulombs.

Electric flux density D is the amount of flux passing through a defined area A that is perpendicular to the direction of the flux:

$$\text{electric flux density, } D = \frac{Q}{A} \text{ coulombs/metre}^2$$

Electric flux density is also called charge density,σ

Permittivity

At any point in an electric field, the electric field strength E maintains the electric flux and produces a particular value of electric flux density D at that point. For a field established in vacuum (or for practical purposes in air), the ratio D/E is a constant $\varepsilon_0$, i.e.

$$\frac{D}{E} = \varepsilon_0$$

where $\varepsilon_0$ is called the permittivity of free space or the free space constant. The value of $\varepsilon_0$ is $8.85 \times 10^{-12}$ F/m.

When an insulating medium, such as mica, paper, plastic or ceramic, is introduced into the region of an electric field the ratio of D/E is modified:

$$\frac{D}{E} = \varepsilon_0 \varepsilon_r$$

where $\varepsilon_r$, the relative permittivity of the insulating material, indicates its insulating power compared with that of vacuum:

$$\text{relative permittivity } \varepsilon_r = \frac{\text{flux density in material}}{\text{flux density in vacuum}}$$

$\varepsilon_r$ has no unit. Typical values of $\varepsilon_r$ include air, 1.00; polythene, 2.3; mica, 3–7; glass, 5–10; water, 80; ceramics, 6–1000.

The product $\varepsilon_0\varepsilon_r$ is called the **absolute permittivity**, $\varepsilon$, i.e.,

$$\boxed{\varepsilon = \varepsilon_0\varepsilon_r}$$

The insulating medium separating charged surfaces is called a dielectric. Compared with conductors, dielectric materials have very high resistivities. They are therefore used to separate conductors at different potentials, such as capacitor plates or electric power lines.

**The parallel plate capacitor**

For a parallel-plate capacitor, as shown in Figure (a)below, experiments show that capacitance C is proportional to the area A of a plate, inversely proportional to the plate spacing d (i.e., the dielectric thickness) and depends on the nature of the dielectric:

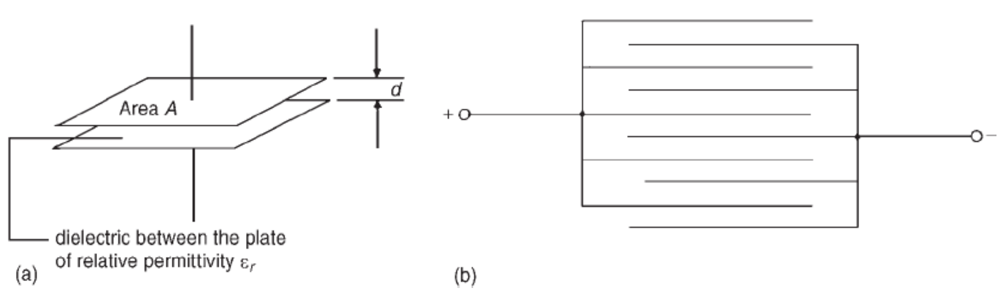
$$\boxed{\textbf{Capacitance, } C = \frac{\varepsilon_0\varepsilon_r A}{d} \textbf{ farads}}$$

where $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m (constant)

$\varepsilon_r$ = relative permittivity

$A$ = area of one of the plates, in $m^2$, and

$d$ = thickness of dielectric in m




Another method used to increase the capacitance is to interleave several plates as shown in Figure (b)above. Ten plates are shown, forming nine capacitors with a capacitance nine times that of one pair of plates.

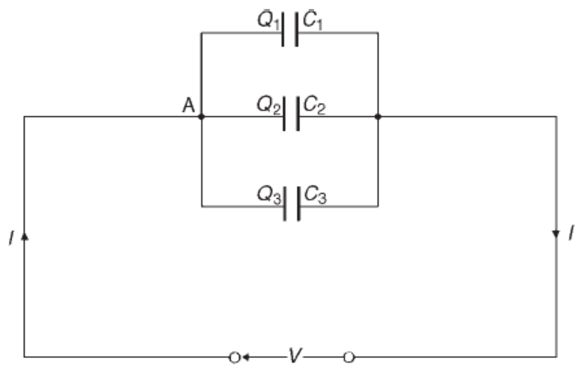
If such an arrangement has n plates then capacitance C α (n -1).

Thus capacitance $$\boxed{C = \frac{\varepsilon_0\varepsilon_r A(n-1)}{d} \textbf{ farads}}$$

## Capacitors connected in parallel and series

### (a) Capacitors connected in parallel

Figure below shows three capacitors, $C_1$, $C_2$ and $C_3$, connected in parallel with a supply voltage V applied across the arrangement.



Total charge, $Q_T = Q_1 + Q_2 + Q_3$

When the charging current I reaches point A it divides, some flowing into $C_1$, some flowing into $C_2$ and some into $C_3$. Hence the total charge $Q_T$ $(= I \times t)$ is divided between the three capacitors. The capacitors each store a charge and these are shown as $Q_1$, $Q_2$ and $Q_3$ respectively. Hence

$Q_T = Q_1 + Q_2 + Q_3$

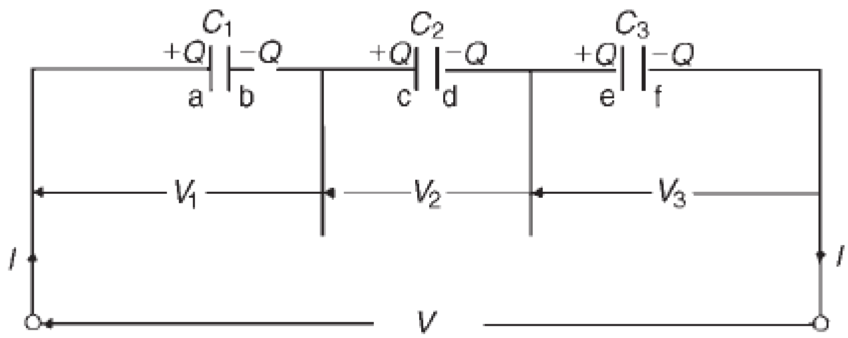
But $Q_T = CV$, $Q_1 = C_1V$, $Q_2 = C_2V$ and $Q_3 = C_3V$
Therefore $CV = C_1V + C_2V + C_3V$ where C is the total equivalent circuit capacitance, i.e. $C = C_1 + C_2 + C_3$
It follows that for n parallel-connected capacitors,

$$C = C_1 + C_2 + C_3 \ldots + C_n,$$

i.e. the equivalent capacitance of a group of parallel-connected capacitors is the sum of the capacitances of the individual capacitors. (Note that this formula is similar to that used for resistors connected in series)

**(b)Capacitors connected in series**

Figure below shows three capacitors, $C_1$, $C_2$ and $C_3$, connected in series across a supply voltage V. Let the p.d. across the individual capacitors be $V_1$, $V_2$ and $V_3$ respectively as shown.




Charge on each capacitor = Q

Let the charge on plate 'a' of capacitor $C_1$ be +Q coulombs. This induces an equal but opposite charge of -Q coulombs on plate 'b'. The conductor between plates 'b' and 'c' is electrically isolated from the rest of the circuit so that an equal but opposite charge of +Q coulombs must appear on plate 'c', which, in turn, induces an equal and opposite charge of -Q coulombs on plate 'd', and so on.

Hence when capacitors are connected in series the charge on each is the same.

In a series circuit: $V = V_1 + V_2 + V_3$

Since $V = \frac{Q}{C}$ then $\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

where $C$ is the total equivalent circuit capacitance,

i.e. $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

It follows that for $n$ series-connected capacitors:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n},$$

i.e. for series-connected capacitors, the reciprocal of the equivalent capacitance is equal to the sum of the reciprocals of the individual capacitances. (Note that this formula is similar to that used for resistors connected in parallel)

For the special case of two capacitors in series:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2 + C_1}{C_1 C_2}$$

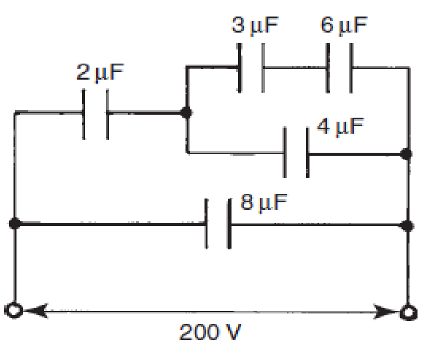
$$\text{Hence} \quad \boxed{C = \frac{C_1 C_2}{C_1 + C_2}} \qquad \left(\text{i.e. } \frac{\text{product}}{\text{sum}}\right)$$

**Series/Parallel combinations**

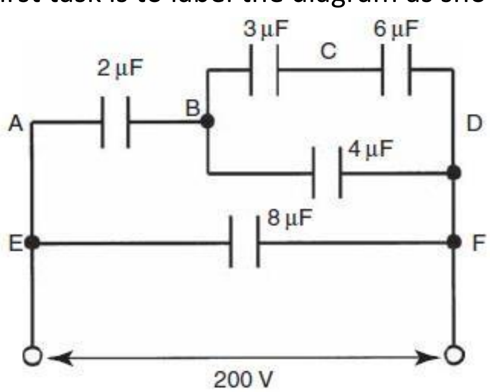
The techniques required for the solution of this type of circuit are again best demonstrated by means of a worked example.

**Examples**

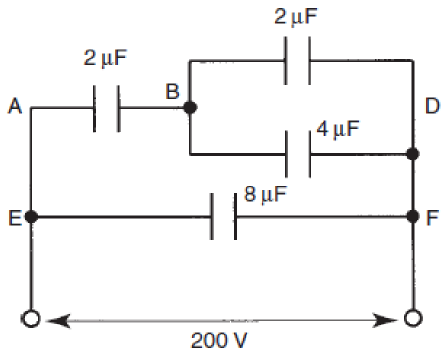
Q1. For the circuit shown in Fig.below, determine (a) the charge drawn from the supply, (b) the charge on the 8μF capacitor, (c) the p.d. across the 4μF capacitor, and (d) the p.d. across the 3μF capacitor.



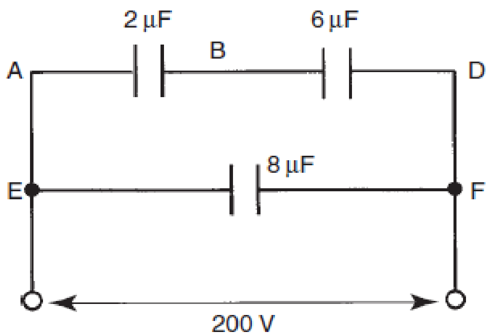

The first task is to label the diagram as shown in Fig. below



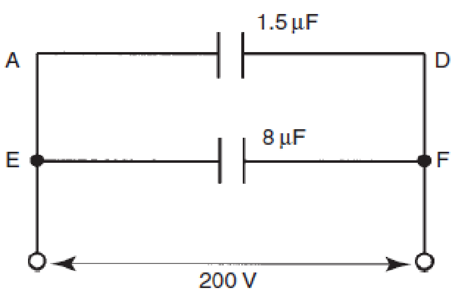

**(a)** $C_{BCD} = \frac{3 \times 6}{3 + 6} = 2\mu F$




$$C_{BD} = 2 + 4 = 6\,\mu\text{F}$$




$$C_{AD} = \frac{6 \times 2}{6 + 2} = 1.5\,\mu\text{F}$$

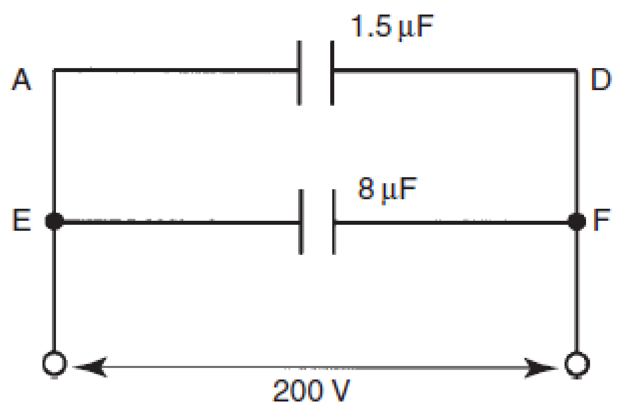



$$C = C_{AD} + C_{EF} = 1.5 + 8$$

so $C = 9.5\,\mu\text{F}$

$$Q = VC \text{ coulomb} = 200 \times 9.5 \times 10^{-6}$$

hence, $Q = 1.9\text{mC}$ **Ans**




**(b)** $Q_{EF} = VC_{EF}$ coulomb $= 200 \times 8 \times 10^{-6}$

so $Q_{EF} = 1.6\,\text{mC}$ **Ans**

**(c)** Total charge $Q = 1.9\,\text{mC}$ and $Q_{EF} = 1.6\,\text{mC}$

so $Q_{AD} = 1.9 - 1.6 = 0.3\,\text{mC}$

$Q_{AB} = Q_{BD} = 0.3\,\text{mC}$.

Thus, $V_{BD}$ = p.d. across 4 μF capacitor

$$V_{BD} = \frac{Q_{BD}}{C_{BD}}$$

$$= \frac{0.3 \times 10^{-3}}{6 \times 10^{-6}} \text{ volt}$$

so $V_{BD} = 50\,\text{V}$ **Ans**

**(d)** $Q_{BCD} = V_{BD}C_{BCD}$

$= 50 \times 2 \times 10^{-6}$

$= 100\,\mu\text{C}$

and this will be the charge on both the 3 μF and 6 μF capacitors, i.e.

$$Q_{BC} = Q_{CD} = 100\,\mu\text{C}$$

$$\text{Thus } V_{BC} = \frac{Q_{BC}}{C_{BC}} \text{ volt}$$

$$= \frac{1 \times 10^{-4}}{3 \times 10^{-6}}$$

$$\text{so } V_{BC} = 33.33\,\text{V}\ \textbf{Ans}$$

Q2. Two parallel plates having a pd of 200 V between them are spaced 0.8 mm apart. What is the electric field strength? Find also the flux density when the dielectric between the plates is (a) air, and (b) polythene of relative permittivity 2.3

**Solution**

$$\textbf{Electric field strength } E = \frac{V}{D} = \frac{200}{0.8 \times 10^{-3}} = \mathbf{250\ kV/m}$$

$$\textbf{Electric field strength } E = \frac{V}{D} = \frac{200}{0.8 \times 10^{-3}} = \mathbf{250\ kV/m}$$

(a) For air: $\varepsilon_r = 1$

$$\frac{D}{E} = \varepsilon_0 \varepsilon_r. \quad \text{Hence}$$

$$\textbf{electric flux density } D = E\varepsilon_0\varepsilon_r$$

$$= (250 \times 10^3 \times 8.85 \times 10^{-12} \times 1)\ \text{C/m}^2$$

$$= \mathbf{2.213\ \mu C/m^2}$$

(b) For polythene, $\varepsilon_r = 2.3$

$$\textbf{Electric flux density } D = E\varepsilon_0\varepsilon_r$$

$$= (250 \times 10^3 \times 8.85 \times 10^{-12} \times 2.3)\ \text{C/m}^2$$

$$= \mathbf{5.089\ \mu C/m^2}$$

**Q3.** Capacitances of 1 μF, 3 μF, 5 μF and 6 μF are connected in parallel to a direct voltage supply of 100 V. Determine (a) the equivalent circuit capacitance, (b) the total charge and (c) the charge on each capacitor.

**Solution**

(a) The equivalent capacitance C for four capacitors in parallel is given by:

$$C = C_1 + C_2 + C_3 + C_4$$

i.e. $C = 1 + 3 + 5 + 6 = \mathbf{15\ \mu F}$

(b) Total charge $Q_T = CV$ where C is the equivalent circuit capacitance

i.e. $Q_T = 15 \times 10^{-6} \times 100 = 1.5 \times 10^{-3}\ C = \mathbf{1.5\ mC}$

(c) The charge on the 1 μF capacitor $Q_1 = C_1V = 1 \times 10^{-6} \times 100$

$= \mathbf{0.1\ mC}$

The charge on the 3 μF capacitor $Q_2 = C_2V = 3 \times 10^{-6} \times 100$

$= \mathbf{0.3\ mC}$

The charge on the 5 μF capacitor $Q_3 = C_3V = 5 \times 10^{-6} \times 100$

$= \mathbf{0.5\ mC}$

The charge on the 6 μF capacitor $Q_4 = C_4V = 6 \times 10^{-6} \times 100$

$= \mathbf{0.6\ mC}$

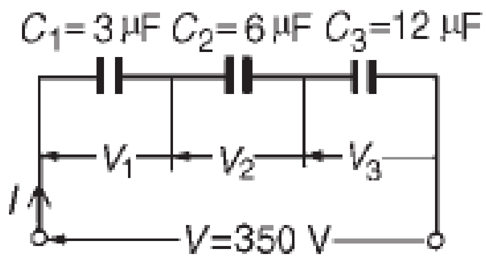
[Check: In a parallel circuit $Q_T = Q_1 + Q_2 + Q_3 + Q_4$

$Q_1 + Q_2 + Q_3 + Q_4 = 0.1 + 0.3 + 0.5 + 0.6 = 1.5\ \text{mC} = Q_T$]

**Q4.** Capacitances of 3 μF, 6 μF and 12 μF are connected in series across a 350 V supply. Calculate (a) the equivalent circuit capacitance, (b) the charge on each capacitor and (c) the pd across each capacitor.

**Solution**

The circuit diagram is shown in Figure below




(a) The equivalent circuit capacitance C for three capacitors in series is given by:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

i.e. $$\frac{1}{C} = \frac{1}{3} + \frac{1}{6} + \frac{1}{12} = \frac{4+2+1}{12} = \frac{7}{12}$$

**Hence the equivalent circuit capacitance** $C = \frac{12}{7} = 1\frac{5}{7}\ \mu F$

(b) Total charge $Q_T = CV$,

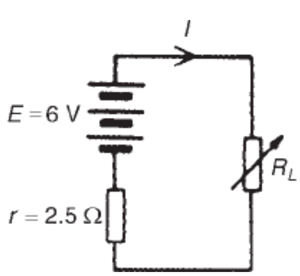
hence $$Q_T = \frac{12}{7} \times 10^{-6} \times 350 = 600\ \mu C \text{ or } 0.6\ mC$$

**Since the capacitors are connected in series 0.6 mC is the charge on each of them.**

(c) The voltage across the 3 μF capacitor, $V_1 = \frac{Q}{C_1} = \frac{0.6 \times 10^{-3}}{3 \times 10^{-6}}$

$$= \mathbf{200\ V}$$

**Q5.** The circuit diagram of Figure below shows dry cells of source e.m.f. 6 V, and internal resistance 2.5Ω. If the load resistance $R_L$ is varied from 0 to 5Ω in 0.5Ω steps, calculate the power dissipated by the load in each case. Plot a graph of $R_L$ (horizontally) against power (vertically) and



determine the maximum power dissipated

**Solution**

When $R_L = 0$, current $I = \dfrac{E}{r + R_L} = \dfrac{6}{2.5} = 2.4\ A$ and power dissipated in $R_L$, $P = I^2 R_L$, i.e. $P = (2.4)^2(0) = 0$ W

When $R_L = 0.5\ \Omega$, current $I = \dfrac{E}{r + R_L} = \dfrac{6}{2.5 + 0.5} = 2$ A

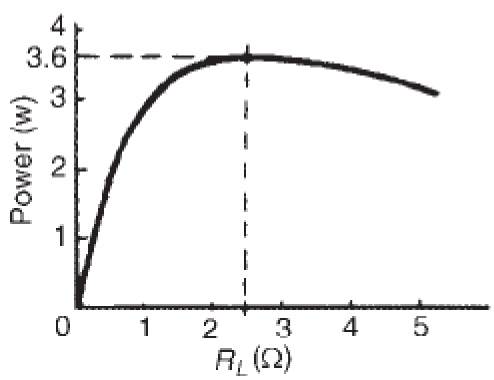
and $\quad P = I^2 R_L = (2)^2(0.5) = 2$ W

When $R_L = 1.0\Omega$, current $I = \dfrac{6}{2.5 + 1.0} = 1.714$ A

and $\quad P = (1.714)^2(1.0) = 2.94$ W

With similar calculations the following table is produced:

| $R_L(\Omega)$ | 0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 4.5 | 5.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $I = \dfrac{E}{r + R_L}$ | 2.4 | 2.0 | 1.714 | 1.5 | 1.333 | 1.2 | 1.091 | 1.0 | 0.923 | 0.857 | 0.8 |
| $P = I^2 R_L (W)$ | 0 | 2.00 | 2.94 | 3.38 | 3.56 | 3.60 | 3.57 | 3.50 | 3.41 | 3.31 | 3.20 |




A graph of RL against P is shown in Figure above. The maximum value of power is 3.60 W which occurs when $R_L$ is 2.5Ω, i.e. maximum power occurs when $R_L$ = r, which is what the maximum power transfer Figure above theorem states.

**Applications of Capacitors**

1. Energy storage
2. Pulse power and weapon
3. Power condition
4. Power factor correction
5. Suppression and coupling
6. Motor starter
7. Signal processing
8. Sensing
9. Oscillators
10. Safety

## LO 1.3 – Apply magnetism and electromagnetism

- Content/Topic 1: Introduction to magnetism

Magnetism plays an important role in electricity. Electrical appliances like generator, motor, measuring instruments, transformer are based on the electromagnetic principal and also the important components of television, Radio, Aeroplane are working on the same principle.

Magnetic materials
Are classified based on the property called permeability as:

Dia magnetic material: The materials whose permeability is below unity.
EX: Lead, Gold, Copper, Glass, Mercury

Para magnetic materials: The materials with permeability above unity. The force of attraction by a magnet towards these materials is low

EX: Copper sulphate, Oxygen, Platinum, Aluminium.

Ferro magnetic materials: The materials with permeability thousands of times more than that of paramagnetic materials.
They are very much attracted by the magnet.

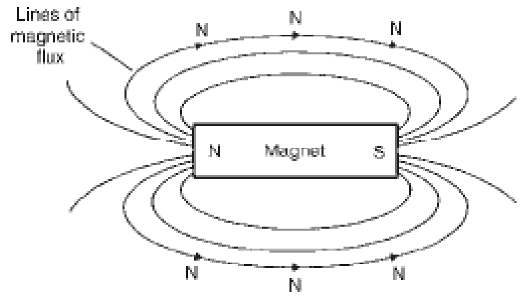
EX: Iron, cobalt, Nickel.

**Magnetic fields**

A permanent magnet is a piece of ferromagnetic material (such as iron, nickel or cobalt) which has properties of attracting other pieces of these materials. A permanent magnet will position itself in a north and south direction when freely suspended. The north-seeking end of the magnet is called the north pole, N, and the south-seeking end the south pole, S.

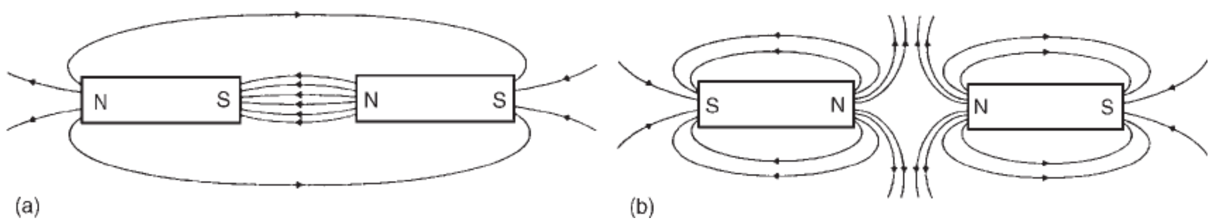
The area around a magnet is called the magnetic field and it is in this area that the effects of the magnetic force produced by the magnet can be detected. A magnetic field cannot be seen, felt, smelt or heard and therefore is difficult to represent. Michael Faraday suggested that the magnetic field could be represented pictorially, by imagining the field to consist of lines of magnetic flux, which enables investigation of the distribution and density of the field to be carried out. The distribution of a magnetic field can be investigated by using some iron fillings. A bar magnet is placed on a flat surface covered by, say, cardboard, upon which is sprinkled some iron filings.

If the cardboard is gently tapped the filings will assume a pattern similar to that shown in figure below.




If a number of magnets is different strength are used, it is found that the stronger the field the closer are the lines of magnetic flux and vice versa. Thus a magnetic field has the property of exerting a force, demonstrated in this case by causing the iron filings to move into the pattern shown. The strength of the magnetic field decreases as we move away from the magnet. It should be realized, of course, that the magnetic field is three dimensional in its effect, and not acting in one plane as appears to be the case in this experiment.

If a compass is placed in the magnetic field in various positions, the direction of the lines of flux may be determined by noting the direction of the compass pointer. The direction of a magnetic field at any point is taken as that in which the north-seeking pole of a compass needle points when suspended in the field. The direction of a line of flux is form the north pole to the south pole on the outside of the magnet and is then assumed to continue through the magnet back to the point at which it emerged at the north pole. Thus such lines of flux always form complete closed loops or paths, they never intersect and always have a definite direction. The laws of magnetic attraction and repulsion can be demonstrated by using two bar magnets. In figure (a) below, with unlike poles adjacent, attraction takes place. Lines of flux are imagined to contract and the magnets try to pull together. The magnetic field is strongest in between the two magnets, shown by the lines of flux being close together. In figure (b) below, with similar poles adjacent (i.e. two north pole), repulsion occurs, i.e. the two north poles try to push each other apart, since magnetic flux lines running side in the same direction repel.



**Magnetic flux and flux density**

Magnetic flux is the amount of magnetic field (or the number of lines of force) produced by a magnetic source. The symbol for magnetic flux is $\Phi$ (Greek letter 'phi'). The unit of magnetic flux is the welber, Wb

Magnetic flux density is the amount of flux passing through a defined area that is perpendicular to the direction of the flux:

$$\text{Magnetic flux density} = \frac{\textit{magnetic flux}}{\textit{area}}$$

The symbol for magnetic flux density is B. The unit of magnetic flux density is the tesla, T, where 1T = 1Wb/m$^2$ Hence

$$B = \frac{\Phi}{A} \text{ tesla}, \text{ where } A(m^2) \text{ is the area}$$

| |
|---|
| Problem 1. A magnetic pole face has a rectangular section having dimensions 200 mm by 100 mm. If the total flux emerging from the pole is 150 µWb, calculate the flux density. |

$$\text{Flux } \Phi = 150\ \mu\text{Wb} = 150 \times 10^{-6}\ \text{Wb}$$

$$\begin{aligned}\text{Cross sectional area } A = 200 \times 100 &= 20000\ \text{mm}^2 \\ &= 20000 \times 10^{-6}\ \text{m}^2\end{aligned}$$

$$\begin{aligned}\text{Flux density } B = \frac{\Phi}{A} &= \frac{150 \times 10^{-6}}{20000 \times 10^{-6}} \\ &= \mathbf{0.0075\ T\ or\ 7.5\ mT}\end{aligned}$$

Problem 2. The maximum working flux density of a lifting electromagnet is 1.8 T and the effective area of a pole face is circular in cross-section. If the total magnetic flux produced is 353 mWb, determine the radius of the pole face.

Flux density $B = 1.8$ T; flux $\Phi = 353$ mWb $= 353 \times 10^{-3}$ Wb

Since $B = \dfrac{\Phi}{A}$, cross-sectional area $A = \dfrac{\Phi}{B} = \dfrac{353 \times 10^{-3}}{1.8}$ m$^2$

$= 0.1961$ m$^2$

The pole face is circular, hence area $= \pi r^2$, where $r$ is the radius.

Hence $\pi r^2 = 0.1961$

from which $r^2 = \dfrac{0.1961}{\pi}$ and radius $r = \sqrt{\left(\dfrac{0.1961}{\pi}\right)} = 0.250$ m

i.e. **the radius of the pole face is 250 mm**

**Magnetomotive force and magnetic field strength**

Magnetomotive force (mmf) is the cause of the existence of a magnetic flux in a magnetic circuit.

mmf, $F_m$ = NI amperes

Where N is the number of conductors (or turns) and I is the current in amperes. The unit of mmf is sometimes expressed as 'amperes-turns'. However since 'turns' have no dimensions, the SI unit of mmf is the ampere.

Magnetic field strength (or magnetizing force)

$H = \frac{NI}{l}$ ampere per metre (A/m)

Where l is the mean length of the flux path in metres.

Thus mmf = NI = Hl amperes

**Permeability and B-H curves**

For air, or any no-magnetic medium, the ratio of magnetic flux density to magnetizing force is a constant, i.e. B/H = a constant.

This constant is $\mu_0$, the permeability of free space (or the magnetic space constant) and is equal to $4\pi x 10^{-7}$ H/m, i.e., for air, or any non-magnetic medium, the ratio $B/H = \mu_0$

For all medium other than free space $B/H = \mu_0 \mu_r$

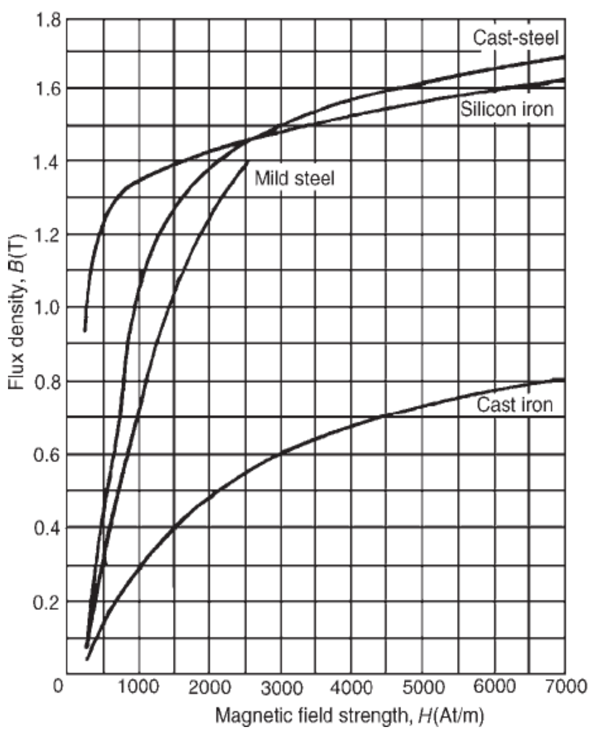
Where $\mu_r$ is the relative permeability, and is defined as

$$\mu_r = \frac{\textbf{\textit{flux density in material}}}{\textbf{\textit{flux density in a vacuum}}}$$

$\mu_r$ varies with the type of magnetic material and, since it is a ratio of flux densities, it has no unit.

From its definition, $\mu_r$ for a vacuum is 1. $\mu_O \mu_r = \mu$ called the absolute permeability.

By plotting measured values of flux density B against magnetic field strength H, a magnetization curve (or B-H curve) is produced. For no-magnetic materials this is a straight line. Typical curves for four magnetic materials are shown below.



The relative permeability of a ferromagnetic material is proportional to the slope of the B-H curve and thus varies with the magnetic field strength. The approximate range of values of relative permeability $\mu_r$ for some common magnetic materials are:

Cast iron $\mu_r$ = 100-250

Mild steel $\mu_r$ = 200-800

Silicon iron $\mu_r$ = 1000-5000

Mast steel $\mu_r$ = 300-900

Mumetal $\mu_r$ = 200-5000

Stalloy $\mu_r$ = 500-6000

**Reluctance**

Reluctance S (or Rm) is the 'magnetic resistance' of a magnetic circuit to the presence of magnetic flux.

$$\text{Reluctance } S = \frac{F_M}{\Phi} = \frac{NI}{\Phi} = \frac{Hl}{BA} = \frac{l}{(B/H)A} = \frac{l}{\mu_O \mu_r A}$$

The unit of reluctance is 1/H (or $H^{-1}$) or A/Wb

Ferromagnetic materials have a low reluctance and can be used as magnetic screens to prevent magnetic fields affecting materials within the screen.

**Composite series magnetic circuits**

For a series magnetic circuit having n parts, the total reluctance S is given by:

$S = S_1 + S_2 + \dots\dots + S_n$

Comparison between electrical and magnetic quantities

| Electrical circuit | Magnetic circuit |
|---|---|
| 1.Current = $\dfrac{e.m.f.}{resistance}$ | Flux = $\dfrac{mmf}{reluctance}$ |
| 2. E.M.F. (Volts) | MMF (ampere-turns) |
| 3. Current I (amperes) | Flux $\Phi$ (webers) |
| 4. Current density (A/m$^2$) | Flux density B (Wb/m$^2$) |
| 5. Resistance R = $\rho\dfrac{l}{A} = \dfrac{l}{\rho A}$ | Reluctance S = $\dfrac{l}{\mu A}\left(= \dfrac{l}{\mu_o \mu_r A}\right)$ |
| 6. Conductance (= 1/resistance) | Permeance (= 1/reluctance) |
| 7. Resistivity | Reluctivity |
| 8. Conductivity (=1/resistivity) | Permeability (= 1/reluctivity) |
| 9. Total e.m.f. = $IR_1 + IR_2 + IR_3 + \dots$ | Total mmf = $\Phi S_1 + \Phi S_2 + \Phi S_3 + \dots\dots$ |

**Hysteresis and hysteresis loss**

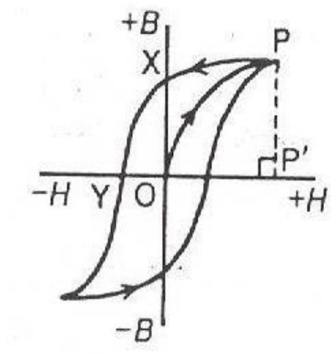
Hysteresis is the 'lagging' effect of flux density B whenever there are changes in the magnetic field strength H. When an initially unmagnetized ferromagnetic material is subjected to a varying magnetic field strength H, the flux density B produced in the material varies as shown in figure below, the arrows indicating the direction of the cycle. Figure below is known as a hysteresis loop.

From figure below, distance OX indicates the residual flux density or remanence, OY indicates the coercive force, and PP'is the saturation flux density.

Hysteresis results in a dissipation of energy which appears as a heating of the magnetic material. The energy loss associated with hysteresis is proportional to the area of the hysteresis loop.

The area of a hysteresis loop varies with the type of material. The area, and thus the energy loss, is much greater for hard materials than soft materials.

For AC-excited devices the hysteresis loop is repeated every cycle of alternating current. Thus a hysteresis loop with a large area (as with hard steel) is often unsuitable since the energy loss would be considerable.
Silicon steel has a narrow hysteresis loop, and thus small hysteresis loss, and is suitable for transformer cores and rotating machine armatures.




**Applications of magnetism**

**Magnets** are used to make a tight seal on the doors to refrigerators and freezers. They power speakers in stereos, earphones, and televisions. **Magnets** are used to store data in computers, and are important in scanning machines called MRIs (magnetic resonance imagers), which doctors use to look inside people's bodies.

| |
|---|
| Problem 3. A magnetizing force of 8000 A/m is applied to a circular magnetic circuit of mean diameter 30 cm by passing a current through a coil wound on the circuit. If the coil is uniformly wound around the circuit and has 750 turns, find the current in the coil. |

$H = 8000$ A/m; $l = \pi d = \pi \times 30 \times 10^{-2}$ m; $N = 750$ turns

Since $H = \dfrac{NI}{l}$ then, $I = \dfrac{Hl}{N} = \dfrac{8000 \times \pi \times 30 \times 10^{-2}}{750}$

Thus, **current $I$ = 10.05 A**

Problem 4. A flux density of 1.2 T is produced in a piece of cast steel by a magnetizing force of 1250 A/m. Find the relative permeability of the steel under these conditions.

For a magnetic material:

$$B = \mu_0 \mu_r H$$

i.e. $$u_r = \frac{B}{\mu_0 H} = \frac{1.2}{(4\pi \times 10^{-7})(1250)} = \mathbf{764}$$

Problem 5. Determine the magnetic field strength and the mmf required to produce a flux density of 0.25 T in an air gap of length 12 mm.

For air: $B = \mu_0 H$ (since $\mu_r = 1$)

**Magnetic field strength** $H = \dfrac{B}{\mu_0} = \dfrac{0.25}{4\pi \times 10^{-7}} = \mathbf{198\ 940\ A/m}$

$$\textbf{mmf} = Hl = 198\ 940 \times 12 \times 10^{-3} = \mathbf{2387\ A}$$

**Problem 6.** A coil of 300 turns is wound uniformly on a ring of non-magnetic material. The ring has a mean circumference of 40 cm and a uniform cross sectional area of 4 $cm^2$. If the current in the coil is 5 A, calculate (a) the magnetic field strength, (b) the flux density and (c) the total magnetic flux in the ring.

(a) Magnetic field strength $H = \frac{NI}{l} = \frac{300 \times 5}{40 \times 10^{-2}} = \mathbf{3750\ A/m}$

(b) For a non-magnetic material $\mu_r = 1$, thus flux density $B = \mu_0 H$

i.e. $\mathbf{B} = 4\pi \times 10^{-7} \times 3750 = \mathbf{4.712\ mT}$

(c) Flux $\Phi = BA = (4.712 \times 10^{-3})(4 \times 10^{-4}) = \mathbf{1.885\ \mu Wb}$

**Problem 7.** An iron ring of mean diameter 10 cm is uniformly wound with 2000 turns of wire. When a current of 0.25 A is passed through the coil a flux density of 0.4 T is set up in the iron. Find (a) the magnetizing force and (b) the relative permeability of the iron under these conditions.

$l = \pi d = \pi \times 10$ cm $= \pi \times 10 \times 10^{-2}$ m; $N = 2000$ turns; $I = 0.25$ A; $B = 0.4$ T

(a) $\mathbf{H} = \frac{NI}{l} = \frac{2000 \times 0.25}{\pi \times 10 \times 10^{-2}} = \frac{5000}{\pi} = \mathbf{1592\ A/m}$

(b) $B = \mu_0 \mu_r H$, hence $\boldsymbol{\mu_r} = \frac{B}{\mu_0 H} = \frac{0.4}{(4\pi \times 10^{-7})(1592)} = \mathbf{200}$

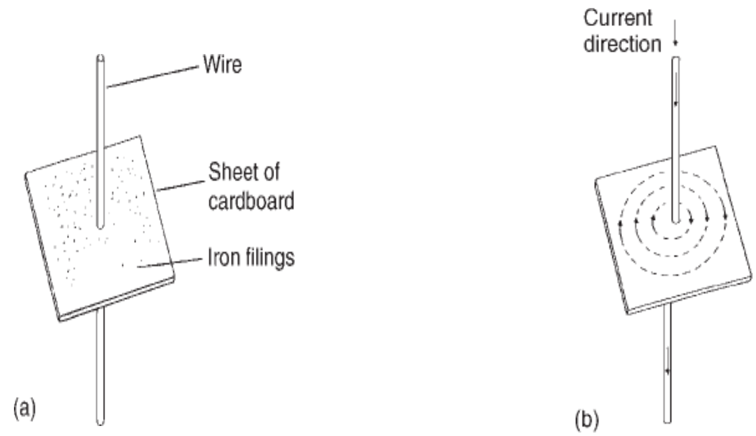
- <u>Content/Topic 2: Introduction to electromagnetism</u>

**Magnetic field due to an electric current**

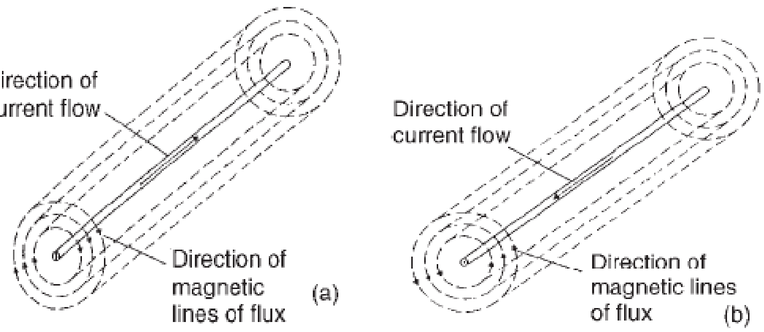
Magnetic fields can be set up not only by permanent magnets, but also by electric currents.

Let a piece of wire be arranged to pass vertically through a horizontal sheet of cardboard, on which is placed some iron fillings, as shown in figure (a) below.

If a current is now passed through the wire, then the iron fillings will form a definite circular field pattern with the wire at the centre, when the cardboard is gently tapped. By placing a compass in different positions the lines of flux are seen to have a definite direction as shown in figure (b) below.
If the current direction is reversed, the direction of the lines of flux is also reversed. The effect on both the iron fillings and the compass needle disappears when the current is switched off. The magnetic field is thus produced by the electric current. The magnetic flux produced has the same properties as the flux produced by a permanent magnet. If the current is increased the strength of the field increases and, as for the permanent magnet, the field strength decreases as we move away from the current-carrying conductor. In figures (a) and (b) below, the effect of only a small part of the magnetic field is shown.



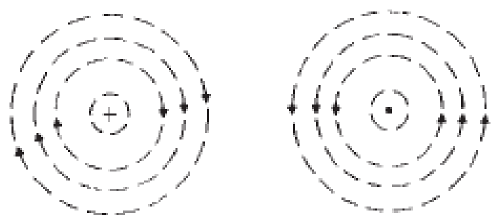

If the whole length of the conductor is similarly investigated it is found that the magnetic field around a straight conductor is in form of concentric cylinders as in figure (a) and (b) below, the field direction depending on the direction of the current flow.




When dealing with magnetic fields formed by electric current it is usual to portray the effect as shown in figures (a) and (b) below. The convention adopted is:

(i) Current flowing away from the viewer, i.e. into the paper, is indicated by ⊕. This may be thought of as the feathered end of the shaft of an arrow. See figure (a) below.

(ii) Current flowing towards the viewer, i.e. out of the paper, is indicated by ⊙ . This may be thought of as the point of an arrow. See figure (b) below.

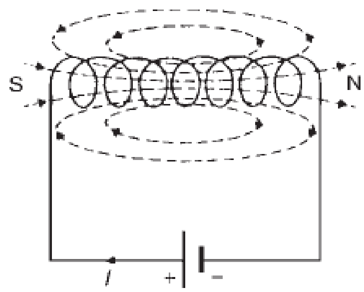


(a) Current flowing away from viewer (b) Current flowing towards viewer

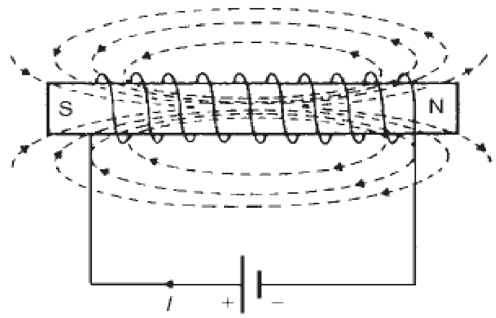
he direction of the magnetic lines of the flux is best remembered by the screw rule. This states that:

*'If a normal right-hand thread screw is screwed along the conductor in the direction of the current, the direction of rotation of the screw is in the direction of the magnetic field'.*

A magnetic field set up by a long coil, or solenoid, is shown in figure (a) below and is seen to be similar to that of a bar magnet. If the solenoid is wound on an iron bar, as shown in figure (b) below, an even stronger magnetic field is produced, the iron becoming magnetized and behaving like a permanent magnet. The direction of the magnetic field produced by the current I in the solenoid may be found by either of two methods, i.e. the screw rule.



(a) Magnetic field of a solenoid



(b) Magnetic field of an iron cored solenoid

**Force on a current – carrying conductor**

If a current-carrying conductor is placed in a magnetic field produced by permanent magnets, then the fields due to the current-carrying conductor and the permanent magnets interact and cause a force to be exerted on the conductor. The force on the current-carrying conductor in a magnetic field depends upon:

(a) the flux density of the field, B teslas

(b) the strength of the current, I amperes

(c) the length of the conductor perpendicular to the magnetic field, l metres

(d) the directions of the field and the current.

When the magnetic field, the current and the conductor are mutually at right angles then:

Force F = BIl newtons

When the conductor and the field are at an angle $\varphi^o$ to each other then: Force

F = BIl sin$\boldsymbol{\varphi}$ newtons

The direction of the force exerted on a conductor can be pre-determined by using Fleming's left-hand rule (often called the motor rule) which states:

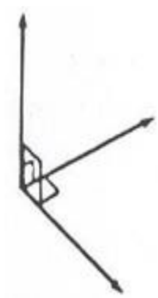
*Let the thumb, first finger and second finger of the left hand be exerted such that they are all at right-angles to each other, (as shown below). If the first finger points in the direction of the magnetic field, the second finger points in the direction of the current, then the thumb will point* *in the direction of the motion of the conductor.*

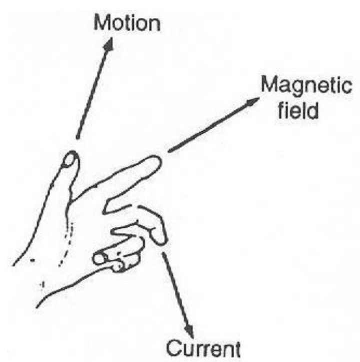
Summarizing:

First finger → Field

Second finger → Current

Thumb → Motion






**Force on a charge**

When a charge of Q coulombs is moving at a velocity of $v$ m/s in a magnetic field of flux density B teslas, the charge moving perpendicular to the field, then the magnitude of the force F exerted on the charge is given by:

$$F = Q\boldsymbol{v}B \text{ newtons}$$

**Applications of electromagnetism**

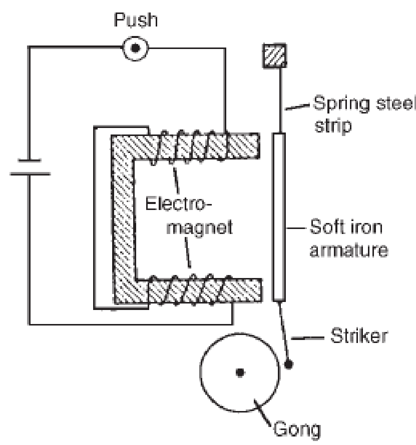
The solenoid is very important in electromagnetic theory since the magnetic field inside the solenoid is practically uniform for a particular current, and is also versatile, in such that a variation of the current can alter the strength of the magnetic field.

An electromagnet, based on the solenoid, provides the basis of many items of electrical equipment, examples of which include electrical bells, relays, lifting magnets and telephone receivers.

**(i) Electric bell**

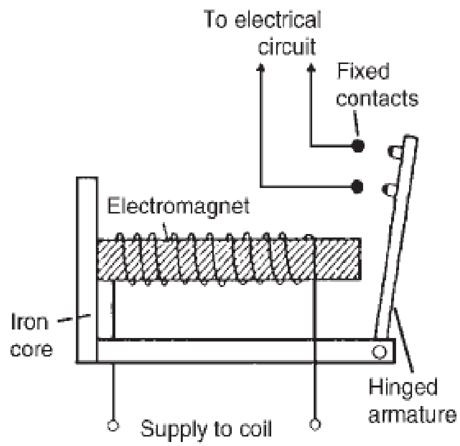
There are various types of electric bell, including the single-stroke bell, the tremble bell, the buzzer and a continuously ringing bell, but all depend on the attraction exerted by an electromagnet on a soft iron armature. A typical single stroke bell circuit is shown below. When the push button is operated a current pass through the coil. Since the iron-cored coil is energized the soft iron armature is attracted to the electromagnet. The armature also carries a striker which hits

the gong. When the circuit is broken the coil becomes demagnetized and the spring steel strip pulls the armature back to its original position. The striker will only operate when the push is operated.



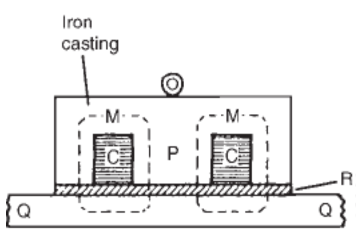

**(ii) Relay**

A relay is similar to an electric bell except that contacts are opened or closed by operation instead of a gong being struck. A typical simple relay is shown below, which consists of a coil wound on a soft iron core. When the coil is energized the hinged soft iron armature is attracted to the electromagnet and pushes against two fixed contacts so that they are connected together, thus closing some other electrical circuit.

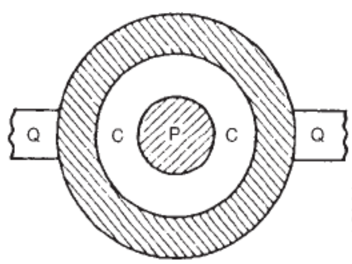



### *(iii)* Lifting magnet

Lifting magnets, incorporating large electromagnets, are used in iron and steel works for lifting scrap metal. A typical robust lifting magnet, capable of exerting large attractive forces, is shown in the elevation and plan view of figure below where a coil, C, is wound round a central core, P, of the iron casting. Over the face of the electromagnet is placed a protective non-magnetic sheet of material, R. The load, Q, which must be of magnetic material is lifted when the coils are energized, the magnetic flux paths, M, being shown by the broken lines.



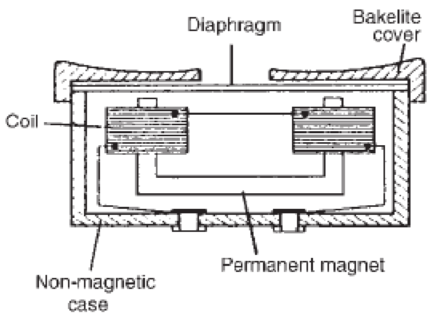

Sectional elevation through a diameter




Plan view

### *(iv)* Telephone receiver

Whereas a transmitter or microphone changes sound waves in corresponding electrical signals, a telephone receiver converts electrical waves back into sound waves. A typical telephone receiver is shown below and consist of permanent magnet with coils wound on its poles. A thin, flexible diaphragm of magnetic material is held in position near to the magnetic poles but not touching them. Variation in current from the transmitter varies the magnetic field and the diaphragm consequently vibrates. The vibration produces sound variations corresponding to those transmitted.




- Content/Topic 3: Introduction to electromagnetic induction

When a conductor is moved across a magnetic field so as cut through the lines of force (or flux), an electromotive force (e.m.f.) is produced in the conductor. If the conductor forms parts of a closed circuit then the e.m.f. produced causes an electric current to flow round the circuit. Hence an e.m.f. (and current) is 'induced' in the conductor as a result of its movement across the magnetic field. This effect is known as 'electromagneticinduction'

Figure(a) below shows a coil of wire connected to a centre-zero galvanometer, which is a sensitive ammeter with the zero-current position in the centre of thescale.

(a) When the magnet is moved at constant speed towards the coil (figure (a) below), a deflection is the galvanometer showing that a current has been produced in the coil.

(b) When the magnet is moved at the same speed as in (a) but away from the coil the same deflection is noted but is in the opposite direction (figure (b) below).

(c) When the magnet is held stationary, even within the coil, no deflection isrecorded.

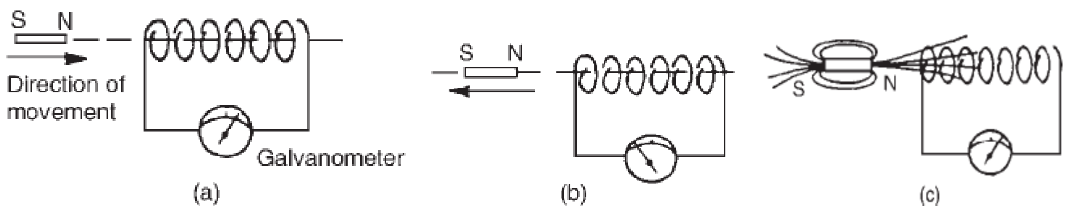
(d) When the coil is moved at the same speed as in (a) and the magnet held stationary the same galvanometer deflection is noted.

(e) When the relative speed is, say, doubled, the galvanometer deflection isdoubled.

(f) When a stronger magnet is used, a greater galvanometer deflection is noted.

(g) When the number of turns of wire of the coil is increased, a greater galvanometer deflection is noted.

Figure (c) below shows the magnetic field associated with the magnet. As the magnet is moved towards the coil, the magnetic flux of the magnet moves across, or cuts, the coil. It is the relative movement of the magnetic flux and the coil that causes an e.m.f. and thus current, to be induced in the coil. This effect is known as electromagnetic induction.




**Laws of electromagneticinduction**

**Faraday's laws of electromagnetic induction**

*(i) 'An induced e.m.f. is set up whenever the magnetic field linking that circuitchanges.'*

(ii) '*The magnitude of the induced e.m.f. in any circuit is proportional to the rate of change of the magnetic flux linking the circuit.*'

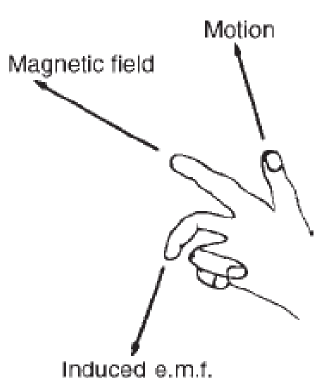
**Lenz's law states**

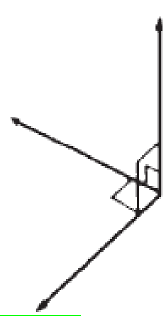
'The direction of an e.m.f. is always such that it tends to set up a current opposing the motion or the change of flux responsible for inducing that e.m.f.'

An alternating method to Lenz's law of determining relative directions is given by Fleming's Right-hand rule (often called the generator rule) which states:

Let the thumb, first finger and second finger of the right hand be extended such that they are all at right angles to each other (as shown in figure below).

If the first finger points in the direction of the magnetic field, the thumb points in the direction of motion of the conductor relative to the magnetic field, then the second finger will point the direction of the induced e.m.f.



Summarizing

First finger → Field

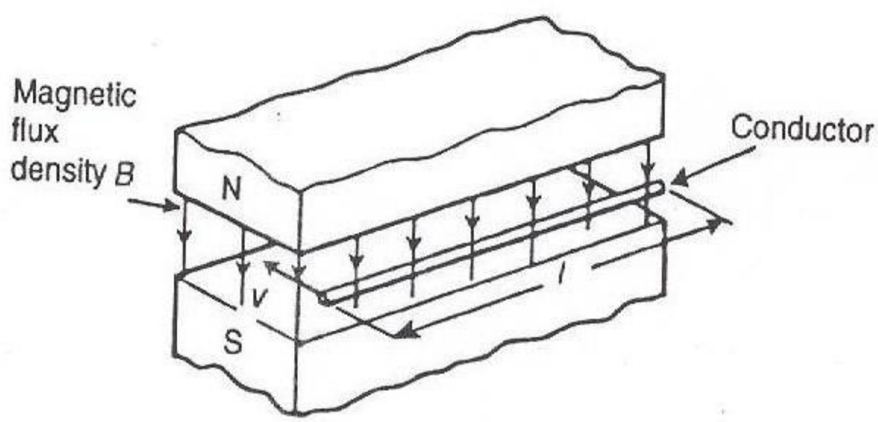
Thumb → Motion

Second finger → E.m.f.

In a generator, conductors forming an electric circuit are made to move through a magnetic field. By Faraday's law an e.m.f. is induced in the conductor and thus a source of e.m.f. is created. A generator converts mechanical energy into electrical energy. The induced e.m.f. E set up between the ends of the conductor shown in figure below is given by:

$$E = Blv \text{ volts}$$

Where B, the flux density, is measured in teslas, l, the length of conductor in the magnetic field, is measured in metres, and v, the conductor velocity, is measured in metres per second.



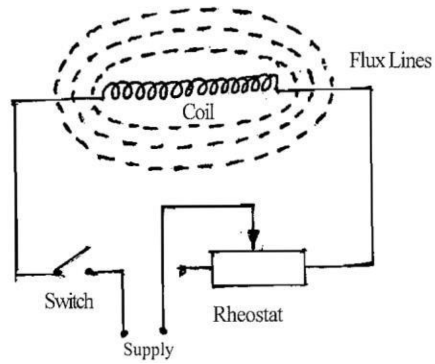

If the conductor moves at an angle $\theta^{\circ}$ to the magnetic field (instead of at $90^{\circ}$ as assumed above) then

$$E = Blv\sin\theta \text{ volts}$$

**Inductance**

Inductance is the name given to the property of a circuit whereby there is an e.m.f. induced into the circuit by the change of flux linkages produced by a current change.

When the e.m.f. is induced in the same circuit as that in which the current is changing, the property is called self inductance, L




When the e.m.f. is induced in a circuit by a change of flux due to current changing in an adjacent circuit, the property is called mutual inductance, M.

The unit of inductance is the henry, H.

'*A circuit has an inductance of one henry when an e.m.f. of one volt is induced in it by a current changing at the rate of one ampere per second.*'

Induced e.m.f. in a coil of N turns,

$$E = -N \frac{d\Phi}{dt} \text{ volts}$$

Where $d\Phi$ is the change in flux in webers, and dt is the the time taken for the flux to change in seconds (i.e., $\frac{d\Phi}{dt}$ is the rate of change of flux).

Induced e.m.f. in a coil of inductance L henrys,

$$E = -L \frac{dI}{dt} \text{ volts}$$

Where dI is the change in current in amperes and dt is the time taken for the current to change in seconds (i.e. dI/dt is the rate of change of current). The minus sign in each of the above two equations remind us of its direction (given by Lenz's law).

**Uses of self-induction**

- In the fluorescent tubes for starting purpose and to reduce the voltage.

- In regulators, to give reduced voltage to fans.

- In auto-transformers.

- In smooth choke which is used in welding plant.

- In rectifiers to keep are stationary.

**Inductors**

A component called an inductor is used when the property of inductance is required in a circuit. The basic form of an inductor is simply a coil of wire.

Factor which affect the inductance of an inductor include:

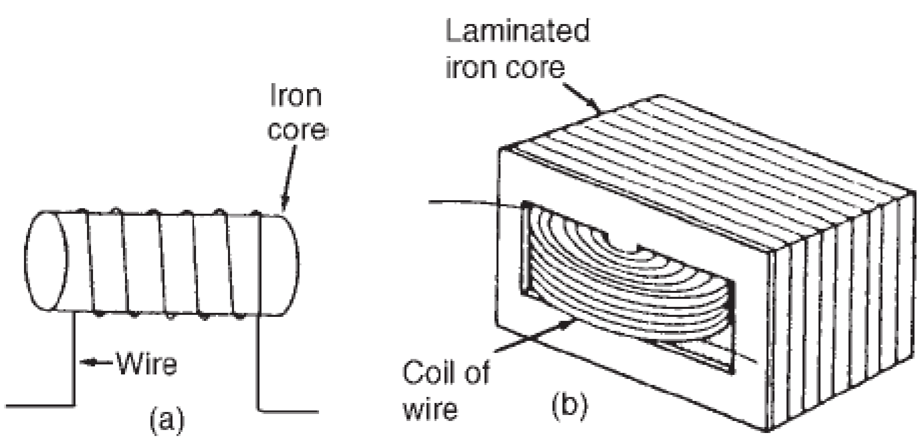
*(i) The number of turns of wire: the more turns the higher the higher inductance.*

*(ii) The cross-sectional area of the coil of wire: the greater the cross-sectional area the higher the inductance.*

*(iii) The presence of a magnetic core: when the coil is wound on an iron core the same current sets up a more concentrated magnetic field and the inductance is increased.*

*(iv) The way the turns are arranged: a short thick coil of wire has a higher inductance than a long thin one.*

Two examples of practical inductor are shown in figure below,

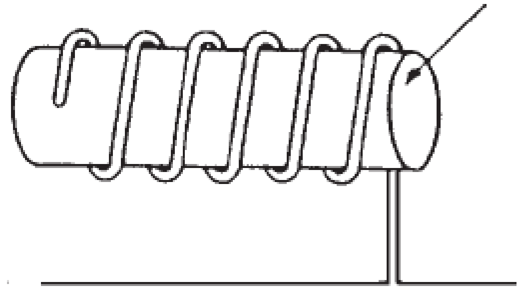


and the standard electrical circuit diagram symbols for air-cored and iron-cored inductors area shown in figure below.

Air-cored inductor

Iron-cored inductor

An iron-cored inductor is often called a choke since, when used in a.c. circuits, it has a choking effect, limiting the current flowing through it. Inductance is often undesirable in a circuit. To reduce inductance to a minimum the wire may be bent back on itself, as shown in figure below, so that the magnetizing effect of one conductor is neutralized by that of the adjacent conductor. The wire may be coiled around an insulator, as shown, without increasing the inductance. Standard resistors may be no-inductively wound in this manner.




**Energy stored**

An inductor possesses an ability to store energy. The energy stored, W, in the magnetic field of an inductor is given by:

$$W = \frac{1}{2} LI^2 \text{ joules}$$

**Coefficient of Self-induction (L)**

It may be defined in any one of the three ways given below :

**(i) First Method for L**

The coefficient of self-induction of a coil is defined as the weber-turns per ampere in the coil

By 'weber-turns' is meant the product of flux in webers and the number of turns with which the flux is linked. In other words, it is the flux-linkages of the coil. Consider a solenoid having N turns and carrying a current of I amperes. If the flux produced is Φ webers, the weber-turns are NΦ. Hence, weber-turns per ampere are N Φ/I.

By definition,

$$L = \frac{N\Phi}{I} \text{ henrys}$$

The unit of self − induction is henry

(ii) **Second Method for L**

We have seen that flux produced in a solenoid is

$$\Phi = \frac{NI}{l/\mu_0\mu_r A} \quad \therefore \frac{\Phi}{I} = \frac{N}{I/\mu_0\mu_r A} \quad \text{Now } L = N\frac{\Phi}{I} = N \cdot \frac{N}{I/\mu_0\mu_r A} H$$

$$L = \frac{N^2}{l/\mu_0\mu_r A} = \frac{N^2}{S} \text{H} \quad \text{or} \quad L = \frac{\mu_0\mu_r A N^2}{l} \text{H}$$

**Coefficient of Mutual Inductance (M)**

It can also be defined in three ways as given below :

**(*i*) First Method for M**

Let there be two magnetically-coupled coils having $N1$ and $N2$ turns respectively. Coefficient of mutual inductance between the two coils is defined as *the weber-turns in one coil due to one ampere current in the other.*

Let a current $I_1$ ampere when flowing in the first coil produce a flux $\Phi_1$ webers in it. It is supposed that whole of this flux links with the turns of the second coil. Then, flux-linkages i.e., webers-turns in the second coil for unit current in the first coil are $N_2\ \Phi_1/I_1$. Hence, by definition

$$M = \frac{N_2\,\Phi_1}{I_1}$$

**(*ii*) Second Method for M**

We will now deduce an expression for coefficient of mutual inductance in terms of the dimensions of the two coils.

$$\text{Flux in the first coil} \quad \Phi_1 = \frac{N_2 I_1}{l/\mu_0\mu_r A}\ \text{Wb}\ ; \quad \text{Flux/ampere} = \frac{\Phi_1}{I_1} = \frac{N_1}{l/\mu_0\mu_r A}$$

Assuming that whole of this flux (it usually is some percentage of it) is linked with the other coil having $N_2$ turns, the weber-turns in it due to the flux/ampere in the first coil is

$$M = \frac{N_2\Phi_1}{I_1} = \frac{N_2N_1}{l/\mu_0\mu_r A} \qquad \therefore\ M = \frac{\mu_0\mu_r A\ N_1N_2}{l}\ \text{H}$$

Also

$$M = \frac{N_1N_2}{l/\mu_0\mu_r A} = \frac{N_1N_2}{\text{reluctance}} = \frac{N_1N_2}{S}\ \text{H}$$

**(iii) Third Method for M**

Mutually induced e.m.f. in the second coil,

$$E_2 = -M\frac{dI_1}{dt}\ volts$$

Where M is the mutual inductance between two coils, in henrys, and $\frac{dI_1}{dt}$ is the rate of change of current in the first coil. The phenomenon of mutual inductance is used in transformers.

**Uses of mutual induction**

- It is used in ignition coil which is used in motor car.

- It is also used in inductance furnace.

- It is used for the principle of transformers.

Problem 1· A section through a magnetic circuit of uniform cross-sectional area 2 $cm^2$ is shown in Figure 7.5. The cast steel core has a mean length of 25 cm. The air gap is 1 mm wide and the coil has 5000 turns. The $B$–$H$ curve for cast steel is shown on page 78. Determine the current in the coil to produce a flux density of 0.80 T in the air gap, assuming that all the flux passes through both parts of the magnetic circuit.

For the cast steel core, when $B = 0.80$ T, $H = 750$ A/m (from page 78)

Reluctance of core $S_1 = \dfrac{l_1}{\mu_0 \mu_r A_1}$ and since $B = \mu_0 \mu_r H$,

then $\mu_r = \dfrac{B}{\mu_0 H}$. Thus $S_1 = \dfrac{l_1}{\mu_0 \left(\dfrac{B}{\mu_0 H}\right) A} = \dfrac{l_1 H}{BA} = \dfrac{(25 \times 10^{-2})(750)}{(0.8)(2 \times 10^{-4})}$

$$= 1\,172\,000/\text{H}$$

For the air gap: Reluctance, $S_2 = \dfrac{l_2}{\mu_0 \mu_r A_2} = \dfrac{l_2}{\mu_0 A_2}$

(since $\mu_r = 1$ for air)

$$= \frac{1 \times 10^{-3}}{(4\pi \times 10^{-7})(2 \times 10^{-4})}$$

$$= 3\,979\,000/\text{H}$$

Total circuit reluctance $S = S_1 + S_2 = 1\,172\,000 + 3\,979\,000$

$$= 5\,151\,000/\text{H}$$

Flux $\Phi = BA = 0.80 \times 2 \times 10^{-4} = 1.6 \times 10^{-4}$ Wb

$$S = \frac{\text{mmf}}{\Phi}, \quad \text{thus mmf} = S\Phi$$

Hence $\quad NI = S\Phi$

and current $I = \dfrac{S\Phi}{N} = \dfrac{(5\,151\,000)(1.6 \times 10^{-4})}{5000} = \mathbf{0.165\ A}$

Problem **2**. A conductor 350 mm long carries a current of 10 A and is at right-angles to a magnetic field lying between two circular pole faces each of radius 60 mm. If the total flux between the pole faces is 0.5 mWb, calculate the magnitude of the force exerted on the conductor.

$l = 350$ mm $= 0.35$ m; $I = 10$ A;

Area of pole face $A = \pi r^2 = \pi(0.06)^2$ m$^2$;

$\Phi = 0.5$ mWb $= 0.5 \times 10^{-3}$ Wb

Force $F = BIl$, and $B = \dfrac{\Phi}{A}$

hence force $F = \left(\dfrac{\Phi}{A}\right) Il = \dfrac{(0.5 \times 10^{-3})}{\pi(0.06)^2}(10)(0.35)$ newtons

**i.e. force = 0.155 N**

Problem **3.** An electron in a television tube has a charge of $1.6 \times 10^{-19}$ coulombs and travels at $3 \times 10^{7}$ m/s perpendicular to a field of flux density 18.5 μT. Determine the force exerted on the electron in the field.

From above, force $F = QvB$ newtons, where

$Q$ = charge in coulombs = $1.6 \times 10^{-19}$ C;

$v$ = velocity of charge = $3 \times 10^{7}$ m/s;

and $B$ = flux density = $18.5 \times 10^{-6}$ T

Hence force on electron $F = 1.6 \times 10^{-19} \times 3 \times 10^{7} \times 18.5 \times 10^{-6}$

$$= 1.6 \times 3 \times 18.5 \times 10^{-18}$$

$$= 88.8 \times 10^{-18} = \mathbf{8.88 \times 10^{-17}\ N}$$

Problem **4.** At what velocity must a conductor 75 mm long cut a magnetic field of flux density 0.6 T if an e.m.f. of 9 V is to be induced in it? Assume the conductor, the field and the direction of motion are mutually perpendicular.

Induced e.m.f. $E = Blv$, hence velocity $v = \dfrac{E}{Bl}$

Hence $v = \dfrac{9}{(0.6)(75 \times 10^{-3})} = \dfrac{9 \times 10^{3}}{0.6 \times 75} = \mathbf{200\ m/s}$

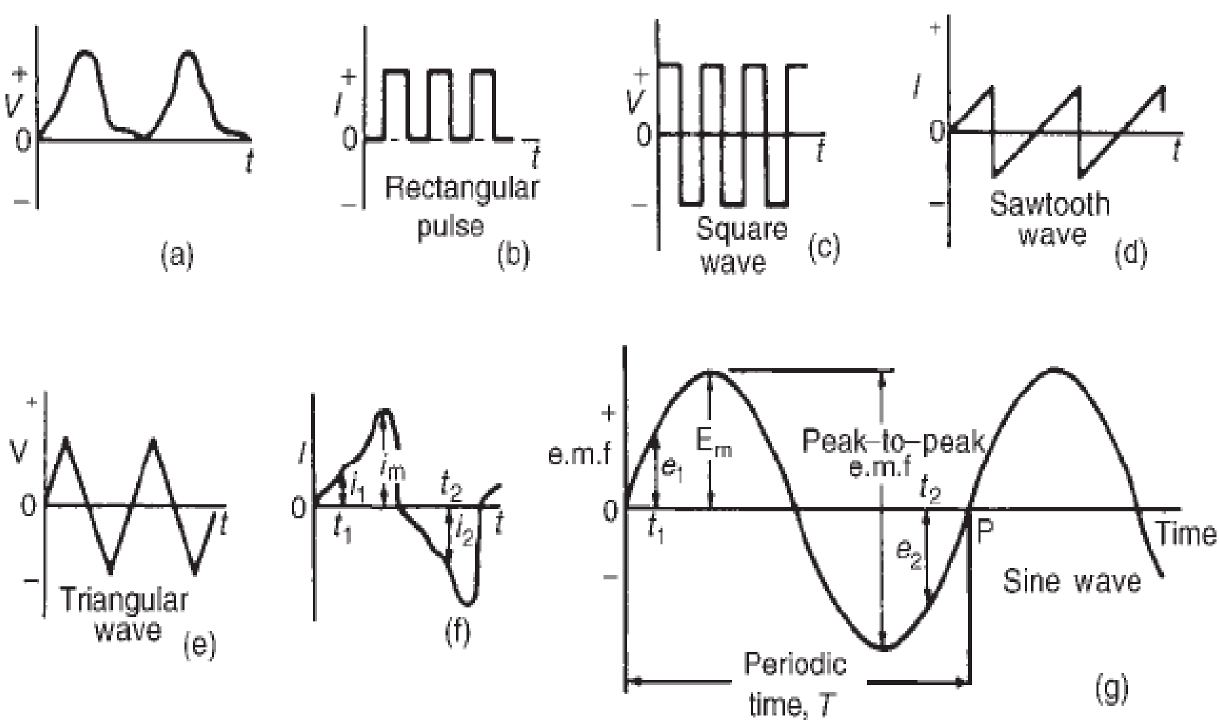
## Learning Unit 2 – Analyse characteristics of AC signals

### LO 2.1 – Identify waveform signals Perform

Electricity is produced by generators at power stations and then distributed by a vast network of transmission lines (called the National Grid system) to industry and for domestic use. It is easier and cheaper to generate alternating current (a.c.) than direct current (d.c.) and a.c. is more conveniently distributed than d.c. since its voltage can be readily altered using transformers.
Whenever d.c. is needed in preference to a.c., devices called rectifiers are used for conversion.

- Content/Topic 1: Introduction to alternating voltages and currents waveforms

If values of quantities which vary with time t are plotted to a base of time, the resulting graph is called a waveform. Some typical waveforms are shown in Figures below Waveforms (a) and (b) are unidirectional waveforms, for, although they vary considerably with time, they flow in one direction only (i.e. they do not cross the time axis and become negative). Waveforms (c) to (g) are called alternating waveforms since their quantities are continually changing in direction (i.e. alternately positive and negative).



- Content/Topic 2: Types of waveform distortion

Waveform distortion is defined as a steady-state deviation from an ideal sine wave of power frequency principally characterized by the spectral content of the deviation.
There are five primary types of waveform distortion: 1. DC Offset 2. Harmonics 3. Interharmonics 4. Notching 5. Noise.

**DC offsets** are instances where direct current (DC) overlaps an alternating current (AC) distribution system. This overlapping of two different types of current can cause overheating in the equipment.

**Harmonics** occur when some loads affect the main waveform of voltage. In this situation, the new loads prevent the waveform from reaching its highest and lowest levels. Harmonics can cause circuit breakers to trip and transformers to overheat.

**Interharmonics** is a condition where a signal affects the main voltage waveform. It can cause display monitors to flicker and equipment to overheat. Interharmonics can also cause communication issues.

**Notching** is an intermittent disturbance that can affect voltage. It normally happens when light dimmers or arc welders are being used. It results in data loss and issues with the transmission of data.

**Noise** is any unnecessary current or voltage affecting the waveform of the main power supply. This waveform distortion can cause data issues and equipment to malfunction.

## LO 2.2 – Interpret waveforms signals

- Content/Topic 1: Components of AC waveforms

A waveform of the type shown in Figure (g) above is called a sine wave. It is the shape of the waveform of e.m.f. produced by an alternator and thus the mains electricity supply is of 'sinusoidal' form.

1. **Period**

One complete series of values is called a cycle (i.e. from O to P in Figure (g) above).

The time taken for an alternating quantity to complete one cycle is called the period or the periodic time, *T*, of the waveform.

**2. Frequency**

The number of cycles completed in one second is called the frequency, *f*, of the supply and is measured in hertz, Hz. The standard frequency of the electricity supply in Rwanda is 50 Hz.

Relationship between Period and frequency

$$T = \frac{1}{f} \quad or \quad f = \frac{1}{T}$$

Where:

- *f = the frequency in Hz*
- *T = the periodic time in Second*

- <u>Content/Topic 2: Amplitude</u>

Instantaneous values are the values of the alternating quantities at any instant of time. They are represented by small letters, i, *v*, e etc., (see Figures (f) and (g) below).

The largest value reached in a half cycle is called the peak value or the maximum value or the crest value or the amplitude of the waveform. Such values are represented by $V_m$, $I_m$, etc. (see Figures (f) and (g) below).

A peak-to-peak value of e.m.f. is shown in Figure 14.3(g) and is the difference between the maximum and minimum values in a cycle.

The average or mean value of a symmetrical alternating quantity, (such as a sine wave), is the average value measured over a half cycle,
(since over a complete cycle the average value is zero).

$$Average\ or\ mean\ value = \frac{area\ under\ the\ curve}{length\ of\ base}$$

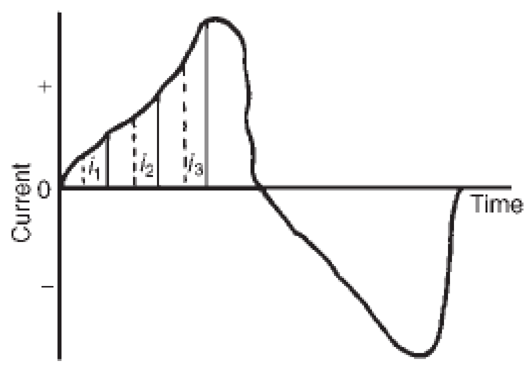
The area under the curve is found by approximate methods such as the trapezoidal rule, the mid-ordinate rule or Simpson's rule. Average values are represented by $V_{AV}$, $I_{AV}$, etc.

$$\textit{For a sine wave, } \boldsymbol{mean} \textit{ value} = 0.637 x \ \boldsymbol{maximum} \textit{ value}$$

$$\left(i.e. \ \frac{2}{\pi} \ x \ \boldsymbol{maximum} \textit{ value}\right)$$

The effective value of an alternating current is that current which will produce the same heating effect as an equivalent direct current. The effective value is called the root mean square (rms) value and whenever an alternating quantity is given, it is assumed to be the rms value.

For example, the domestic mains supply in Great Britain is 240 V and is assumed to mean '240 V rms'. The symbols used for rms values are I,V, E, etc. For a non-sinusoidal waveform as shown in Figure below the rms value is given by:




$$\boldsymbol{I} = \sqrt{\frac{\boldsymbol{i_1^2 + i_2^2 + \cdots i_n^2}}{\boldsymbol{n}}}$$

Where *n* is the number of intervals used

$$\textit{For a sine wave, } \boldsymbol{rms} \textit{ value} = 0.707 \ x \ \boldsymbol{maximum} \textit{ value}$$

$$\left(i.e. \ \frac{1}{\sqrt{2}} \ x \ \boldsymbol{maximum} \textit{ value}\right)$$

$$\textit{Form factor} = \frac{\textit{rms value}}{\textit{average value}}$$

For a sine wave, form factor = 1.11

$$\textit{Peak factor} = \frac{\textit{maximum value}}{\textit{rms value}}$$

For a sine wave, peak factor = 1.41

The values of form and peak factors give an indication of the shape of waveforms.

- Content/Topic 3: Application of waveform

Applications of sine wave mainly involves in the following

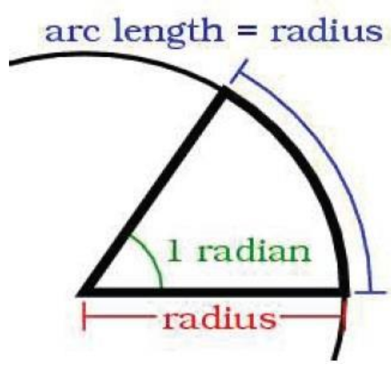
1. House appliances
2. Industries
3. AC motors
4. Refrigerators
5. Television
6. Radio

## LO 2.3 – Determine instantaneous equation of AC voltage and currents

- <u>Content/Topic 1: Introduction to trigonometric functions</u>

Trigonometry ( From Greek trigonon "triangle"+metron "measure" Relationship Between Degrees and Radians

A radian is defined as an angle subtended at the center of a circle for θ which the arc length is equal to the radius of that circle (see Fig.below).




The circumference of the circle is equal to 2πR, where R is the radius of the circle. Consequently, 360°=2π radians. Thus,

$$1 \text{ radian} = 360°/2\pi \approx 57.296°$$
$$1° = (2\pi/360) \text{ radians} \approx 0.01745 \text{ radians}$$

**The Unit Circle**

In mathematics, a unit circle is defined as a circle with a radius of 1. Often, especially in applications to trigonometry, the unit circle is centered at the origin (0,0) in the coordinate plane.

The equation of the unit circle in the coordinate plane is

$$x^2 + y^2 = 1.$$

As mentioned above, the unit circle is taken to be 360°, or 2π radians. We can divide the coordinate plane, and therefore, the unit circle, into 4 quadrants.

The first quadrant is defined in terms of coordinates by $x>0$, $y>0$, or, in terms of angles, by $0°<\theta<90°$, or $0<\theta<\pi/2$.

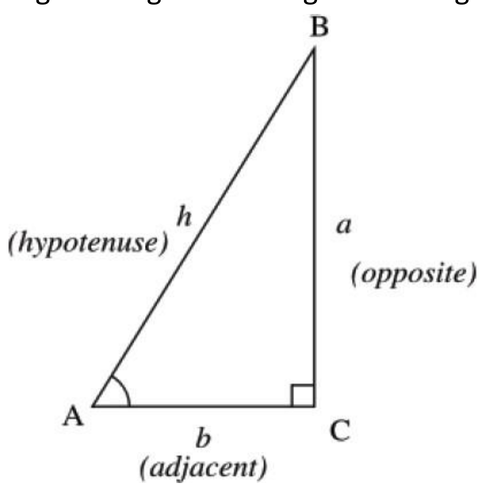
The second quadrant is defined by $x<0$, $y>0$, or $90°<\theta<180°$, or $\pi/2<\theta<\pi$.

The third quadrant is defined by $x<0$, $y<0$, or $180°<\theta<270°$, or $\pi<\theta<3\pi/2$.

Finally, the fourth quadrant is defined by $x>0$, $y<0$, or $270°<\theta<360°$, or $3\pi/2<\theta<2\pi$.

Definitions of Trigonometric Functions For a Right Triangle

A right triangle is a triangle with a right angle (90°) (See Fig.below).




For every angle in the triangle, there is the side of the triangle adjacent θ to it (from here on denoted as “adj”), the side opposite of it (from here on denoted as “opp”), and the hypotenuse

$$\text{sine of } \theta = \sin\theta = \frac{opp}{hyp}$$

$$\text{cosine of } \theta = \cos\theta = \frac{adj}{hyp}$$

$$\text{tangent of } \theta = \tan\theta = \frac{sin\theta}{cos\theta} = \frac{opp}{adj}$$

$$\text{cotangent of } \theta = \cot\theta = \frac{1}{tan\theta} = \frac{cos\theta}{sin\theta} = \frac{adj}{opp}$$

$$\text{secant of } \theta = \sec\theta = \frac{1}{cos\theta} = \frac{hyp}{adj}$$

$$\text{cosecant of } \theta = \csc\theta = \frac{1}{sin\theta} = \frac{hyp}{opp}$$

(from here on denoted as "hyp"), which is the longest side of the triangle located opposite of the right angle. For angle θ, the trigonometric functions are defined as follows:

Definitions of Trigonometric Functions for a Unit Circle

In the unit circle, one can define the trigonometric functions cosine and sine as follows. If (x,y) is a point on the unit cirlce, and if the ray from the origin (0,0) to that point (x,y) makes an angle θ with the positive x-axis, (such that the counterclockwise direction is considered positive), then,

cosθ = x/1 = x

sinθ = y/1 = y

Then, each point (x,y) on the unit circle can be written as (cosθ, sinθ). Combined with the equation $x^2 + y^2 = 1$, the definitions above give the relationship sin2θ+cos2θ=1. In addition, other trigonometric functions can be defined in terms of x and y:

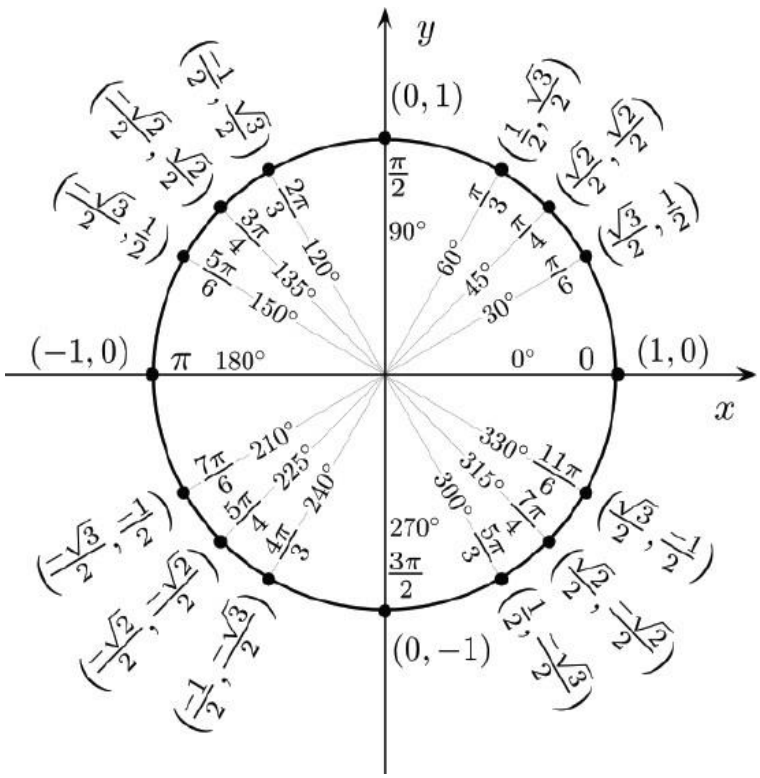
tanθ = sinθ/cosθ = y/x

cotθ = cosθ/sinθ = x/y

secθ = 1/cosθ = 1/x

cscθ = 1/sinθ = 1/y

Fig.below below shows a unit circle in the coordinate plane, together with some useful values of angle θ, and the points (x,y)=(cosθ, sinθ), that are most commonly used (also see table in the following section).




Exact Values for Trigonometric Functions of Most Commonly Used Angles

| θ in degrees | θ in radians | sinθ | cosθ | tanθ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 |
| 30 | $\frac{\pi}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ |
| 45 | $\frac{\pi}{4}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | 1 |
| 60 | $\frac{\pi}{3}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ |
| 90 | $\frac{\pi}{2}$ | 1 | 0 | undefined |
| 180 | π | 0 | -1 | 0 |
| 270 | $\frac{3\pi}{2}$ | -1 | 0 | undefined |
| 360 | 2π | 0 | 1 | 0 |

Trigonometric Functions of Negative Angles

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

Some Useful Relationships Among Trigonometric Functions

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta - \tan^2\theta = 1$$
$$\csc^2\theta - \cot^2\theta = 1$$

Sum, Difference and Product of Trigonometric Functions

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$$

Graphs of Trigonometric Functions (Fig.below, a-f)

Note: In each graph in Fig.below, the horizontal axis (x) is measured in radians.

"Sine." "Cosine." "Tangent." "Cotangent." "Secant." "Cosecant."

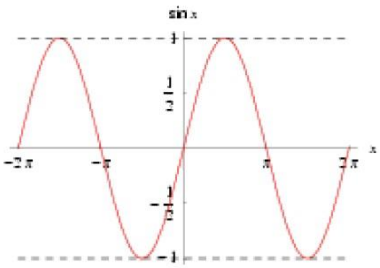



Fig. a. Graph of sin(x).

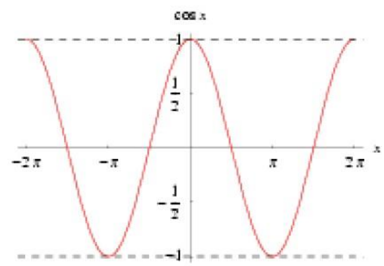



Fig. b. Graph of cos(x).

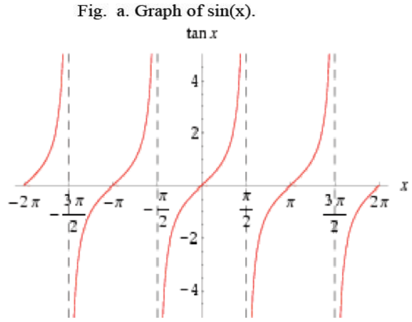



Fig. c. Graph of tan(x).

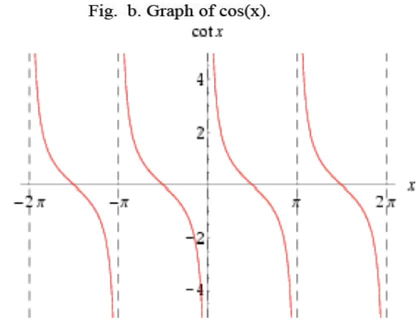



Fig. d. Graph of cot(x).

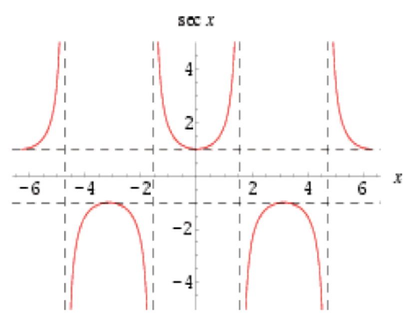



Fig. e. Graph of sec(x).

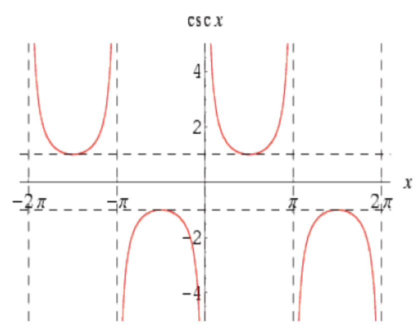



Fig. f. Graph of csc(x).

**Inverse Trigonometric Functions**

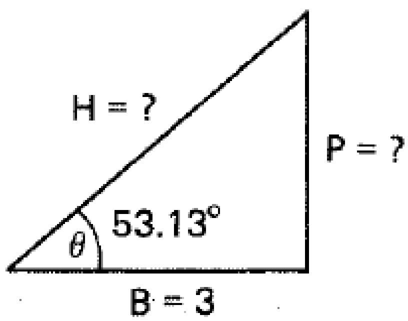
If x=sin(y), then y=sin-1(x), i.e. s is the angle whose sine is y. In other words, x is the inverse sine of y. Another name for inverse sine is arcsine, and the notation used is y=arcsin(x). Similarly, we can define inverse cosine, inverse tangent, inverse cotangent, inverse secant and inverse cosecant.

All of the inverse functions are many-valued functions of x (for each value of x, there are many corresponding values of y), which are collections of single-valued functions (for each value of x, there is only one corresponding value of y) called branches. For many purposes a particular branch is required. This is called the principal branch and the values for this branch are called principal values.

| | |
|---|---|
| $x = \sin(y)$ | $y = \sin^{-1}(x) = \arcsin(x)$ |
| $x = \cos(y)$ | $y = \cos^{-1}(x) = \arccos(x)$ |
| $x = \tan(y)$ | $y = \tan^{-1}(x) = \arctan(x)$ |
| $x = \cot(y)$ | $y = \cot^{-1}(x) = \text{arccot}(x)$ |
| $x = \sec(y)$ | $y = \sec^{-1}(x) = \text{arcsec}(x)$ |
| $x = \csc(y)$ | $y = \csc^{-1}(x) = \text{arccsc}(x)$ |

Examples

1. From the values shown in Fig. below, calculate P and H:




$$\cos\theta = \frac{B}{H}$$

Transposing,

$$H = \frac{B}{\cos\theta} = \frac{3}{\cos 53.13^\circ}$$

From tables or calculator, $\cos 53.13^\circ = 0.6$

$$\therefore H = \frac{3}{0.6} = 5$$

Now we can use sin or tan to find P:

$$\tan\theta = \frac{P}{B}$$

Transposing,

$$P = B.\tan\theta$$

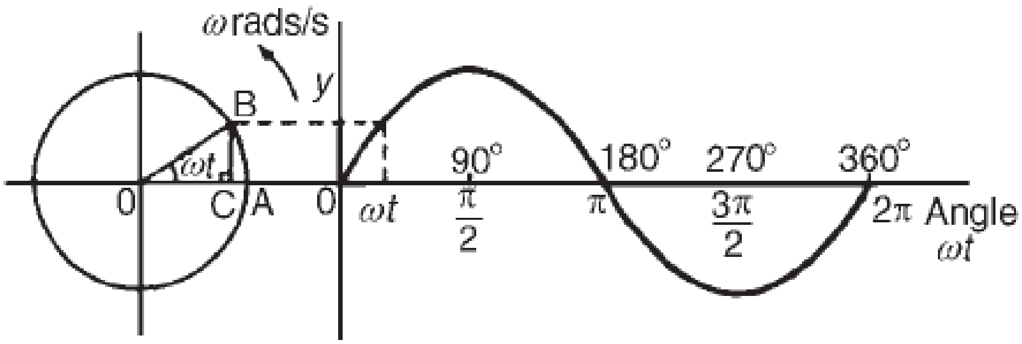
$$\tan\theta = \tan 53.13^\circ = 1.333$$

$$\therefore P = 3 \times 1.333 = 4\ (3.999)$$

**The equation of a sinusoidal waveform**

In Fig below, OA represents a vector that is free to rotate anticlockwise about 0 at an angular velocity of ω rad/s. A rotating vector is known as a phasor.
After time t seconds the vector OA has turned through an angle ωt. If the line BC is constructed perpendicular to OA as shown, then

$$Sin\omega t = \frac{BC}{OB} \text{ i.e. } BC = OBSin\omega t$$



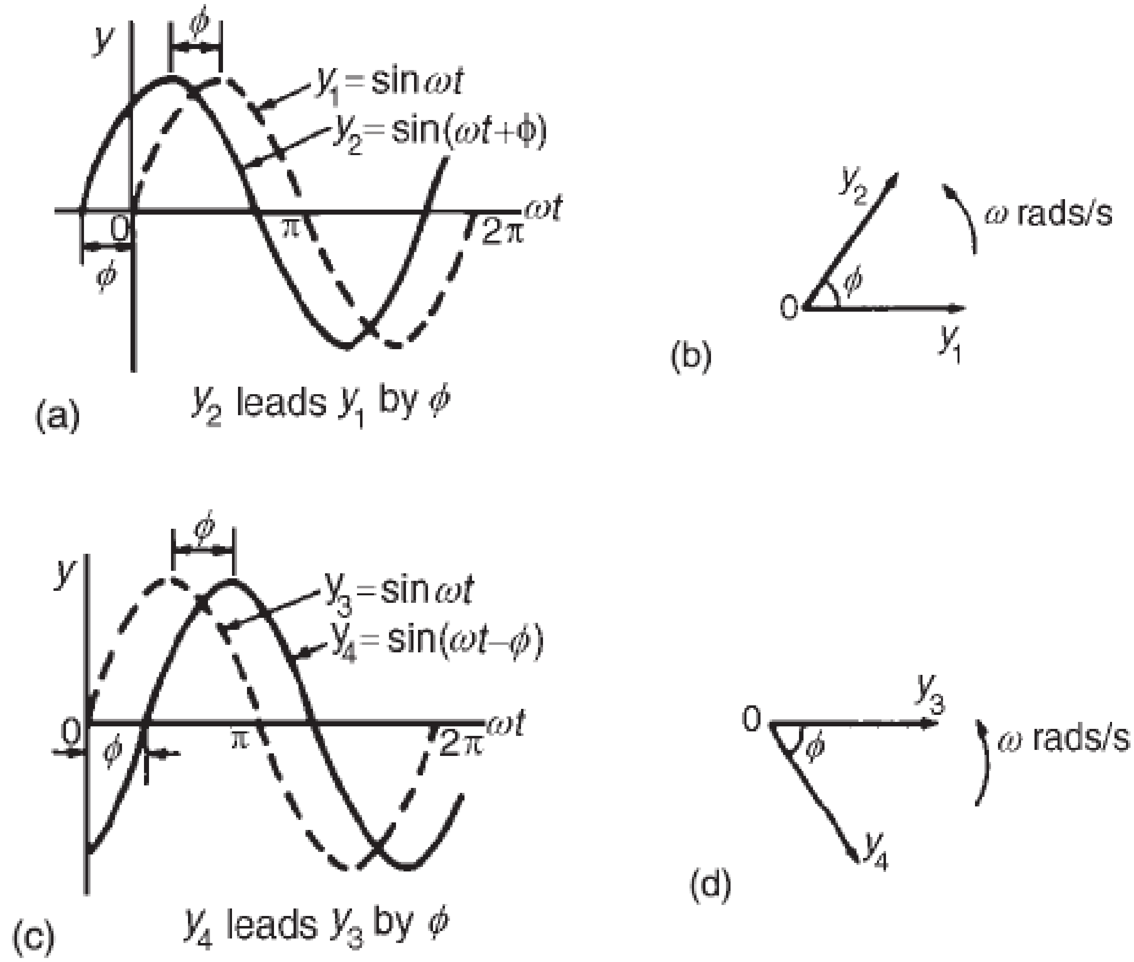

If all such vertical components are projected on to a graph of y against angle ωt (in radians), a sine curve results of maximum value OA. Any quantity which varies sinusoidally can thus be represented as a phasor. A sine curve may not always start at 0°.

To show this a periodic function is represented by $y = \sin(\omega t \pm \phi)$, where $\phi$ is the phase (or angle) difference compared with $y= \sin\omega t$.
In Figure (a) below, $y_2= \sin(wt+\phi)$ starts $\phi$ radians earlier than $y_1= \sin\omega t$ and is thus said to lead $y_1$ by $\phi$ radians.

Phasors $y_1$ and $y_2$ are shown in Figure (b) below at the time when t = 0.
In Figure (c) below, $y_4 = \sin(\omega t - \phi)$ starts $\phi$ radians later than $y_3 = \sin\omega t$ and is thus said to lag $y_3$ by $\phi$ radians. Phasors $y_3$ and $y_4$ are shown in Figure (d) below at the time when t = 0. Given the general sinusoidal voltage,

$v = V_m \sin(\boldsymbol{w}t \pm \phi)$ then

*(i)* Amplitude or maximum value = $V_m$
*(ii)* Peak to peak value = $2V_m$
*(iii)* Angular velocity = $\boldsymbol{w}$ *rad/s*
*(iv)* Periodic time, $T = 2\boldsymbol{\pi}/\boldsymbol{w}$ *seconds*
*(v)* Frequency, $f = w/2\boldsymbol{\pi}$ *Hz (sin* $\boldsymbol{w}= 2\boldsymbol{\pi}f$*)*
(vi) $\phi$ = angle of lag or lead (compared with $v=V_m\sin\omega t$)




**Combination of waveforms**

The resultant of the addition (or subtraction) of two sinusoidal quantities may be determined either:

(a) by plotting the periodic functions graphically.
(b) by resolution of phasors by drawing or calculation

## Content/Topic 2: Instantaneous values

The instantaneous value of an alternating voltage or current is the value of voltage or current at one particular instant. The value may be zero if the particular instant is the time in the cycle at which the polarity of the voltage is changing.

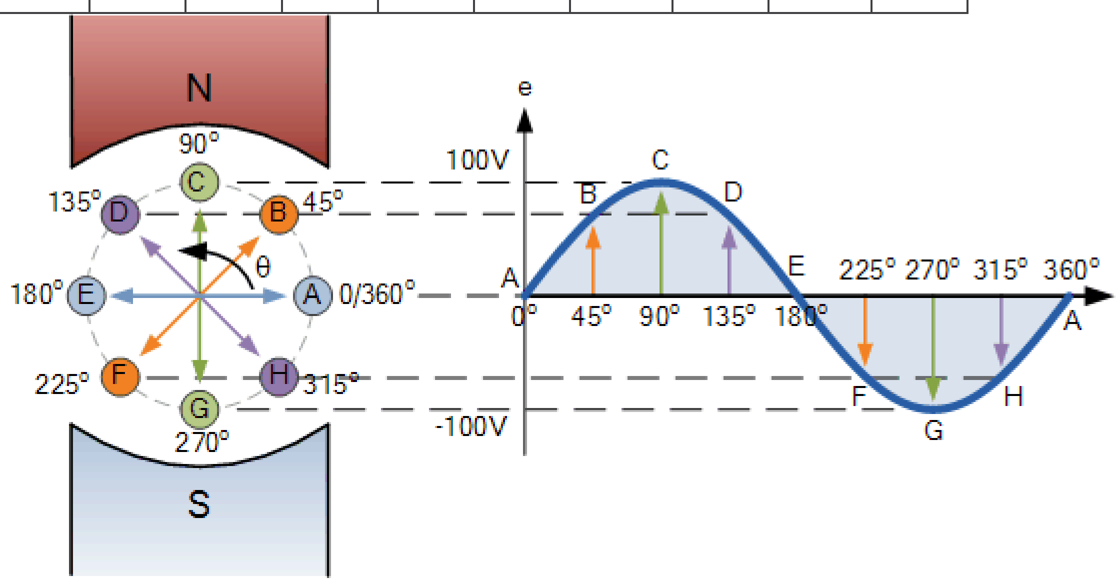
Let an **instantaneous voltage V** be represented by v = VM sin 2πft volts. This is a waveform which varies sinusoidally with time t, has a frequency f, and a maximum value, VM. Alternating voltages are usually assumed to have wave-shapes which are sinusoidal where only one frequency is present.

## Content /Topic3: Graphical interpretation of instantaneous equation of voltage and current

$V_{MAX}$ value of 100V. Plotting the instantaneous values at shorter intervals, for example at every 30° (12 points) or 10° (36 points) for example would result in a more accurate sinusoidal waveform construction.

**Sinusoidal Waveform Construction**

| Coil Angle ( θ ) | 0 | 45 | 90 | 135 | 180 | 225 | 270 | 315 | 360 |
|---|---|---|---|---|---|---|---|---|---|
| e = Vmax.sinθ | 0 | 70.71 | 100 | 70.71 | 0 | -70.71 | -100 | -70.71 | -0 |



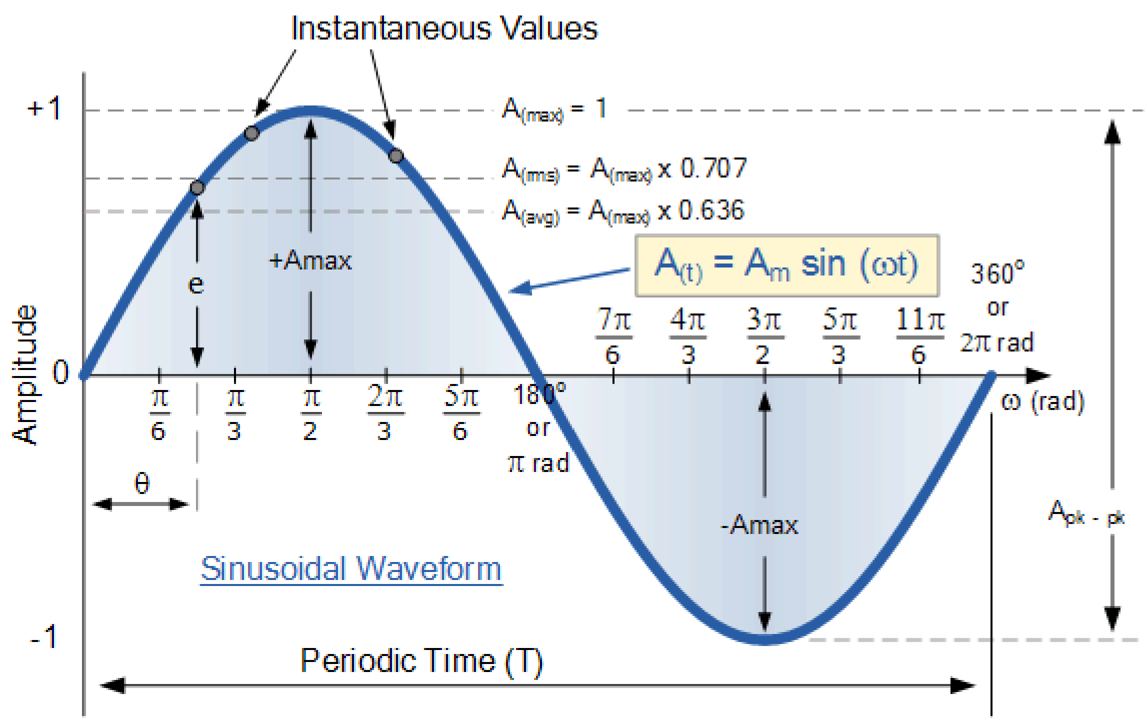
The points on the sinusoidal waveform are obtained by projecting across from the various positions of rotation between 0º and 360º to the ordinate of the waveform that corresponds to the angle, θ and when the wire loop or coil rotates one complete revolution, or 360º, one full waveform is produced.

From the plot of the sinusoidal waveform we can see that when θ is equal to 0º, 180º or 360º, the generated EMF is zero as the coil cuts the minimum amount of lines of flux. But when θ is equal to 90º and 270º the generated EMF is at its maximum value as the maximum amount of flux is cut.

Therefore a sinusoidal waveform has a positive peak at 90º and a negative peak at 270º. Positions B, D, F and H generate a value of EMF corresponding to the formula: e = Vmax.sinθ.

Then the generalised format used for analysing and calculating the various values of a **Sinusoidal Waveform** is as follows:

**A Sinusoidal Waveform**



## Worked Example 6.1

**Q** An alternating voltage is represented by the expression $v = 35\sin(314.2t)$ volt. Determine, (a) the maximum value, (b) the frequency, (c) the period of the waveform, and (d) the value 3.5 ms after it passes through zero, going positive.

**A**

**(a)** $v = 35\sin(314.2t)$ volt

and comparing this to the standard,

$v = V_m \sin(2\pi ft)$ volt we can see that:

$V_m = 35$ V **Ans**

**(b)** Again, comparing the two expressions:

$$2\pi f = 314.2$$

$$\text{so, } f = \frac{314.2}{2\pi} = 50 \text{ Hz } \textbf{Ans}$$

**(c)** $$T = \frac{1}{f} = \frac{1}{50} \text{ second}$$

so, $T = 20$ ms **Ans**

**(d)** When $t = 3.5$ ms; then:

$$v = 35\sin(2\pi \times 50 \times 3.5 \times 10^{-3}) \text{ volt}$$

$$= 35\sin(1.099)^*$$

$$= 35 \times 0.891$$

therefore, $v = 31.19$ V **Ans**

**Q** A sinusoidal alternating voltage has an average value of 3.5V and a period of 6.67 ms. Write down the standard (trigonometrical) expression for this voltage.

**A**

$V_{av} = 3.5\text{V}; T = 6.67 \times 10^{-3}\text{s}$

The standard expression is of the form $v = V_m \sin(2\pi ft)$ volt

$$V_{av} = 0.637\ V_m \text{ volt}$$

$$\text{so, } V_m = \frac{V_{av}}{0.637} \text{ volt} = \frac{3.5}{0.637}$$

$$V_m = 5.5\ V$$

$$f = \frac{1}{T} \text{ hertz} = \frac{1}{6.67 \times 10^{-3}} \text{Hz}$$

and, $f = 150$ Hz

$$v = 5.5 \sin(2\pi \times 150 \times t) \text{ volt}$$

so, $v = 5.5 \sin(300\pi t)$ volt **Ans**

## Worked Example 6.4

**Q** **For the waveform specified in Example 6.3 above, after the waveform passes through zero, going positive, determine its instantaneous value (a) 0.5 ms later, (b) 4.5 ms later, and (c) the time taken for the voltage to reach 3 V for the first time.**

**A**

(a) $t = 0.5 \times 10^{-3}\text{s}$; (b) $t = 4.5 \times 10^{-3}\text{s}$; (c) $v = 3\text{V}$

**(a)**

$$v = V_m \sin(300\pi \times 0.5 \times 10^{-3}) \text{ volt}$$

$$= 5.5 \sin 0.4712$$

$$= 5.5 \times 0.454$$

thus, $v = 2.5$ V **Ans**

**(b)**

$$v = 5.5 \sin(300\pi \times 4.5 \times 10^{-3}) \text{ volt}$$

$$= 5.5 \sin 4.241$$

$$= 5.5 \times (-0.891)$$

and, $v = -4.9$ V **Ans**

**(c)** $3 = 5.5 \sin(300\pi t)$ volt

so, $\sin(300\pi t) = \dfrac{3}{5.5} = 0.5455$

$300\pi t = \sin^{-1} 0.5455 = 0.5769$ rad

$t = \dfrac{0.5769}{300\pi} = 6.12 \times 10^{-4}$

and, $t = 0.612$ ms **Ans**

A sketch graph illustrating these answers is shown in Fig. 6.6.

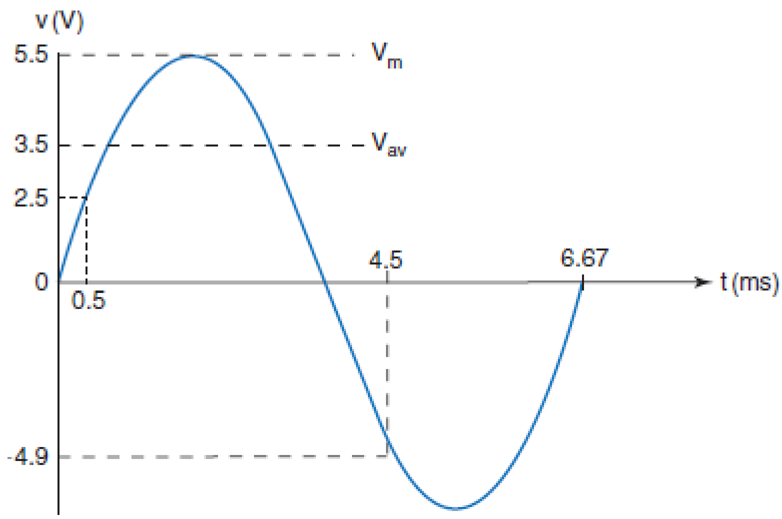



Fig. 6.6

## Worked Example 6.10

**Q** **Three alternating currents are specified below. Determine the frequency, and for each current, determine its phase angle, and amplitude.**

$$i_1 = 5 \sin(80\pi t + \pi/6) \text{ amp}$$
$$i_2 = 3 \sin 80\pi t \text{ amp}$$
$$i_3 = 6 \sin(80\pi t - \pi/4) \text{ amp}$$

**A**

All three waveforms have the same value of $\omega$, namely $80\pi$ rad/s. Thus all three have the same frequency:

$$\omega = 2\pi f = 80\pi \text{ rad/s}$$
$$\text{therefore, } f = \frac{80\pi}{2\pi} = 40 \text{ Hz } \textbf{Ans}$$

Since zero phase angle is quoted for $i_2$, then this is the reference waveform, of amplitude 3 A **Ans**

$I_{m1} = 5$ A, and leads $i_2$ by $\pi/6$ rad (30°) **Ans**
$I_{m3} = 6$ A, and lags $i_2$ by $\pi/4$ rad (45°) **Ans**

## Worked Example 6.11

**Q** **Four currents are as shown below. Draw to scale the corresponding phasor diagram.**

$$i_1 = 2.5 \sin(\omega t + \pi/4) \text{ amp}; \quad i_2 = 4 \sin(\omega t - \pi/3) \text{ amp};$$
$$i_3 = 6 \sin \omega t \text{ amp}; \quad i_4 = 3 \cos \omega t \text{ amp}$$

**A**

Before the diagram is drawn, we need to select a reference waveform (if one exists). The currents $i_1$ and $i_2$ do not meet this criterion, since they both have an associated phase angle.

This leaves the other two currents. Neither of these has a phase angle shown. However, $i_3$ is a sinewave, whilst $i_4$ is a *cosine* waveform. Now, a cosine wave *leads* a sinewave by 90°, or $\pi/2$ radian.

Therefore, $i_4$ may also be expressed as $i_4 = 3 \sin(\omega t + \pi/2)$ amp. Thus $i_3$ is chosen as the reference waveform, and will therefore be drawn along the horizontal axis.

The resulting phasor diagram is shown in Fig. 6.21.

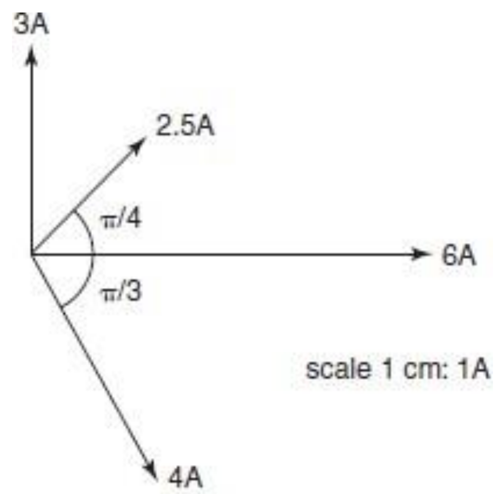



Fig. 6.21

## Worked Example 6.12

**Q** The phasor diagram representing four alternating currents is shown in Fig. 6.22, where the length of each phasor represents the amplitude of that waveform. Write down the standard expression for each waveform.

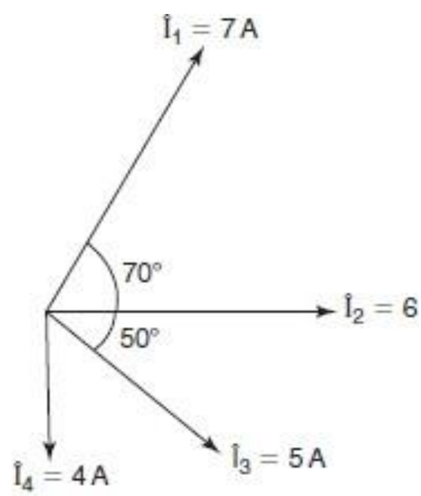



Fig. 6.22

**A**

$$I_{m1} = 7\text{ A};\ \phi_1 = +70^\circ = \frac{70\pi}{180} = 1.22\text{ rad}$$

$$I_{m2} = 6\text{ A};\ \phi_2 = 0^\circ = 0\text{ rad}$$

$$I_{m3} = 5\text{ A};\ \phi_3 = -50^\circ = \frac{-50\pi}{180} = -0.873\text{ rad}$$

$$I_{m4} = 4\text{ A};\ \phi_4 = -90^\circ = \frac{-90\pi}{180} = -1.571\text{ rad}$$

hence, $i_1 = 7\sin(\omega t + 1.22)$ amp **Ans**

$i_2 = 6\sin \omega t$ amp **Ans**

$i_3 = 5\sin(\omega t - 0.873)$ amp **Ans**

$i_4 = 4\sin(\omega t - 1.571)$ amp **Ans**

## Worked Example 6.13

**Q** Determine the phasor sum of the two voltages specified below.

$$v_1 = 25\sin(314t + \pi/3),\text{ and } v_2 = 15\sin(314t - \pi/6)\text{ volt}$$

**A**

Figure 6.27 shows the sketch of the phasor diagram.

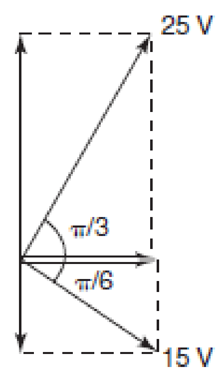



Fig. 6.27

***Note:*** Always sketch a phasor diagram.

$$\text{H.C.} = 25\cos\pi/3 + 15\cos(-\pi/6)$$
$$= (25\times 0.5) + (15\times 0.866)$$
$$= 12.5 + 12.99$$
$$\text{so, H.C.} = 25.49\ \text{V}$$

$$\text{V.C.} = 25\sin\pi/3 + 15\sin(-\pi/6)$$
$$= (25\times 0.866) + (15\times(-0.5))$$
$$= 21.65 - 7.5$$
$$\text{so, V.C.} = 14.15\ \text{V}$$

Figure 6.28 shows the phasor diagram for H.C., V.C. and $V_m$.

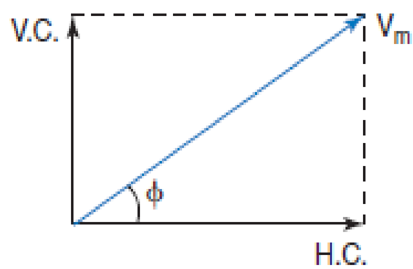



Fig. 6.28

$$V_m = \sqrt{\text{H.C.}^2 + \text{V.C.}^2} = \sqrt{25.49^2 + 14.15^2}$$
$$\text{so, } V_m = 29.15\ \text{V}$$
$$\tan\phi = \frac{\text{V.C.}}{\text{H.C.}} = \frac{14.15}{25.49} = 0.555^*$$
$$\text{so, } \phi = \tan^{-1} 0.555 = 0.507\ \text{rad}$$

therefore, $v = 29.15\sin(314t + 0.507)$ volt **Ans**

## Worked Example 6.14

**Q** **Calculate the phasor sum of the three currents listed below.**

$$i_1 = 6\sin\omega t\ \text{amp}$$
$$i_2 = 8\sin(\omega t - \pi/2)\ \text{amp}$$
$$i_3 = 4\sin(\omega t + \pi/6)\ \text{amp}$$

## A

The relevant phasor diagrams are shown in Figs. 6.29 and 6.30.

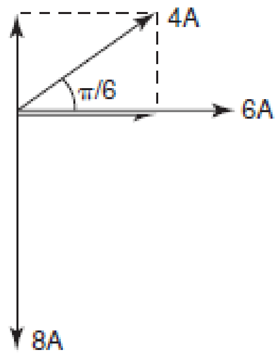



Fig. 6.29

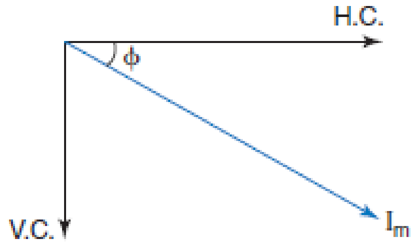



Fig. 6.30

$$\text{H.C.} = 6\cos 0 + 8\cos(-\pi/2) + 4\cos\pi/6$$
$$= (6\times 1) + (8\times 0) + (4\times 0.866)$$
$$= 6 + 3.46$$

so, $\text{H.C.} = 9.46\ \text{A}$

$$\text{V.C.} = 6\sin 0 + 8\sin(-\pi/2) + 4\sin\pi/6$$
$$= (6\times 0) + (8\times[-1]) + (4\times 0.5)$$
$$= -8 + 2$$

so, $\text{V.C.} = -6\ \text{A}$

$$I_m = \sqrt{\text{H.C.}^2 + \text{V.C.}^2} = \sqrt{9.46^2 + (-6)^2}$$

so, $I_m = 11.2\ \text{A}$

$$\phi = \tan^{-1}\frac{\text{V.C.}}{\text{H.C.}} = \tan^{-1}\frac{-6}{9.46} = \tan^{-1} - 0.6342$$

so, $\phi = -0.565$ rad

therefore, $i = 11.2\sin(\omega t - 0.565)$ amp **Ans**

Problem 9. An alternating voltage is given by $v = 282.8 \sin 314t$ volts. Find (a) the rms voltage, (b) the frequency and (c) the instantaneous value of voltage when $t = 4$ ms

(a) The general expression for an alternating voltage is

$$v = V_m \sin(\omega t \pm \phi).$$

Comparing $v = 282.8 \sin 314t$ with this general expression gives the peak voltage as 282.8 V

Hence the rms voltage = 0.707 × maximum value

$$= 0.707 \times 282.8 = \mathbf{200\ V}$$

(b) Angular velocity, $\omega = 314$ rad/s, i.e. $2\pi f = 314$

Hence frequency, $f = \dfrac{314}{2\pi} = \mathbf{50\ Hz}$

(c) When $t = 4$ ms, $v = 282.8 \sin(314 \times 4 \times 10^{-3})$

$$= 282.8 \sin(1.256) = \mathbf{268.9\ V}$$

(Note that 1.256 radians $= \left[1.256 \times \dfrac{180}{\pi}\right]^\circ = 71.96^\circ = 71^\circ 58'$

Hence $v = 282.8 \sin 71^\circ 58' = \mathbf{268.9\ V}$)

Problem 10. An alternating voltage is given by

$v = 75\sin(200\pi t - 0.25)$ volts.

Find (a) the amplitude, (b) the peak-to-peak value, (c) the rms value, (d) the periodic time, (e) the frequency, and (f) the phase angle (in degrees and minutes) relative to $75\sin 200\pi t$

Comparing $v = 75\sin(200\pi t - 0.25)$ with the general expression $v = V_m \sin(\omega t \pm \phi)$ gives:

(a) Amplitude, or peak value = **75 V**

(b) Peak-to-peak value $= 2 \times 75 =$ **150 V**

(c) The rms value $= 0.707 \times$ maximum value $= 0.707 \times 75 =$ **53 V**

(d) Angular velocity, $\omega = 200\pi$ rad/s

Hence periodic time, $T = \frac{2\pi}{\omega} = \frac{2\pi}{200\pi} = \frac{1}{100} =$ **0.01 s or 10 ms**

(e) Frequency, $f = \frac{1}{T} = \frac{1}{0.01} =$ **100 Hz**

(f) Phase angle, $\phi = 0.25$ radians lagging $75\sin 200\pi t$

$0.25 \text{ rads} = \left(0.25 \times \frac{180}{\pi}\right)^{\circ} = 14.32^{\circ} = 14^{\circ}19'$

Hence phase angle = **14°19′ lagging**

Problem 11. An alternating voltage, $v$, has a periodic time of 0.01 s and a peak value of 40 V. When time $t$ is zero, $v = -20$ V. Express the instantaneous voltage in the form $v = V_m \sin(\omega t \pm \phi)$

Amplitude, $V_m = 40$ V

Periodic time $T = \dfrac{2\pi}{\omega}$ hence angular velocity,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.01} = 200\pi \text{ rad/s}$$

$v = V_m \sin(\omega t + \phi)$ thus becomes $v = 40\sin(200\pi t + \phi)$ V

When time $t = 0$, $v = -20$ V

i.e. $-20 = 40\sin\phi$

so that $\sin\phi = \dfrac{-20}{40} = -0.5$

Hence $\phi = \arcsin(-0.5) = -30° = \left(-30 \times \dfrac{\pi}{180}\right)$ rads $= -\dfrac{\pi}{6}$ rads

**Thus $v = 40\sin\left(200\pi t - \dfrac{\pi}{6}\right)$ V**

Problem 12. The current in an a.c. circuit at any time $t$ seconds is given by: $i = 120\sin(100\pi t + 0.36)$ amperes. Find:

(a) the peak value, the periodic time, the frequency and phase angle relative to $120\sin 100\pi t$

(b) the value of the current when $t = 0$

(c) the value of the current when $t = 8$ ms

(d) the time when the current first reaches 60 A, and

(e) the time when the current is first a maximum

(a) Peak value = **120 A**

$$\text{Periodic time } T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} \text{ (since } \omega = 100\pi)$$

$$= \frac{1}{50} = \mathbf{0.02\ s\ or\ 20\ ms}$$

$$\text{Frequency, } f = \frac{1}{T} = \frac{1}{0.02} = \mathbf{50\ Hz}$$

$$\text{Phase angle} = 0.36 \text{ rads} = \left(0.36 \times \frac{180}{\pi}\right)^{\circ} = \mathbf{20^{\circ}38'\ leading}$$

(b) When $t = 0$, $i = 120\sin(0 + 0.36) = 120\sin 20^{\circ}38' = \mathbf{49.3\ A}$

(c) When $t = 8$ ms, $i = 120\sin\left[100\pi\left(\frac{8}{10^3}\right) + 0.36\right]$

$$= 120\sin 2.8733 (= 120\sin 164^{\circ}38') = \mathbf{31.8\ A}$$

(d) When $i = 60$ A, $60 = 120\sin(100\pi t + 0.36)$

$$\text{thus } \frac{60}{120} = \sin(100\pi t + 0.36)$$

$$\text{so that } (100\pi t + 0.36) = \arcsin 0.5 = 30^{\circ} = \frac{\pi}{6} \text{ rads} = 0.5236 \text{ rads}$$

$$\text{Hence time, } t = \frac{0.5236 - 0.36}{100\pi} = \mathbf{0.521\ ms}$$

(e) When the current is a maximum, $i = 120$ A

Thus $120 = 120\sin(100\pi t + 0.36)$

$$1 = \sin(100\pi t + 0.36)$$

$$(100\pi t + 0.36) = \arcsin 1 = 90^\circ = \frac{\pi}{2} \text{ rads} = 1.5708 \text{ rads}$$

Hence time, $t = \dfrac{1.5708 - 0.36}{100\pi} = \mathbf{3.85\ ms}$

> Problem 13. The instantaneous values of two alternating currents are given by $i_1 = 20\sin\omega t$ amperes and $i_2 = 10\sin(\omega t + \pi/3)$ amperes. By plotting $i_1$ and $i_2$ on the same axes, using the same scale, over one cycle, and adding ordinates at intervals, obtain a sinusoidal expression for $i_1 + i_2$

$i_1 = 20\sin\omega t$ and $i_2 = 10\sin\left(\omega t + \dfrac{\pi}{3}\right)$ are shown plotted in Figure 14.9

Ordinates of $i_1$ and $i_2$ are added at, say, 15° intervals (a pair of dividers are useful for this).

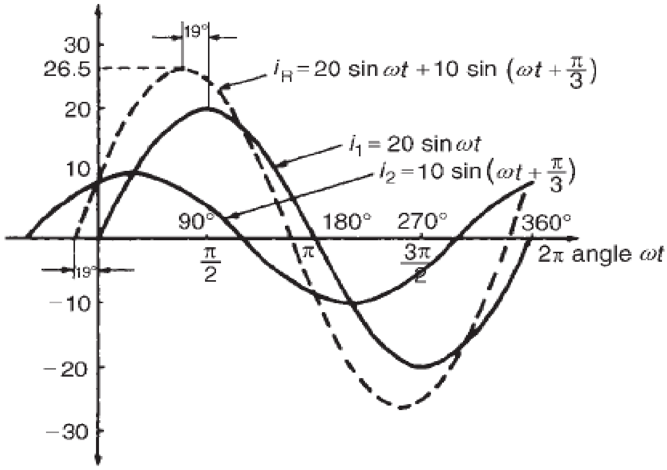
For example,

at 30°, $i_1 + i_2 = 10 + 10 = 20$ A
at 60°, $i_1 + i_2 = 8.7 + 17.3 = 26$ A
at 150°, $i_1 + i_2 = 10 + (-5) = 5$ A, and so on.

The resultant waveform for $i_1 + i_2$ is shown by the broken line in Figure 14.9. It has the same period, and hence frequency, as $i_1$ and $i_2$. The amplitude or peak value is 26.5 A.

The resultant waveform leads the curve $i_1 = 20\sin\omega t$ by 19°

i.e. $\left(19 \times \dfrac{\pi}{180}\right)$ rads $= 0.332$ rads




Hence the sinusoidal expression for the resultant $i_1 + i_2$ is given by:

$$i_R = i_1 + i_2 = \mathbf{26.5 \sin(\omega t + 0.332)\ A}$$

**Example 11.41.** *The voltage applied to a purely inductive coil of self-inductance 15.9 mH is given by the equation, v = 100 sin 314 t + 75 sin 942 t + 50 sin 1570 t. Find the equation of the resulting current wave.*

**Solution.** Here ω= 314 rad/s $\therefore X_1 = \omega L = (15.9 \times 10^{-3}) \times 314 = 5\ \Omega$

$$X_3 = 3\omega L = 3 \times 5 = 15\ \Omega;\ X_5 = 5\omega L = 5 \times 5 = 25\ \Omega$$

Hence, the current equation is

$$i = (100/5) \sin(314\,t - \pi/2) + (75/15) \sin(942t - \pi/2) + (50/25) \sin(1570t - \pi/2)$$

or $i =$ **20 sin (314$t$ −π/2) + 5 sin (942$t$ −π/2) + 2 sin (1570 $t$ −π/2)**

# Learning Unit 3 – Study the behaviour of different AC circuits

## LO 3.1 – Analyze A.C circuits

<u>Content/Topic 1: Series, parallel and mixed connection of resistors, capacitors and inductors</u>

**Series circuits**

**Series circuits** are sometimes referred to as *current*-coupled or daisy chain-coupled. The current in a series circuit goes through every component in the circuit. Therefore, all of the components in a series connection carry the same current.

A series circuit has only one path in which its current can flow. Opening or breaking a series circuit at any point causes the entire circuit to "open" or stop operating. For example, if even one of the light bulbs in an older-style string of Christmas tree lights burns out or is removed, the entire string becomes inoperable until the bulb is replaced.

**Current**

$$I = I_1 = I_2 = \cdots = I_n$$

In a series circuit, the current is the same for all of the elements.

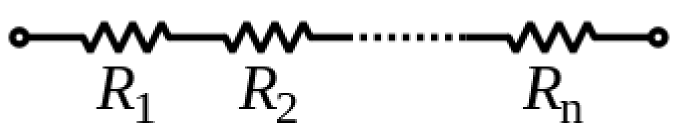
**Voltage**

$$V = V_1 + V_2 + \cdots + V_n$$

In a series circuit, the voltage is the sum of the voltage drops of the individual components (resistance units).

**Resistance units**

The total resistance of two or more resistors connected in series is equal to the sum of their individual resistances:




$$R_{\text{total}} = R_s = R_1 + R_2 + \cdots + R_n$$

Rs=>Resistance in series

**Electrical conductance** presents a reciprocal quantity to resistance. Total conductance of a series circuits of pure resistances, therefore, can be calculated from the following expression:

$$\frac{1}{G_{\text{total}}} = \frac{1}{G_1} + \frac{1}{G_2} + \cdots + \frac{1}{G_n}.$$

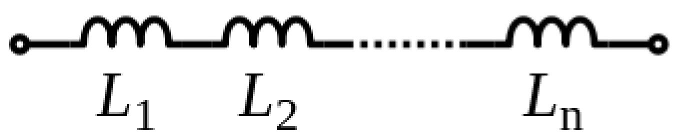
.

For a special case of two resistances in series, the total conductance is equal to:

$$G_{\text{total}} = \frac{G_1 G_2}{G_1 + G_2}.$$

**Inductors**

**Inductors** follow the same law, in that the total inductance of non-coupled inductors in series is equal to the sum of their individual inductances:




$$L_{\text{total}} = L_1 + L_2 + \cdots + L_n$$

However, in some situations, it is difficult to prevent adjacent inductors from influencing each other, as the magnetic field of one device coupled with the windings of its neighbours. This influence is defined by the mutual inductance M. For example, if two inductors are in series, there are two possible equivalent inductances depending on how the magnetic fields of both inductors influence each other.

When there are more than two inductors, the mutual inductance between each of them and the way the coils influence each other complicates the calculation. For a larger number of coils the total combined inductance is given by the sum of all mutual inductances between the various coils including the mutual inductance of each given coil with itself, which we term self-inductance or simply inductance. For three coils, there are six mutual inductances

$M_{12}$, $M_{13}$, $M_{23}$ and $M_{21}$, $M_{31}$ and $M_{32}$.

There are also the three self-inductances of the three coils:

$M_{11}$, $M_{22}$ and $M_{33}$.

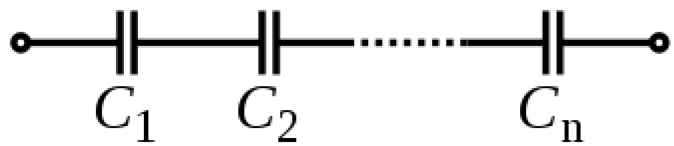
Therefore

$$L_{\text{total}} = (M_{11} + M_{22} + M_{33}) + (M_{12} + M_{13} + M_{23}) + (M_{21} + M_{31} + M_{32})$$

By reciprocity, $M_{ij} = M_{ji}$ so that the last two groups can be combined. The first three terms represent the sum of the self-inductances of the various coils. The formula is easily extended to any number of series coils with mutual coupling. The method can be used to find the self-inductance of large coils of wire of any cross-sectional shape by computing the sum of the mutual inductance of each turn of wire in the coil with every other turn since in such a coil all turns are in series.

**Capacitors**

Capacitors follow the same law using the reciprocals. The total capacitance of capacitors in series is equal to the reciprocal of the sum of the reciprocals of their individual capacitances:




$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}.$$

.

**Cells and batteries**

A battery is a collection of electrochemical cells. If the cells are connected in series, the voltage of the battery will be the sum of the cell voltages. For example, a 12 volt car battery contains six 2-volt cells connected in series. Some vehicles, such as trucks, have two 12 volt batteries in series to feed the 24-volt system.

**Parallel circuits**

If two or more components are connected in parallel, they have the same difference of potential (voltage) across their ends. The potential differences across the components are the same in magnitude, and they also have identical polarities. The same voltage is applied to all circuit components connected in parallel. The total current is the sum of the currents through the individual components, in accordance with Kirchhoff's current law.

**Voltage**

In a parallel circuit, the voltage is the same for all elements.

$$V = V_1 = V_2 = \ldots = V_n$$

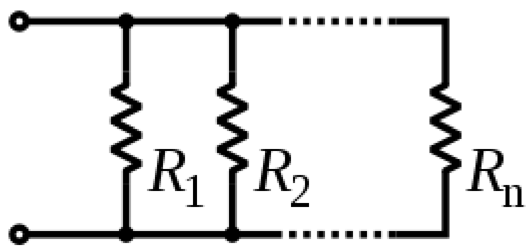
**Current**

The current in each individual resistor is found by Ohm's law. Factoring out the voltage gives

$$I_{\text{total}} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right).$$

**Resistance units**

To find the total resistance of all components, add the reciprocals of the resistances of each component and take the reciprocal of the sum. Total resistance will always be less than the value of the smallest resistance:




$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}.$$

.

For only two resistances, the unreciprocated expression is reasonably simple:

$$R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2}.$$

This sometimes goes by the mnemonic *product over sum*.

For *N* equal resistances in parallel, the reciprocal sum expression simplifies to:

$$\frac{1}{R_{\text{total}}} = N\frac{1}{R}.$$

and therefore to:

$$R_{\text{total}} = \frac{R}{N}.$$

.

To find the current in a component with resistance $R_i$, use Ohm's law again:

$$I_i = \frac{V}{R_i}.$$

.

The components divide the current according to their reciprocal resistances, so, in the case of two resistors,

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}.$$

.

An old term for devices connected in parallel is *multiple*, such as multiple connections for arc lamps.

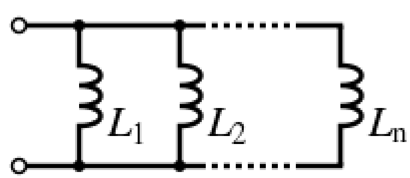
Since electrical conductance **G** is reciprocal to resistance, the expression for total conductance of a parallel circuit of resistors reads:

$$G_{\text{total}} = G_1 + G_2 + \cdots + G_n.$$

The relations for total conductance and resistance stand in a complementary relationship: the expression for a series connection of resistances is the same as for parallel connection of conductances, and vice versa.

**Inductors**

Inductors follow the same law, in that the total inductance of non-coupled inductors in parallel is equal to the reciprocal of the sum of the reciprocals of their individual inductances:




$$\frac{1}{L_{\text{total}}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}.$$

.

If the inductors are situated in each other's magnetic fields, this approach is invalid due to mutual inductance. If the mutual inductance between two coils in parallel is M, the equivalent inductor is:

$$\frac{1}{L_{\text{total}}} = \frac{L_1 + L_2 - 2M}{L_1 L_2 - M^2}$$

If $L_1 = L_2$

$$L_{\text{total}} = \frac{L + M}{2}$$

The sign of **M** depends on how the magnetic fields influence each other. For two equal tightly coupled coils the total inductance is close to that of every single coil. If the polarity of one coil is reversed so that M is negative, then the parallel inductance is nearly zero or the combination is almost non-inductive. It is assumed in the "tightly coupled" case M is very nearly equal to L.
However, if the inductances are not equal and the coils are tightly coupled there can be near short circuit conditions and high circulating currents for both positive and negative values of M, which can cause problems.

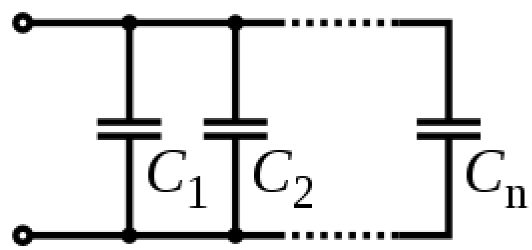
More than three inductors become more complex and the mutual inductance of each inductor on each other inductor and their influence on each other must be considered. For three coils, there are three mutual inductances M12, M13 and 23 . This is best handled by matrix methods and summing the terms of the inverse of the L matrix (3 by 3 in this case).

The pertinent equations are of the form:

$$v_i = \sum_j L_{i,j} \frac{di_j}{dt}$$

**Capacitors**

The total capacitance of capacitors in parallel is equal to the sum of their individual capacitances:




$$C_{\text{total}} = C_1 + C_2 + \cdots + C_n.$$

The working voltage of a parallel combination of capacitors is always limited by the smallest working voltage of an individual capacitor.

**Cells and batteries**

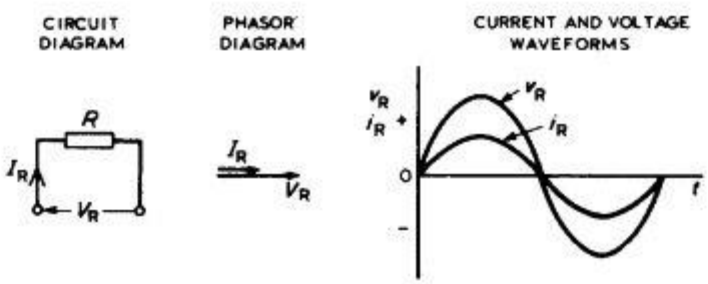
If the cells of a battery are connected in parallel, the battery voltage will be the same as the cell voltage, but the current supplied by each cell will be a fraction of the total current. For example, if a battery comprises four identical cells connected in parallel and delivers a current of 1 ampere, the current supplied by each cell will be 0.25 ampere. If the cells are not identical, cells with higher voltages will attempt to charge those with lower ones, potentially damaging them.

Parallel-connected batteries were widely used to power the valve filaments in portable radios. Lithium-ion rechargeable batteries (particularly laptop batteries) are often connected in parallel to increase the ampere-hour rating. Some solar electric systems have batteries in parallel to increase the storage capacity; a close approximation of total amp-hours is the sum of all amp-hours of in-parallel batteries.

- Content/Topic 2:   Single-phase series AC circuit

**Purely resistive a.c. circuit**

In a purely resistive a.c. circuit, the current *IR* and applied voltage *VR* are in phase

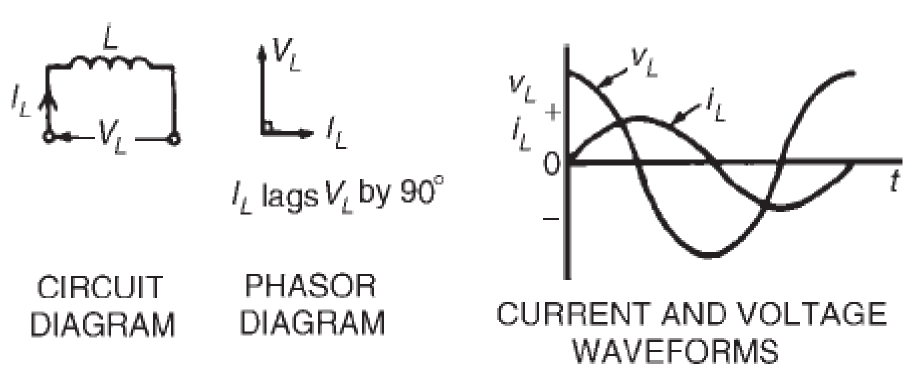



**Purely inductive a.c. circuit**

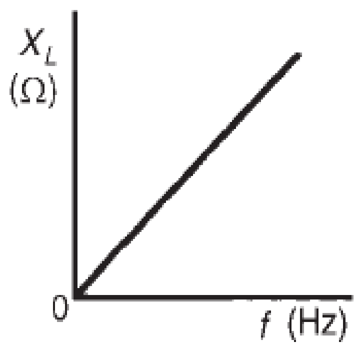
In a purely inductive a.c. circuit, the current $I_L$ lags the applied voltage $V_L$ by 90° (i.e. $\pi/2$ rads).
See Figures below.
In a purely inductive circuit the opposition to the flow of alternating current is called the inductive reactance, $X_L$

$$X_L = \frac{V_L}{I_L} = 2\pi f L \ \Omega$$

where *f* is the supply frequency, in hertz, and *L* is the inductance, in henry's.

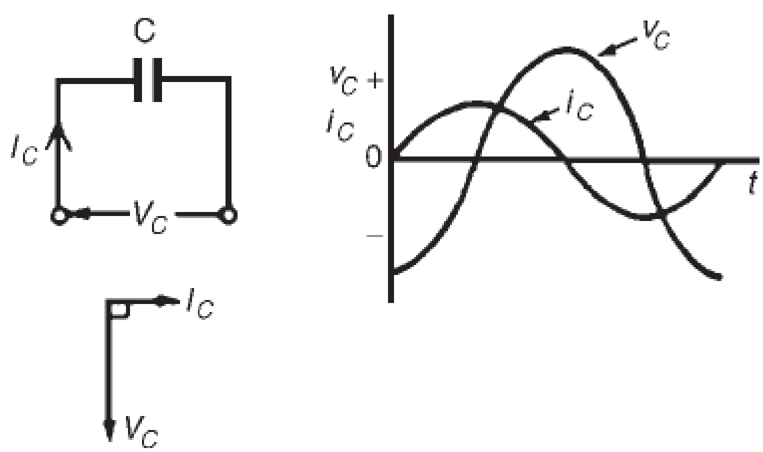


*XL* is proportional to *f* as shown below.




**Purely capacitive a.c. circuit**

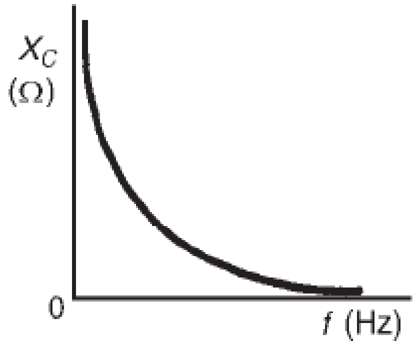
In a purely capacitive a.c. circuit, the current IC leads the applied voltage VC by 90° (i.e. /2 rads). See Figures below.
In a purely capacitive circuit the opposition to the flow of alternating current is called the capacitive reactance, *XC*




$$X_C = \frac{V_C}{I_C} = \frac{1}{2\pi f C}\,\Omega$$

where *C* is the capacitance in farads.

*XC* varies with frequency f as shown below.




**R–L series a.c. circuit**

In an a.c. circuit containing inductance L and resistance R, the applied voltage V is the phasor sum of $V_R$ and $V_L$ (see Figures below), and thus the current I lags the applied voltage V by an angle lying between 0° and 90° (depending on the values of $V_R$ and $V_L$), shown as angle ϕ. In any a.c. series circuit the current is common to each component and is thus taken as the reference phasor.
From the phasor diagram of Figure 15.6, the 'voltage triangle' is derived.

For the *R*–*L* circuit: $V = \sqrt{(V_R^2 + V_L^2)}$ (by Pythagoras' theorem)

and $\tan\phi = \dfrac{V_L}{V_R}$ (by trigonometric ratios)

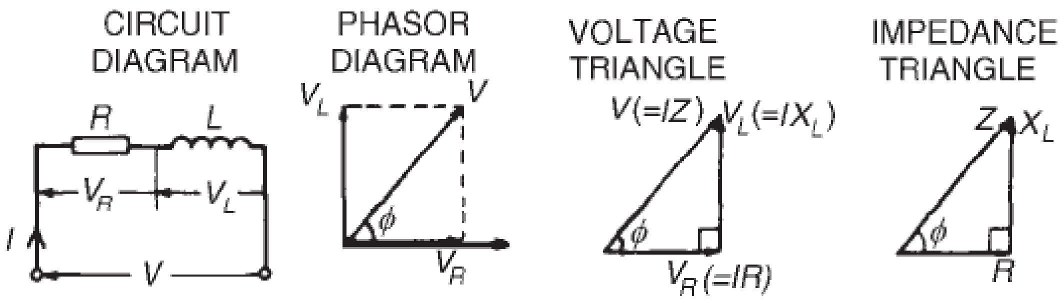
In an a.c. circuit, the ratio $\dfrac{\text{applied voltage } V}{\text{current } I}$ is called the **impedance *Z***, i.e.

$$Z = \frac{V}{I}\,\Omega$$

If each side of the voltage triangle in Figures below is divided by current *I* then the 'impedance triangle' is derived.

For the *R*–*L* circuit: $Z = \sqrt{(R^2 + X_L^2)}$

$\tan\phi = \dfrac{X_L}{R}$, $\sin\phi = \dfrac{X_L}{Z}$ and $\cos\phi = \dfrac{R}{Z}$




**_R–C_ series a.c. circuit**

In an a.c. series circuit containing capacitance C and resistance R, the applied voltage V is the phasor sum of $V_R$ and $V_C$ (see Figures below) and thus the current I leads the applied voltage V by an angle lying between 0° and 90° (depending on the values of $V_R$ and $V_C$), shown as angle ▯. From the phasor diagram of Figures below, the 'voltage triangle' is derived. For the R–C circuit:

$$V = \sqrt{(V_R^2 + V_C^2)} \text{ (by Pythagoras' theorem)}$$

$$\text{and } \tan\alpha = \frac{V_C}{V_R} \text{ (by trigonometric ratios)}$$

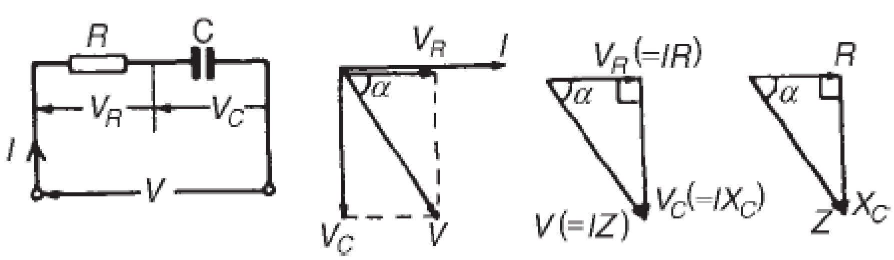
As stated above, in an a.c. circuit, the ratio

(applied voltage V)/(current I) is called the impedance *Z*, i.e. $Z = \frac{V}{I}\ \Omega$

If each side of the voltage triangle in Figures below is divided by current *I* then the 'impedance triangle' is derived.

$$\text{For the } R\text{–}C \text{ circuit: } Z = \sqrt{(R^2 + X_C^2)}$$

$$\tan\alpha = \frac{X_C}{R},\ \sin\alpha = \frac{X_C}{Z} \text{ and } \cos\alpha = \frac{R}{Z}$$

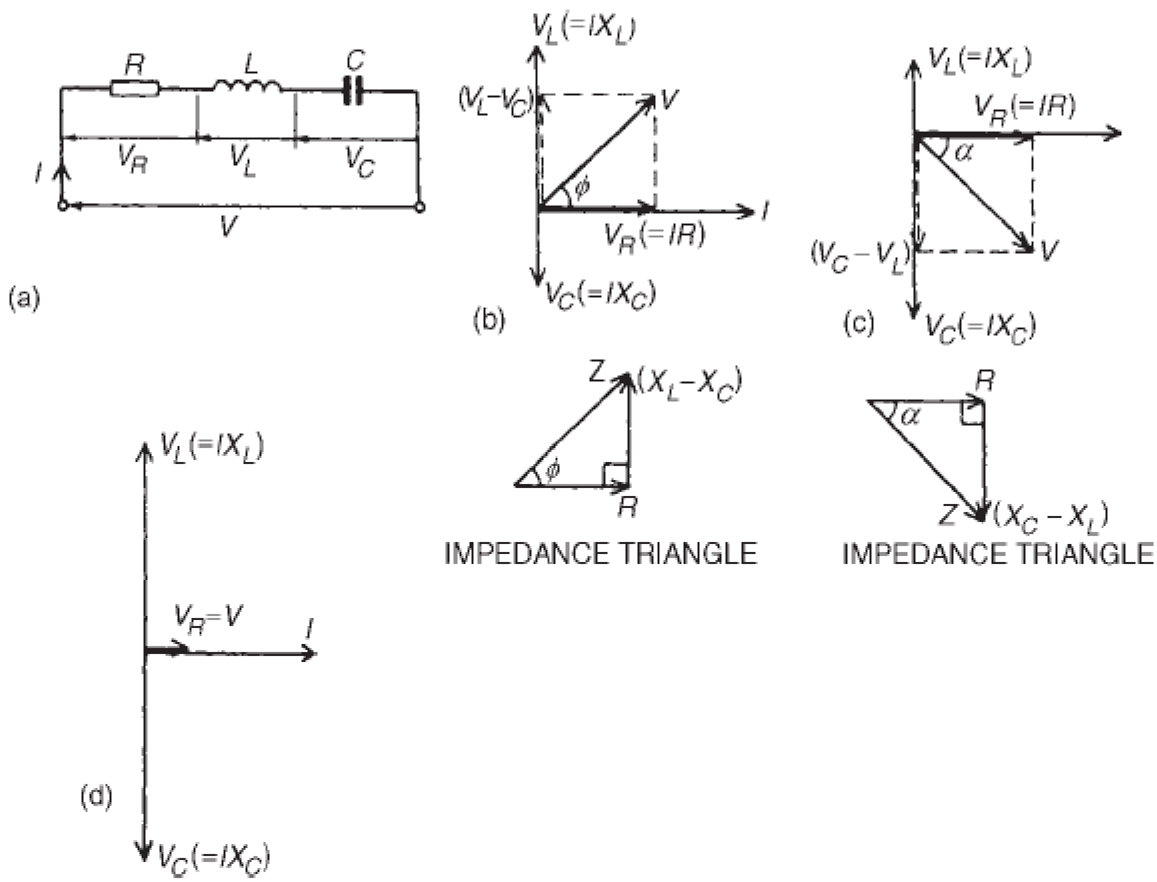


***R–L–C* series a.c. circuit**

In an a.c. series circuit containing resistance R, inductance L and capacitance C, the applied voltage V is the phasor sum of $V_R$, $V_L$ and $V_C$ (see Figures below). $V_L$ and $V_C$ are anti-phase, i.e. displaced by 180°, and there are three phasor diagrams possible each depending on the relative values of $V_L$ and $V_C$

When *XL* > *XC* (Figure (b) below:

$$Z = \sqrt{[R^2 + (X_L - X_C)^2]}$$
$$\text{and } \tan\phi = \frac{(X_L - X_C)}{R}$$




When *XC* > *XL* (Figure(c) below) :

$$Z = \sqrt{[R^2 + (X_C - X_L)^2]}$$
$$\text{and } \tan\alpha = \frac{(X_C - X_L)}{R}$$

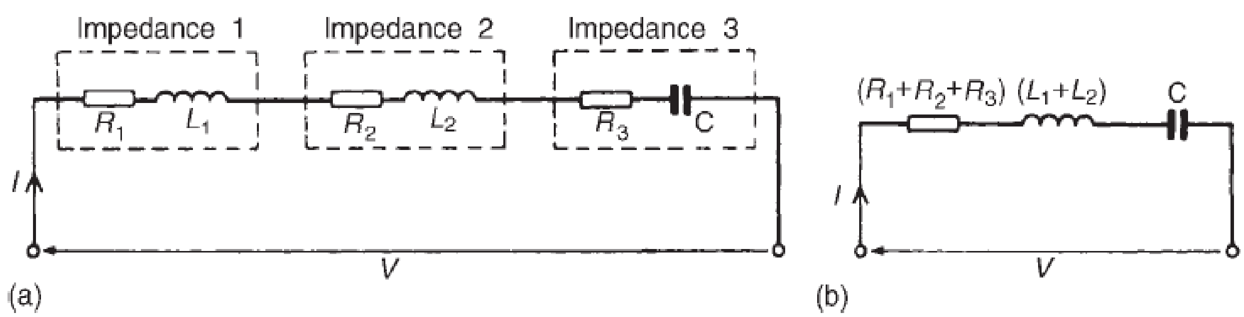
When $XL = XC$ (Figure (d) below), the applied voltage V and the current I are in phase. This effect is called series resonance

**Series connected impedances**

For series-connected impedances the total circuit impedance can be represented as a single L–C–R circuit by combining all values of resistance together, all values of inductance together and all values of capacitance together,

(remembering that for series connected capacitors $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$).

For example, the circuit of Figure (a) below showing three impedances has an equivalent circuit of Figure (b) below.




| *Type of Impedance* | *Value of Impedance* | *Phase angle for current* | *Power factor* |
|---|---|---|---|
| Resistance only | $R$ | 0° | 1 |
| Inductance only | $\omega L$ | 90° lag | 0 |
| Capacitance only | $1/\omega C$ | 90° lead | 0 |
| Resistance and Inductance | $\sqrt{[R^2 + (\omega L)^2]}$ | $0 < \phi < 90°$ lag | $1 >$ p.f. $> 0$ lag |
| Resistance and Capacitance | $\sqrt{[R^2 + (-1/\omega C)^2]}$ | $0 < \phi < 90°$ lead | $1 >$ p.f. $> 0$ lead |
| *R-L-C* | $\sqrt{[R^2 + (\omega L \sim 1/\omega C)^2]}$ | between 0° and 90° lag or lead | between 0 and unity lag or lead |

**Power in a.c. circuits**

The value of power at any instant is given by the product of the voltage and current at that instant.

(a) For a purely resistive a.c. circuit, the average power dissipated, P, is given by:

$$P = VI = I^2R = \frac{V^2}{R} \text{ watts } (V \text{ and } I \text{ being rms values}).$$

(b) For a purely inductive a.c. circuit, the average power is zero.
(c) For a purely capacitive a.c. circuit, the average power is zero.
For an R–L, R–C or R–L–C series a.c. circuit, the average power P is given by:

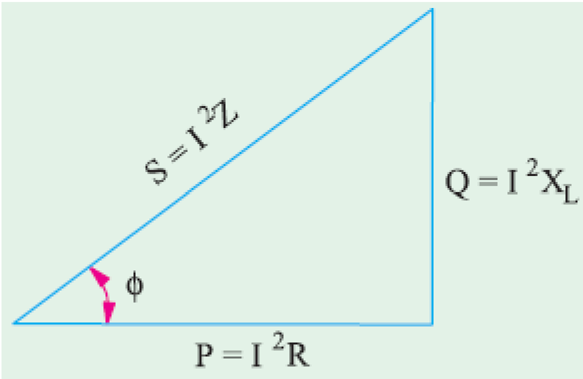
$$\boxed{P = VI\cos\phi \text{ watts}}$$

or

$$\boxed{P = I^2R \text{ watts}} \quad (V \text{ and } I \text{ being rms values})$$

**Power triangle and power factor**

For figure shown below,




**(i) apparent power (*S*)**

It is given by the product of r.m.s. values of applied voltage and circuit current.

∴ $S = VI = (IZ) \cdot I = I^2Z$ volt-amperes ($VA$)

(ii) active power ($P$ or $W$)

It is the power which is actually dissipated in the circuit resistance. $P = I^2R = VI \cos\phi$ watts

(iii) reactive power (Q)

It is the power developed in the inductive reactance of the circuit.

$Q = I^2.X_L = I^2.Z \sin\phi = I.(IZ).\sin\phi = VI \sin\phi$ volt-amperes-reactive (VAR)

$$S^2 = P^2 + Q^2 \text{ or } S = \sqrt{P^2 + Q^2}$$

| | |
|---|---|
| Apparent power, | $S = VI$ voltamperes (VA) |
| True or active power, | $P = VI \cos\phi$ watts (W) |
| Reactive power, | $Q = VI \sin\phi$ reactive voltamperes (var) |

$$\text{Power factor} = \frac{\text{True power } P}{\text{Apparent power } S}$$

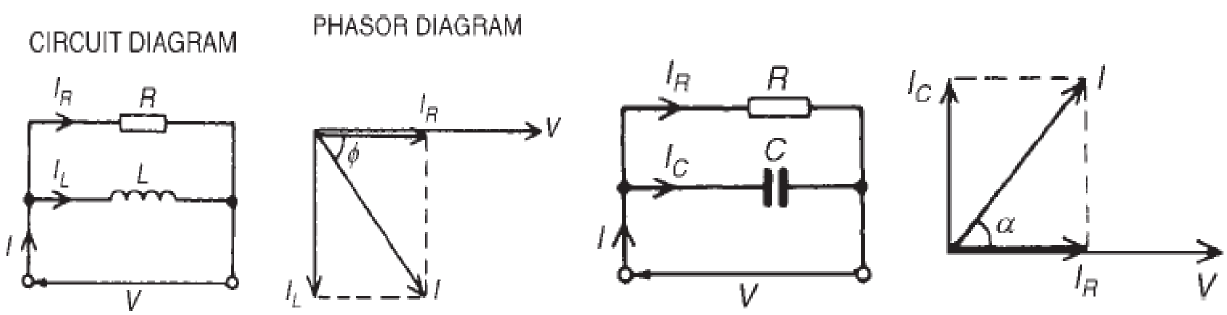
$$\text{p.f.} = \cos\phi = \frac{R}{Z}$$

- <u>Content/Topic 3: Single-phase parallel AC circuit:</u>

**Introduction**

In parallel circuits, such as those shown in Figures below, the voltage is common to each branch of the network and is thus taken as the reference phasor when drawing phasor diagrams.
For any parallel a.c. circuit:
True or active power, $P = VI\cos\phi$ watts (W)
or $P = I_R^2 R$ watts
Apparent power, $S = VI$ voltamperes (VA)
Reactive power, $Q = VI\sin\phi$ reactive voltamperes (var)

$$\text{Power factor} = \frac{\text{true power}}{\text{apparent power}} = \frac{P}{S} = \cos\phi$$



***R–L* parallel a.c. circuit**

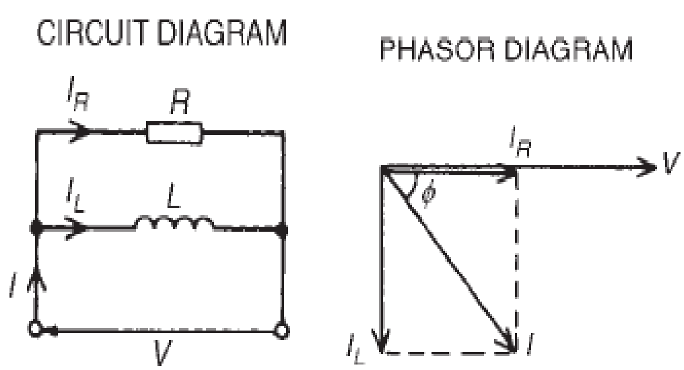
In the two-branch parallel circuit containing resistance R and inductance L shown below, the current flowing in the resistance, $I_R$, is in-phase with the supply voltage V and the current flowing in the inductance, $I_L$, lags the supply voltage by 90°. The supply current I is the phasor sum of $I_R$ and $I_L$ and thus the current I lags the applied voltage V by an angle lying between 0° and 90° (depending on the values of $I_R$ and $I_L$), shown as angle ϕ in the phasor diagram.
From the phasor diagram:

$$I = \sqrt{(I_R^2 + I_L^2)},\ \text{(by Pythagoras' theorem)}$$

$$\text{where } I_R = \frac{V}{R} \text{ and } I_L = \frac{V}{X_L}$$

$$\tan\phi = \frac{I_L}{I_R},\ \sin\phi = \frac{I_L}{I} \text{ and } \cos\phi = \frac{I_R}{I} \text{ (by trigonometric ratios)}$$

$$\text{Circuit impedance, } Z = \frac{V}{I}$$




***R–C* parallel a.c. circuit**

In the two branch parallel circuit containing resistance R and capacitance C shown in Figures below, $I_R$ is in-phase with the supply voltage V and the current flowing in the capacitor, $I_C$, leads V by 90°. The supply current I is the phasor sum of $I_R$ and $I_C$ and thus the current I leads the applied voltage V by an angle lying between 0° and 90° (depending on the values of I= and I=), shown as angle α in the phasor diagram.

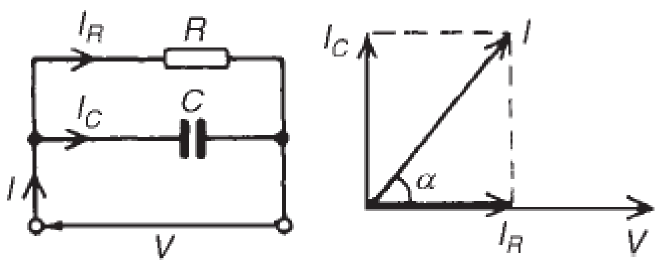
From the phasor diagram:

$$I = \sqrt{(I_R^2 + I_C^2)}, \text{ (by Pythagoras' theorem)}$$

where $I_R = \dfrac{V}{R}$ and $I_C = \dfrac{V}{X_C}$

$\tan\alpha = \dfrac{I_C}{I_R}$, $\sin\alpha = \dfrac{I_C}{I}$ and $\cos\alpha = \dfrac{I_R}{I}$ (by trigonometric ratios)

Circuit impedance $Z = \dfrac{V}{I}$




***L–C*** **parallel a.c. circuit**

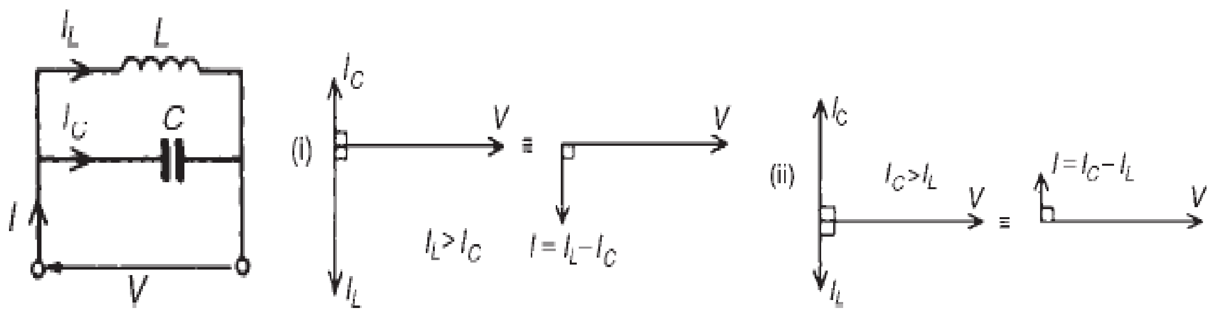
In the two branch parallel circuit containing inductance L and capacitance C shown in Figures below, $I_L$ lags V by 90° and $I_C$ leads V by 90°. Theoretically there are three phasor diagrams possible each depending on the relative values of $I_L$ and $I_C$:

(i) $I_L > I_C$ (giving a supply current, $I = I_L - I_C$ lagging V by 90°)
(ii) $I_C > I_L$ (giving a supply current, $I = I_C - I_L$ leading V by 90°)
(iii) $I_L = I_C$ (giving a supply current, $I = 0$).

The latter condition is not possible in practice due to circuit resistance inevitably being present.

For the $L$–$C$ parallel circuit, $I_L = \dfrac{V}{X_L}$, $I_C = \dfrac{V}{X_C}$

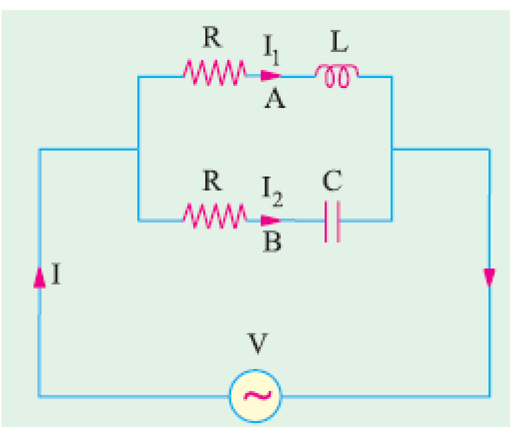
$I$ = phasor difference between $I_L$ and $I_C$, and $Z = \dfrac{V}{I}$




**Solving Parallel Circuits**

## Vector or Phasor Method

Consider the circuits shown in Fig. below. Here, two reactors *A* and *B* have been joined in parallel across an r.m.s. supply of *V* volts. The voltage across two parallel branches *A* and *B* is the same, but currents through them are different.




**For Branch *A*,** $Z_1 = \sqrt{(R_1^2 + X_L^2)}$ ; $I_1 = V/Z_1$ ; $\cos \phi_1 = R_1/Z_1$ or $\phi_1 = \cos^{-1}(R_1/Z_1)$

Current $I_1$ lags behind the applied voltage by $\phi_1$ (Fig.(a) below).

**For Branch B,** $Z_2 = \sqrt{(R_2^2 + X_c^2)}$ ; $I_2 = V/Z_2$ ; $\cos \phi_2 = R_2/Z_2$ or $\phi_2 = \cos^{-1}(R_2/Z_2)$

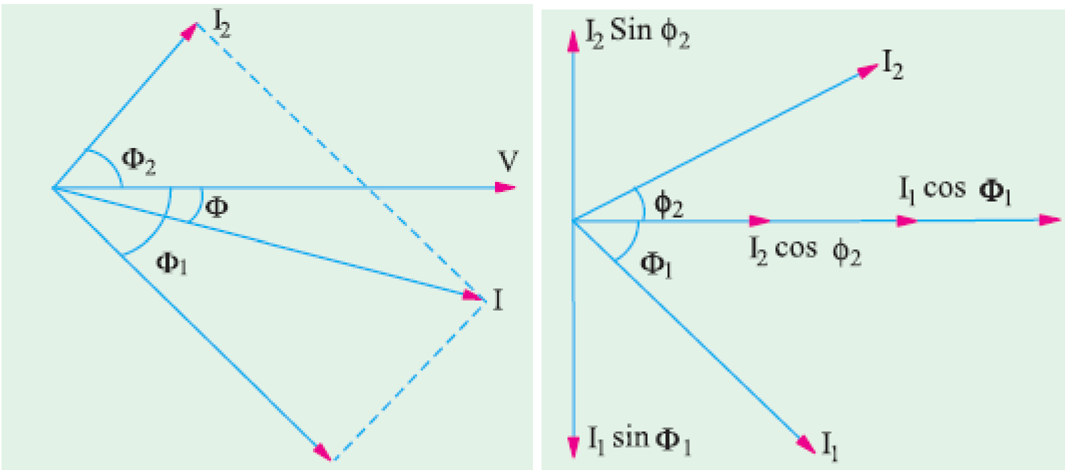
Current $I_2$ leads *V* by $\phi_2$ (Fig.

below). Resultant Current I

The resultant circuit current *I* is the vector sum of the branch currents $I_1$ and $I_2$ and can be found by (*i*) using parallelogram law of vectors, as shown in Fig.(a) below. or (*ii*) resolving $I_2$ into their *X*- and *Y*-components (or active and reactive components respectively)

and then by combining these components, as shown below. Method (*ii*) is preferable, as it is quick and convenient. With reference to Fig.(b) below. (*a*) we have
Sum of the active components of $I_1$ and $I_2$

$$= I_1 \cos \phi_1 + I_2 \cos \phi_2$$




(a) (b)

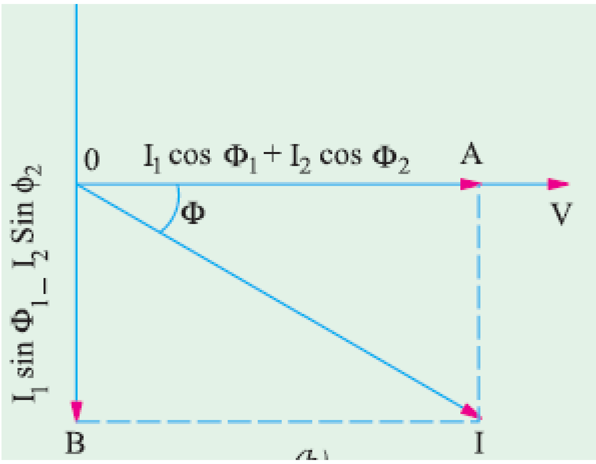
Sum of the reactive components of $I_1$ and $I_2$ = $I_2 \sin \phi_2 - I_1 \sin \phi_1$
If $I$ is the resultant current and $\phi$ its phase, then its active and reactive components must be equal to these *X*-and *Y*-components respectively [Fig. below]
$I \cos \varphi = I_1 \cos \varphi_1 + I_2 \cos \varphi_2$ *and* $I \sin \varphi = I_2 \sin \varphi_2 - I_1 \sin \varphi_1$

$$I = \sqrt{[(I_1 \cos \phi_1 + I_2 \cos \phi_2)^2 + (I_2 \sin \phi_2 - I_1 \sin \phi_1)^2}$$ And

$$tan\varphi = \frac{I_2 sin\varphi_2 - I_1 sin\varphi_1}{I_1 cos\varphi_1 + I_2 cos\varphi_2} = \frac{Y - component}{X - component}$$

If tan $\phi$ is positive, then current leads and if tan $\phi$ is negative, then current lags behind the applied voltage *V*. Power factor for the whole circuit is given by

$$cos\varphi = \frac{I_1 cos\varphi_1 + I_2 cos\varphi_2}{I} = \frac{X - comp.}{I}$$




**Admittance Method**

Admittance of a circuit is defined as *the reciprocal of its impedance.* Its symbol is *Y*.

$$Y = \frac{1}{Z} = \frac{I}{V} \text{ or } Y = \frac{\text{r.m.s. amperes}}{\text{r.m.s. volts}}$$

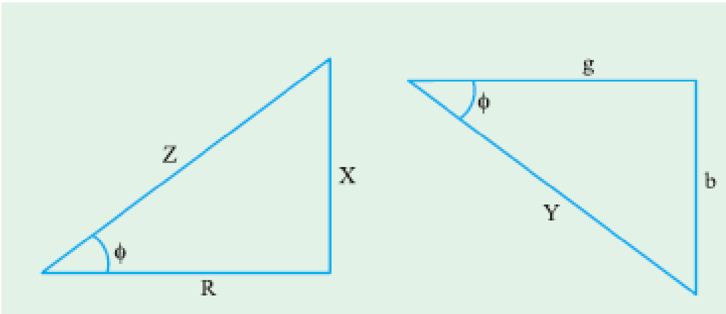
Its unit is Siemens (*S*). A circuit having an impedance of one ohm has an admittance of one Siemens. The old unit was mho (ohm spelled backwards). As the impedance *Z* of a circuit has two components *X* and *R* (Fig. (a).), similarly, admittance *Y* also has two components as shown in Fig. (b). The *X* component is known as *conductance* and *Y*-component as *susceptance*.
Obviously, conductance $g = Y \cos \varphi$
Or $g = 1/Z \times R/Z$ *(Fig.a. below)*

$$g^* = \frac{R}{Z^2} = \frac{R}{R^2 + X^2}$$

*Similarly, susceptance* $b = Y \sin\varphi = 1/Z \times X/Z = X/Z^2 = X/R^2+X^2$ *(Fig.b below).*

The admittance $Y = \sqrt{(g^2 + b^2)}$ just as $Z = \sqrt{(R^2 + X^2)}$

The unit of *g*, *b* and *Y* is Siemens. We will regard the *capacitive susceptance as positive and inductive susceptance as negative.*




(a) (b)

> Problem 6. In a series *R–L* circuit the p.d. across the resistance *R* is 12 V and the p.d. across the inductance *L* is 5 V. Find the supply voltage and the phase angle between current and voltage.

From the voltage triangle of Figure 15.6,

supply voltage $V = \sqrt{(12^2 + 5^2)}$ i.e. $\mathbf{V = 13\ V}$

(Note that in a.c. circuits, the supply voltage is **not** the arithmetic sum of the p.d's across components. It is, in fact, the **phasor sum**.)

$$\tan\phi = \frac{V_L}{V_R} = \frac{5}{12}, \text{ from which } \phi = \arctan\left(\frac{5}{12}\right) = 22.62^\circ$$

$$= \mathbf{22^\circ 37' \text{ lagging}}$$

> Problem 7. A coil has a resistance of 4 Ω and an inductance of 9.55 mH. Calculate (a) the reactance, (b) the impedance, and (c) the current taken from a 240 V, 50 Hz supply. Determine also the phase angle between the supply voltage and current.

$R = 4\ \Omega$; $L = 9.55\ \text{mH} = 9.55 \times 10^{-3}\ \text{H}$; $f = 50\ \text{Hz}$; $V = 240\ \text{V}$

(a) Inductive reactance, $X_L = 2\pi f L = 2\pi(50)(9.55 \times 10^{-3}) = \mathbf{3\ \Omega}$

(b) Impedance, $Z = \sqrt{(R^2 + X_L^2)} = \sqrt{(4^2 + 3^2)} = \mathbf{5\ \Omega}$

(c) Current, $I = \dfrac{V}{Z} = \dfrac{240}{5} = \mathbf{48\ A}$

The circuit and phasor diagrams and the voltage and impedance triangles are as shown in Figure 15.6.

$$\text{Since } \tan\phi = \frac{X_L}{R},\ \ \phi = \arctan\frac{X_L}{R} = \arctan\frac{3}{4} = 36.87^\circ$$

$$= \mathbf{36^\circ 52' \text{ lagging}}$$

> Problem 8. A coil takes a current of 2 A from a 12 V d.c. supply. When connected to a 240 V, 50 Hz supply the current is 20 A. Calculate the resistance, impedance, inductive reactance and inductance of the coil.

$$\text{Resistance } R = \frac{\text{d.c. voltage}}{\text{d.c. current}} = \frac{12}{2} = 6\ \Omega$$

$$\text{Impedance } Z = \frac{\text{a.c. voltage}}{\text{a.c. current}} = \frac{240}{20} = 12\ \Omega$$

$$\text{Since } Z = \sqrt{(R^2 + X_L^2)},\ \text{inductive reactance, } X_L = \sqrt{(Z^2 - R^2)}$$

$$= \sqrt{(12^2 - 6^2)}$$

$$= 10.39\ \Omega$$

Since $X_L = 2\pi f L$, inductance $L = \frac{X_L}{2\pi f} = \frac{10.39}{2\pi(50)} = \mathbf{33.1\ mH}$

This problem indicates a simple method for finding the inductance of a coil, i.e. firstly to measure the current when the coil is connected to a d.c. supply of known voltage, and then to repeat the process with an a.c. supply.

Problem 9. A coil of inductance 318.3 mH and negligible resistance is connected in series with a 200 Ω resistor to a 240 V, 50 Hz supply. Calculate (a) the inductive reactance of the coil, (b) the impedance of the circuit, (c) the current in the circuit, (d) the p.d. across each component, and (e) the circuit phase angle.

$L = 318.3$ mH $= 0.3183$ H; $R = 200\ \Omega$; $V = 240$ V; $f = 50$ Hz

The circuit diagram is as shown in Figure 15.6.

(a) Inductive reactance $X_L = 2\pi f L = 2\pi(50)(0.3183) = \mathbf{100\ \Omega}$

(b) Impedance $Z = \sqrt{(R^2 + X_L^2)} = \sqrt{[(200)^2 + (100)^2]} = \mathbf{223.6\ \Omega}$

(c) Current $I = \frac{V}{Z} = \frac{240}{223.6} = \mathbf{1.073\ A}$

(d) The p.d. across the coil, $V_L = IX_L = 1.073 \times 100 = \mathbf{107.3\ V}$

The p.d. across the resistor, $V_R = IR = 1.073 \times 200 = \mathbf{214.6\ V}$

[Check: $\sqrt{(V_R^2 + V_L^2)} = \sqrt{[(214.6)^2 + (107.3)^2]} = 240$ V, the supply voltage]

(e) From the impedance triangle, angle $\phi = \arctan \frac{X_L}{R} = \arctan\left(\frac{100}{200}\right)$

**Hence the phase angle $\phi = 26.57^\circ = 26^\circ 34'$ lagging**

Problem 10. A coil consists of a resistance of 100 Ω and an inductance of 200 mH. If an alternating voltage, $v$, given by $v = 200 \sin 500t$ volts is applied across the coil, calculate (a) the circuit impedance, (b) the current flowing, (c) the p.d. across the resistance, (d) the p.d. across the inductance and (e) the phase angle between voltage and current.

Since $v = 200 \sin 500t$ volts then $V_m = 200$ V and $\omega = 2\pi f$

$$= 500 \text{ rad/s}$$

Hence rms voltage $V = 0.707 \times 200 = 141.4$ V

Inductive reactance, $X_L = 2\pi f L = \omega L = 500 \times 200 \times 10^{-3} = 100\ \Omega$

(a) Impedance $Z = \sqrt{(R^2 + X_L^2)} = \sqrt{(100^2 + 100^2)} = \mathbf{141.4\ \Omega}$

(b) Current $I = \dfrac{V}{Z} = \dfrac{141.4}{141.4} = \mathbf{1\ A}$

(c) p.d. across the resistance $V_R = IR = 1 \times 100 = \mathbf{100\ V}$

p.d. across the inductance $V_L = IX_L = 1 \times 100 = \mathbf{100\ V}$

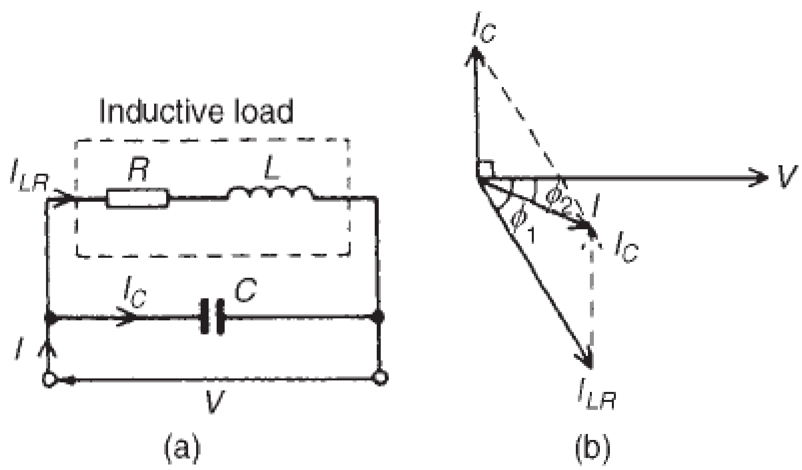
(e) Phase angle between voltage and current is given by: $\tan\phi = \left(\dfrac{X_L}{R}\right)$

from which, $\phi = \arctan(100/100)$, hence $\boldsymbol{\phi} = \mathbf{45^\circ}$ **or** $\dfrac{\boldsymbol{\pi}}{\mathbf{4}}$**rads**

- Content/Topic 4: Power factor improvement in single phase AC circuits

For a particular power supplied, a high power factor reduces the current flowing in a supply system and therefore reduces the cost of cables, switch-gear, transformers and generators. Supply authorities use tariffs which encourage electricity consumers to operate at a reasonably high power factor.

Industrial loads such as a.c. motors are essentially inductive (R–L) and may have a low power factor. One method of improving (or correcting) the power factor of an inductive load is to connect a static capacitor C in parallel with the load (see Figure (a) below). The supply current is reduced from $I_{LR}$ to $I$, the phasor sum of $I_{LR}$ and $I_C$, and the circuit power factor improves from $\cos\varphi_1$ to $\cos\varphi_2$ (see Figure (b) below).



**Example:**
A Single phase 400V, 50Hz, motor takes a supply current of 50A at a P.F (Power factor) of 0.6. The motor power factor has to be improved to 0.9 by connecting a capacitor in parallel with it. Calculate the required capacity of Capacitor in both kVAR and Farads.

**Solution**

Motor Input = P = V x I x Cosθ
= 400V x 50A x 0.6
= 12kW

Actual P.F = $\cos\theta_1$ = 0..6
Required P.F = $\cos\theta_2$ = 0.90
$\theta_1 = \cos^{-1} = (0.60) = 53°.13$; $\tan\theta_1 = \tan(53°.13) = 1.3333$
$\theta_2 = \cos^{-1} = (0.90) = 25°.84$; $\tan\theta_2 = \tan(25°.50) = 0.4843$

Required Capacitor kVAR to improve P.F from 0.60 to 0.90
Required Capacitor kVAR = P ($\tan\theta_1 - \tan\theta_2$)
= 5kW (1.3333– 0.4843)
= **10.188 kVAR**

**To find the required capacity of Capacitance in Faradsto improve P.F from 0.6 to 0.9 (Two Methods)**

**Solution #1 (Using a Simple Formula)**

We have already calculated the required Capacity of Capacitor in kVAR, so we can easily convert it into Farads by using this simple formula
Required Capacity of Capacitor in Farads/Microfarads
**C = kVAR / (2 π f $V^2$) in microfarad**

Putting the Values in the above formula
= (10.188kVAR) / (2 x π x 50 x $400^2$)
= 2.0268 x $10^{-4}$
= 202.7 x $10^{-6}$
**= 202.7µF**

**Solution # 2 (Simple Calculation Method)**

kVAR = 10.188 … (i)

We know that;
$I_C = V / X_C$

Whereas $X_C = 1 / 2 \pi F C$

$I_C = V / (1 / 2 \pi F C)$
$I_C = V 2 F C$
$= (400) x 2\pi x (50) x C$
$I_C = 125663.7 x C$

And,
$kVAR = (V x I_C) / 1000$ … [$kVAR =( V x I)/ 1000$ ]
$= 400 x 125663.7 x C$
$I_C = 50265.48 x C$ … (ii)

Equating Equation (i) & (ii), we get,

$50265.48 x C = 10.188C$
$C = 10.188 / 50265.48$
$C = 2.0268 x 10^{-4}$
$C = 202.7 x 10^{-6}$
C **= 202.7µF**

## LO 3.2 – Analyze resonance in AC circuit

Content/Topic 1: Series RLC circuit resonance

Series resonance

As stated above, for an R–L–C series circuit, when *XL = XC*, the applied voltage V and the current I are in phase. This effect is called series resonance. At resonance:

(i) $V_L = V_C$

(ii) $Z = R$ (i.e. the minimum circuit impedance possible in an $L$–$C$–$R$ circuit)

(iii) $I = \frac{V}{R}$ (i.e. the maximum current possible in an $L$–$C$–$R$ circuit)

(iv) Since $X_L = X_C$, then $2\pi f_r L = \frac{1}{2\pi f_r C}$

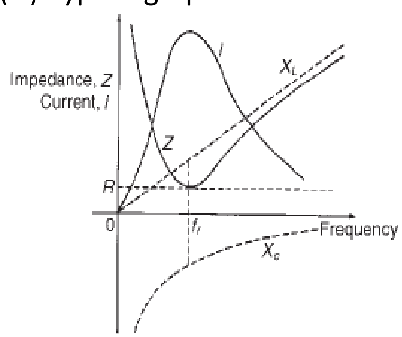
from which, $f_r^2 = \frac{1}{(2\pi)^2 LC}$

and, $$f_r = \frac{1}{2\pi\sqrt{(LC)}} \text{ Hz,}$$

where $f_r$ is the resonant frequency.

(v) The series resonant circuit is often described as an acceptor circuit since it has its minimum impedance, and thus maximum current, at the resonant frequency.

(vi) Typical graphs of current I and impedance Z against frequency are shown in Figure below.



**Properties**

- At resonance, $X_L$&$X_C$ nullify each other.
- Impedance of resonance circuit is very low.
- Current through the circuit depends upon the resistance only. So current will be maximum at this condition.

i.e, if current at resonance is $I_m$ then $I_m = V/R$

- Power factor of the circuit is unit as the circuit is purely resistive.
- As the circuit is purely resistive, both voltage and current are in phase.
- Voltage drop across inductance and capacitance is maximum.
- Frequency at which resonance occurs in a circuit is called resonance frequency.

**Applications of resonance circuit.**

- It is used in radios to tune for a particular radio station.Used in TV receiver.
- Used in oscillators as tank circuit.
- Used in microwave communication equipment.
- Used in Telex and Tele printer.
- Used in IF and RF transformers.
- Used in navy ships.

**Q-factor**

At resonance, if R is small compared with $X_L$ and $X_C$, it is possible for $V_L$ and $V_C$ to have voltages many times greater than the supply voltage.

$$\text{Voltage magnification at resonance} = \frac{\text{voltage across } L \text{ (or } C\text{)}}{\text{supply voltage } V}$$

This ratio is a measure of the quality of a circuit (as a resonator or tuning device) and is called the Q-factor.

Hence Q-factor $= \dfrac{V_L}{V} = \dfrac{IX_L}{IR} = \dfrac{X_L}{R} = \dfrac{\mathbf{2\pi f_r L}}{\mathbf{R}}$

Alternatively, Q-factor $= \dfrac{V_C}{V} = \dfrac{IX_C}{IR} = \dfrac{X_C}{R} = \dfrac{\mathbf{1}}{\mathbf{2\pi f_r CR}}$

At resonance $f_r = \dfrac{1}{2\pi\sqrt{(LC)}}$ i.e. $2\pi f_r = \dfrac{1}{\sqrt{(LC)}}$

Hence $Q$-factor $= \dfrac{2\pi f_r L}{R} = \dfrac{1}{\sqrt{(LC)}}\left(\dfrac{L}{R}\right) = \dfrac{\mathbf{1}}{\mathbf{R}}\sqrt{\left(\dfrac{\mathbf{L}}{\mathbf{C}}\right)}$

Problem 22. A coil of negligible resistance and inductance 100 mH is connected in series with a capacitance of 2 μ$F$ and a resistance of 10 Ω across a 50 V, variable frequency supply. Determine (a) the resonant frequency, (b) the current at resonance, (c) the voltages across the coil and the capacitor at resonance, and (d) the Q-factor of the circuit.

(a) Resonant frequency, $f_r = \dfrac{1}{2\pi\sqrt{(LC)}} = \dfrac{1}{2\pi\sqrt{\left[\left(\dfrac{100}{10^3}\right)\left(\dfrac{2}{10^6}\right)\right]}}$

$$= \frac{1}{2\pi\sqrt{\left(\dfrac{20}{10^8}\right)}} = \frac{1}{\left(\dfrac{2\pi\sqrt{20}}{10^4}\right)} = \frac{10^4}{2\pi\sqrt{20}}$$

$= \mathbf{355.9\ Hz}$

(b) Current at resonance $I = \frac{V}{R} = \frac{50}{10} = \mathbf{5\ A}$

(c) Voltage across coil at resonance,

$$V_L = IX_L = I(2\pi f_r L)$$
$$= (5)(2\pi \times 355.9 \times 100 \times 10^{-3})$$
$$= \mathbf{1118\ V}$$

Voltage across capacitance at resonance,

$$V_C = IX_C = \frac{I}{2\pi f_r C}$$
$$= \frac{5}{2\pi(355.9)(2 \times 10^{-6})}$$
$$= \mathbf{1118\ V}$$

(d) Q-factor (i.e. voltage magnification at resonance) $= \frac{V_L}{V}$ or $\frac{V_C}{V}$

$$= \frac{1118}{50} = \mathbf{22.36}$$

Q-factor may also have been determined by $\frac{2\pi f_r L}{R}$ or $\frac{1}{2\pi f_r CR}$

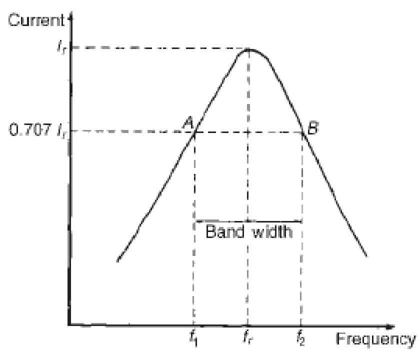
or $\frac{1}{R}\sqrt{\left(\frac{L}{C}\right)}$

- Content/Topic 2: Basic of RLC filters

**Bandwidth and selectivity**

Fig. below shows how current I varies with frequency in an R–L–C series circuit. At the resonant frequency $f_r$, current is a maximum value, shown as $I_r$. Also shown are the points A and B where the current is 0.707 of the maximum value at frequencies $f_1$ and $f_2$. The power delivered to the circuit is $I^2R$. At $I = 0.707I_r$, the power is $(0.707I_r)^2R = 0.5I_r^2R$, i.e., half the power that occurs at frequency $f_r$. The points corresponding to $f_1$ and $f_2$ are called the half-power points. The distance between these points, i.e. $(f_2 - f_1)$, is called the bandwidth. It may be shown that

$$Q = \frac{f_r}{f_2 - f_1} \quad \text{or} \quad (f_2 - f_1) = \frac{f_r}{Q}$$



Bandwidth and half-power points $f_1$ and $f_2$

Selectivity is the ability of a circuit to respond more readily to signals of a particular frequency to which it is tuned than to signals of other frequencies. The response becomes progressively weaker as the frequency departs from the resonant frequency. The higher the Q-factor, the narrower the bandwidth and the more selective is the circuit. Circuits having high Q factors (say, in the order of 100 to 300) are therefore useful in communications engineering. A high Q-factor in a series power circuit has disadvantages in that it can lead to dangerously high voltages across the insulation and may result in electrical breakdown.

- <u>Content/Topic 3: Parallel RLC circuit resonance</u>

We will consider the practical case of a coil in parallel with a capacitor, as shown below. Such a circuit is said to be in electrical resonance when the reactive (or wattless) component of line current becomes zero. The frequency at which this happens is known as *resonant* frequency. The vector diagram for this circuit is shown in Fig. (*b*) below. Net reactive or wattless component = $I_C - I_L \sin \phi_L$

As at resonance, its value is zero, hence

$I_C - I_L \sin \phi_L = 0$ or $I_L \sin \phi_L = I_C$

Now, $I_L = V/Z$; $\sin \phi_L = X_L$ and $I_C = V/X_C$

Hence, condition for resonance becomes

$$\frac{V}{Z} \times \frac{X_L}{Z} = \frac{V}{X_C} \quad \text{or} \quad X_L \times X_C = Z^2$$

$$\text{Now, } X_L = \omega L, X_C = \frac{1}{\omega C}$$

$$\therefore \frac{\omega L}{\omega C} = Z^2 \quad \text{or} \quad \frac{L}{C} = Z^2 . \qquad ..(i)$$

$$\text{or} \quad \frac{L}{C} = R^2 + X_L^2 = R^2 + (2\pi f_0 L)^2$$

$$\text{or} \qquad (2\pi f_0 L)^2 = \frac{L}{C} - R^2 \quad \text{or} \quad 2\pi f_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{or} \quad f_0 = \frac{1}{2} \sqrt{\frac{1}{LC} \quad \frac{R^2}{L^2}}$$

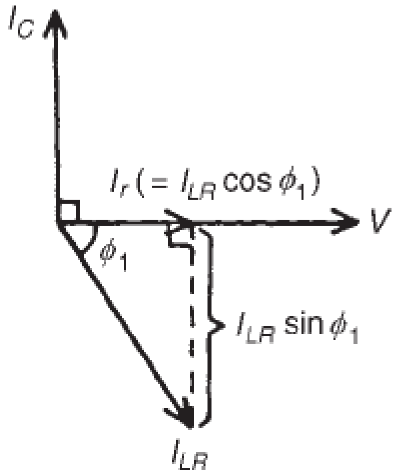
This is the resonant frequency and is given in Hz, *R* is in ohm, *L* is the henry and *C* is the farad. If *R* is the negligible, then

$$f_0 = \frac{1}{2\pi\sqrt{(LC)}}$$

same as for series resonance

**Current at Resonance**

Current at resonance, *Ir = ILRcosφ1 (Figure below)*




$$I = I_L \cos \phi_L = \frac{V}{Z} \cdot \frac{R}{Z} \text{ or } I = \frac{VR}{Z^2}.$$

Putting the value of $Z^2 = L/C$ from (*i*) above, we get $I = \frac{VR}{L/C} = \frac{V}{L/CR}$

$I_r = I$
$I_{LR} = I_L$
$\cos\varphi_1 = \cos\varphi_L$

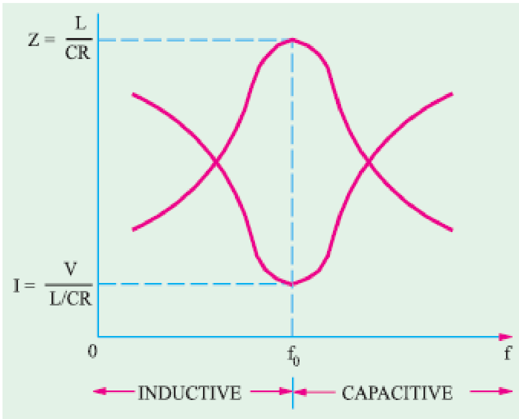
The denominator *L*/*CR* is known as the *equivalent* or *dynamic impedance* of the parallel circuit at resonance. It should be noted that impedance is 'resistive' only. Since current is minimum at resonance, *L*/*CR* must, therefore, represent the maximum impedance of the circuit. In fact, parallel resonance is a condition of maximum impedance or minimum admittance.

Current at resonance is minimum, hence such a circuit (when used in radio work) is sometimes known as *rejector* circuit because it rejects (or takes minimum current of) that frequency to which it resonates.

This resonance is often referred to as current resonance also because the current circulating *between* the two branches is many times greater than the line current taken from the supply.

The phenomenon of parallel resonance is of great practical importance because it forms the basis of tuned circuits in Electronics. The variations of impedance and current with frequency are shown in Fig. below. As seen, at resonant frequency, impedance is maximum and equals *L*/*CR*.

Consequently, current at resonance is minimum and is = *V* / (*L*/*CR*). At off-resonance frequencies, impedance decreases and, as a result, current increases as shown.




Properties

- Impedance of the circuit is maximum.
- Current at resonance is minimum.

i.e $I = \frac{V}{L/CR}$

- Current magnification takes place.
- Voltage and current are in phase.
- Power factor of the circuit is unity.

Q-factor of a Parallel Circuit

Currents higher than the supply current can circulate within the parallel branches of a parallel resonant circuit, the current leaving the capacitor and establishing the magnetic field of the inductor, this then collapsing and recharging the capacitor, and so on.

*It is defined* as *the ratio of the current circulating between its two branches to the line current drawn from the supply or simply, as the current magnification.*
circulating current between capacitor and coil branches is
$I_C$. Hence $Q$-factor = $I_C/I_r$
Now $I_C = V/X_C = V/(1/\omega C) = \omega CV$
and $I_C = V/(L/CR)$

$Q - factor\ at\ resonance = current\ magnification = circulating\ current / Supply\ current$

$$= \frac{I_C}{I_r} = \frac{I_{LR}\sin\phi_1}{I_r}$$

$$= \frac{I_{LR}\sin\phi_1}{I_{LR}\cos\phi_1} = \frac{\sin\phi_1}{\cos\phi_1}$$

$$= \tan\phi_1 = \frac{X_L}{R}$$

i.e. $$\boxed{\textbf{Q-factor at resonance} = \frac{2\pi f_r L}{R}}$$

*fo = fr*

$$f_0 = \frac{1}{2\pi\sqrt{(LC)}}$$

Putting this value above, we get, *Q*-factor is

$$\frac{1}{R}\sqrt{\frac{L}{C}}$$

Note that in a parallel circuit the Q-factor is a measure of current magnification, whereas in a series circuit it is a measure of voltage magnification.
At mains frequencies the Q-factor of a parallel circuit is usually low, typically less than 10, but in radio-frequency circuits the Q-factor can be very high.
It should be noted that in series circuits, *Q*-factor gives the voltage magnification, whereas in parallel circuits, it gives the current magnification.
Again,

$$Q = 2\pi \frac{\text{maximum stored energy}}{\text{energy dissipated/cycle}}$$

Problem 9. A coil of inductance 0.20 H and resistance 60 Ω is connected in parallel with a 20 μF capacitor across a 20 V, variable frequency supply. Calculate (a) the resonant frequency, (b) the dynamic resistance, (c) the current at resonance and (d) the circuit Q-factor at resonance.

(a) Parallel resonant frequency,

$$f_r = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$$

$$= \frac{1}{2\pi}\sqrt{\left(\frac{1}{(0.20)(20 \times 10^{-6})} - \frac{(60)^2}{(0.2)^2}\right)}$$

$$= \frac{1}{2\pi}\sqrt{(250\,000 - 90\,000)}$$

$$= \frac{1}{2\pi}\sqrt{(160\,000)} = \frac{1}{2\pi}(400)$$

$$= \mathbf{63.66\ Hz}$$

(b) Dynamic resistance, $R_D = \frac{L}{RC} = \frac{0.20}{(60)(20 \times 10^{-6})} = \mathbf{166.7\ \Omega}$

(c) Current at resonance, $I_r = \frac{V}{R_D} = \frac{20}{166.7} = \mathbf{0.12\ A}$

(d) Circuit Q-factor at resonance $= \frac{2\pi f_r L}{R} = \frac{2\pi(63.66)(0.2)}{60} = \mathbf{1.33}$

Alternatively, Q-factor at resonance = current magnification (for a parallel circuit) = $I_c/I_r$

$$I_c = \frac{V}{X_c} = \frac{V}{\left(\dfrac{1}{2\pi f_r C}\right)} = 2\pi f_r CV = 2\pi(63.66)(20 \times 10^{-6})(20)$$

$$= 0.16\ A$$

Hence Q-factor $= \dfrac{I_c}{I_r} = \dfrac{0.16}{0.12} = \mathbf{1.33}$, as obtained above

> Problem 10. A coil of inductance 100 mH and resistance 800 Ω is connected in parallel with a variable capacitor across a 12 V, 5 kHz supply. Determine for the condition when the supply current is a minimum: (a) the capacitance of the capacitor, (b) the dynamic resistance, (c) the supply current, and (d) the Q-factor.

(a) The supply current is a minimum when the parallel circuit is at resonance.

Resonant frequency, $f_r = \dfrac{1}{2\pi}\sqrt{\left(\dfrac{1}{LC} - \dfrac{R^2}{L^2}\right)}$

Transposing for C gives: $(2\pi f_r)^2 = \dfrac{1}{LC} - \dfrac{R^2}{L^2}$

$$(2\pi f_r)^2 + \frac{R^2}{L^2} = \frac{1}{LC}$$

$$C = \frac{1}{L\left\{(2\pi f_r)^2 + \dfrac{R^2}{L^2}\right\}}$$

When $L = 100$ mH, $R = 800\ \Omega$ and $f_r = 5000$ Hz,

$$C = \frac{1}{100 \times 10^{-3}\left\{2\pi(5000)^2 + \dfrac{800^2}{(100 \times 10^{-3})^2}\right\}}$$

$$= \frac{1}{0.1[\pi^2 10^8 + (0.64)10^8]} F$$

$$= \frac{10^6}{0.1(10.51 \times 10^8)}\ \mu\text{F} = \mathbf{0.009515\ \mu F\ or\ 9.515\ nF}$$

(b) Dynamic resistance, $R_D = \dfrac{L}{CR} = \dfrac{100 \times 10^{-3}}{(9.515 \times 10^{-9})(800)}$

$$= \mathbf{13.14\ k\Omega}$$

(c) Supply current at resonance, $I_r = \dfrac{V}{R_D} = \dfrac{12}{13.14 \times 10^3} = \mathbf{0.913\ mA}$

(d) Q-factor at resonance $= \dfrac{2\pi f_r L}{R} = \dfrac{2\pi(5000)(100 \times 10^{-3})}{800} = \mathbf{3.93}$

Alternatively, Q-factor at resonance $= \dfrac{I_c}{I_r} = \dfrac{V/X_c}{I_r} = \dfrac{2\pi f_r CV}{I_r}$

$$= \frac{2\pi(5000)(9.515 \times 10^{-9})(12)}{0.913 \times 10^{-3}}$$

$$= \mathbf{3.93}$$

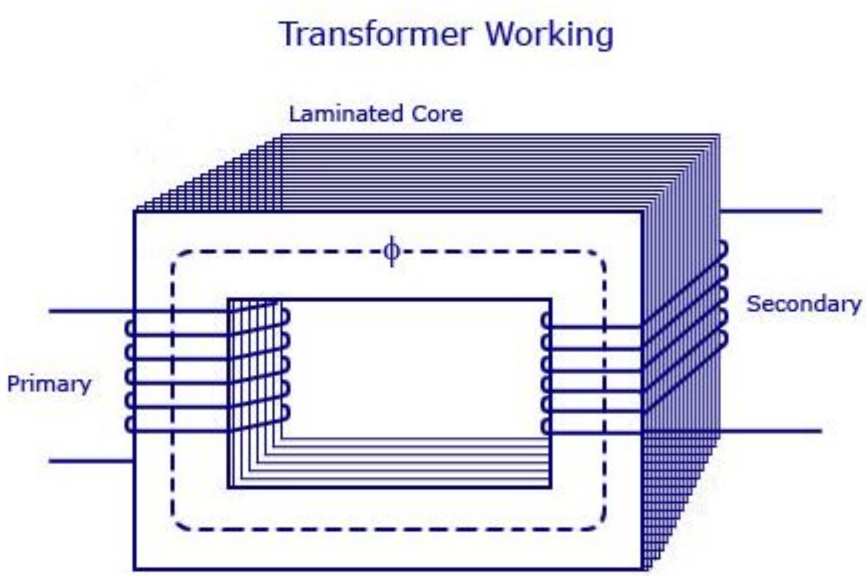
## LO 3.3 – Apply basic concepts of transformer

- <u>Content/Topic 1: Working principle of a transformer</u>

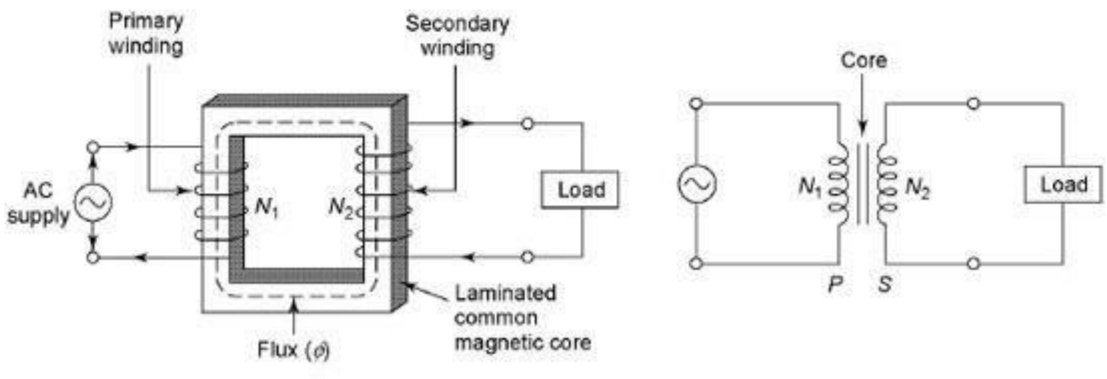
**What is a transformer?**
A transformer is a static or stationary electromagnetic device, consisting of two coils, by means of which electrical power in one circuit is transformed into electrical power of the same frequency in another circuit.

**Transformer – Working Principle**

When alternating **current** flows through the *primary* winding (or coil) of a transformer, a varying **magnetic flux** is induced in the *secondary* winding. This varying magnetic flux **induces** a varying **electromotive force (emf)** or **voltage** in the secondary winding, which in turn produces a current through the secondary winding if a load impedance is connected across it. Thus, a transformer cannot operate with direct current.




As shown above the electrical transformer has primary and secondary windings. The core laminations are joined in the form of strips in between the strips you can see that there are some narrow gaps right through the cross-section of the core. These staggered joints are said to be 'imbricated'. Both the coils have high mutual inductance. A mutual electro-motive force is induced in the transformer from the alternating flux that is set up in the laminated core, due to the coil that is connected to a source of alternating voltage. Most of the alternating flux developed by this coil is linked with the other coil and thus produces the mutual induced electro-motive force. The so produced electro-motive force can be explained with the help of Faraday's laws of Electromagnetic Induction as




(a) Basic transformer (b) Symbolic representation

In brief, we can say the following:

The transformer is a static device.

It transfers electrical power from one circuit to another. During transfer of

power, there is no change of frequency.

It uses electromagnetic induction to transfer electrical power.

The two electrical circuits are in mutual inductive influence of each other.

**CONSTRUCTION OF SINGLE-PHASE TRANSFORMER**

Magnetic core and windings (or coils) are the two basic parts of any transformer. The core is made of silicon or sheet steel with 4 per cent silicon and laminated to reduce eddy current loss. It may be in either square or rectangular shape. It has two parts. The vertical portion on which the coil is wound is called the *limb of the core*, whereas the top and bottom horizontal portions are called the *yoke*.The permeability of the material used for core must have high value ($\mu r$>1,000) to reduce reluctance of the magnetic path.

**WINDINGS (OR COILS)**

The two windings are wound on two limbs, that is, one on the primary and the other on the secondary, as shown below .

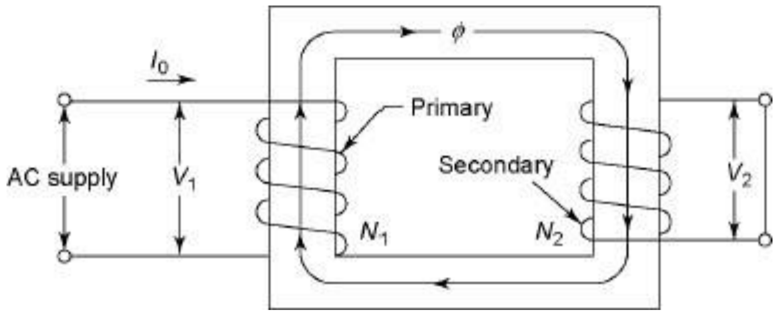



**Figure of** Two-winding Transformer

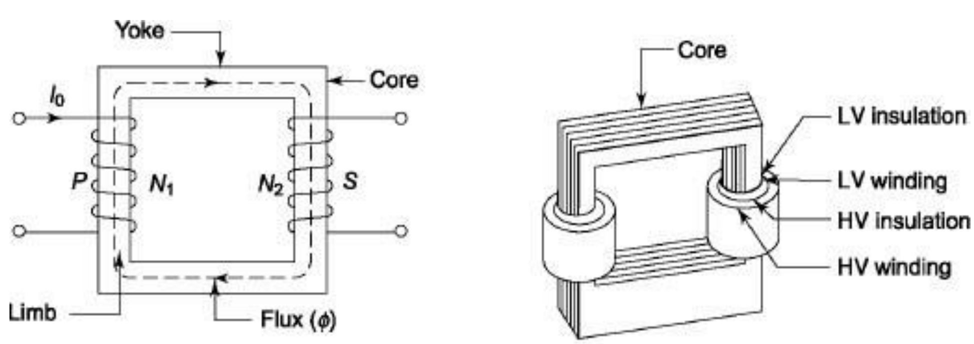
**MAGNETIC CORE**

**Core Type**

Figures below shows the complete magnetic circuit of a core-type transformer in the shape of a hollow rectangle having two limbs. It has a single magnetic circuit. $I_0$ is the no-load current and $\phi$ is the flux produced by it. Number of turns of the primary and secondary are $N_1$ and $N_2$, respectively.

The windings surround the core. The coils used are wound and are of cylindrical type having the general form circular, oval or rectangular.

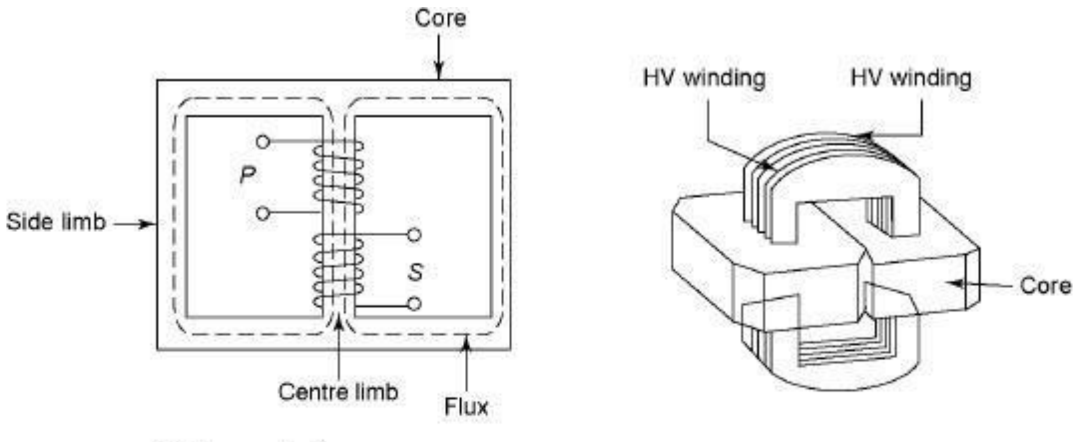
Core-type transformer has a longer mean length of core and a shorter mean length of coil turn. Core has a small cross section of iron; more number of turns is required because the high flux may not reach the core. Core type is used for high-voltage service, since it has sufficient room for insulation.




(a) Representation (b) Construction

**Shell Type**

Shell-type transformer has double magnetic circuit and three limbs as shown in figures below. Both windings are placed on the central limb.




(a) Representation

(b) Construction

**THEORY OF TRANSFORMER**

Figure below shows the elementary diagram of an ideal transformer with secondary side open circuited. It has no ohmic resistance and leakage reactance. There is no loss in an ideal transformer.

Alternating voltage ($V_1$) is applied at the primary and hence alternating current flows in the primary. The primary draws the magnetizing current $I_\mu$ only because it is purely inductive in nature.

$I_\mu$ is small in magnitude and lags behind $V_1$ by an angle 90°. The function of $I_\mu$ is to magnetize the core, and it produces an alternating flux ( $\Phi$ ), which is proportional to $I_\mu$. The alternating flux ( $\Phi$ ) is linked with both primary and secondary windings and causes self-induced emf ($E_1$) in the primary. This self-induced emf ($E_1$) is equal and opposite to $V_1$ at every instant. This induced emf is known as *back emf or counter emf*. Due to mutual induction, an emf $E_2$ is produced in the secondary. This emf is known as ***mutually induced emf***. It is anti-phase with $V_1$ and its magnitude is proportional to the rate of flux as well as the number of turns of the secondary windings.

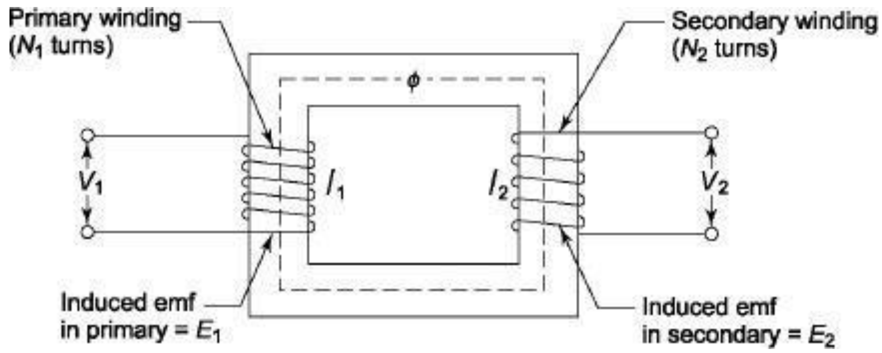
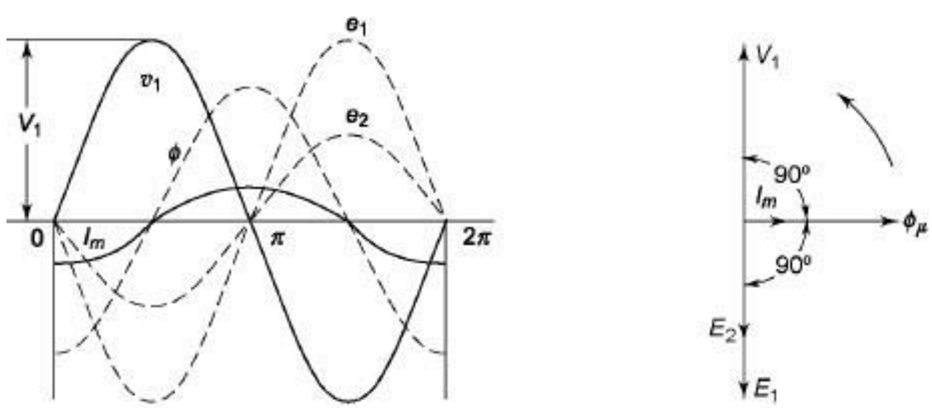



Figure (a) below shows the instantaneous values of applied voltage, induced emfs, flux and magnetizing current by sinusoidal waves, while

Figure (b) below shows the vectorial representation of the effective values of the above quantities.




(a) Instantaneous values of applied voltage, induced emfs, flux and magnetizing current

(b) Vectorial representation of the effective values

- <u>Content/Topic 2: Classification of transformer</u>

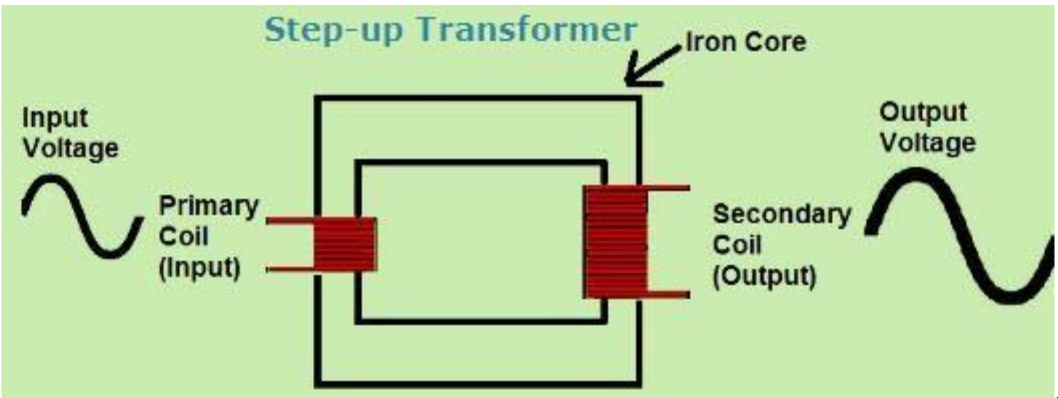
The transformers are classified based on voltage levels, Core medium used, winding arrangements, use and installation place, etc. Here we discuss different types of transformers are the step up and step down Transformer, Distribution Transformer, Potential Transformer, Power Transformer, 1-φ and 3-φ transformer, Auto transformer, etc.

**Transformers Based on Voltage Levels**

These are the most commonly used transformer types for all the applications. Depends upon the voltage ratios from primary to secondary windings, the transformers are classified as step-up and step-down transformers.

**Step-Up Transformer**

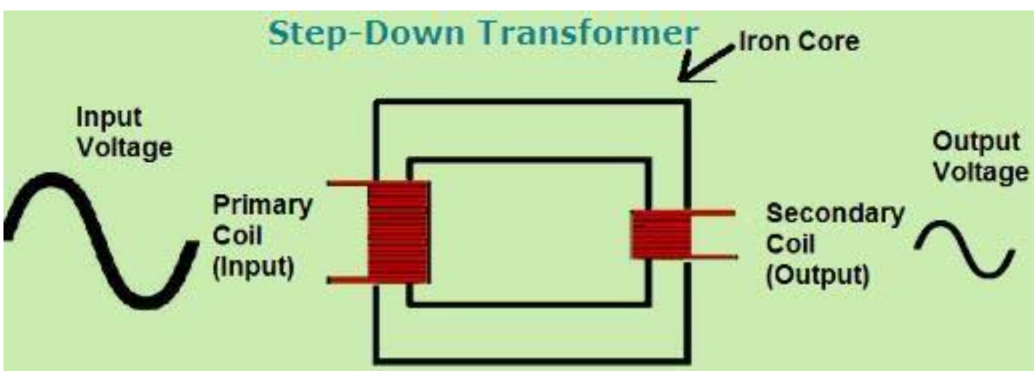
As the name states that, the secondary voltage is stepped up with a ratio compared to primary voltage. This can be achieved by increasing the number of windings in the secondary than the primary windings as shown in the figure. In power plant, this transformer is used as connecting transformer of the generator to the grid.



Step-up Transformer

**Step-Down Transformer**

It used to step down the voltage level from lower to higher level at secondary side as shown below so that it is called as a step-down transformer. The winding turns more on the primary side than the secondary side.



Step-Down Transformer

In distribution networks, the step-down transformer is commonly used to convert the high grid voltage to low voltage that can be used for home appliances.

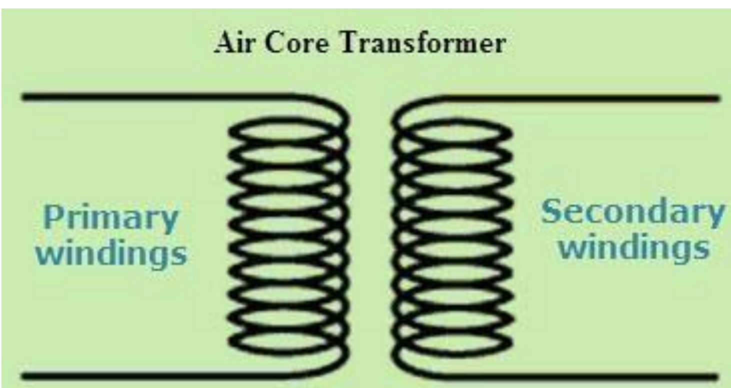
**Transformer Based on the Core Medium Used**

Based on the medium placed between the primary and secondary winding the transformers are classified as Air core and Iron core

**Air Core Transformer**

Both the primary and secondary windings are wound on a non-magnetic strip where the flux linkage between primary and secondary windings is through the air.

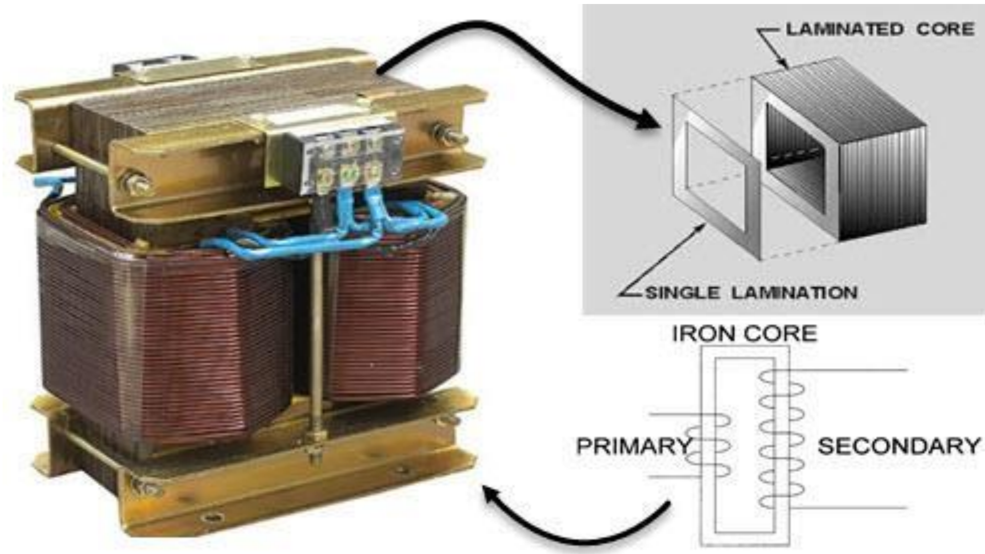
Compared to iron core the mutual inductance is less in air core, i.e. the reluctance offered to the generated flux is high in the air medium. But the hysteresis and eddy current losses are completely eliminated in air-core type transformer.



Air Core Transformer

**Iron Core Transformer**

Both the primary and secondary windings are wound on multiple iron plate bunch which provide a perfect linkage path to the generated flux. It offers less reluctance to the linkage flux due to the conductive and magnetic property of the iron. These are widely used transformers in which the efficiency is high compared to the air core type transformer.

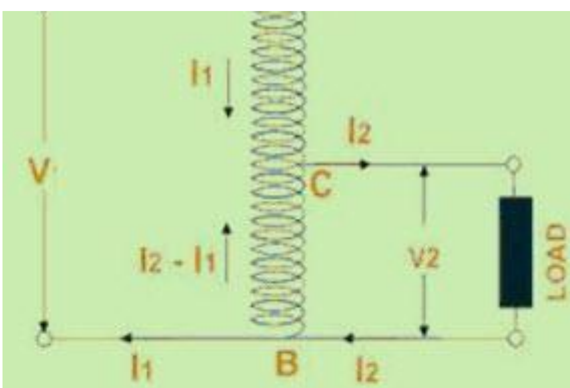



Iron Core Transformer

**Transformers Based on Winding Arrangement**

**AutoTransformer**

Standard transformers have primary and secondary windings placed in two different directions, but in autotransformer windings, the primary and the secondary windings are connected to each other in series both physically and magnetically as shown in the figure below.



Auto Transformer

On a single common coil which forms both primary and secondary winding in which voltage is varied according to the position of secondary tapping on the body of the coil windings.

**Transformers Based on Usage**

According to the necessity, these are classified as the power transformer, distribution transformer measuring transformer, and protection transformer.

**Power Transformer**

The power transformers are big in size. They are suitable for high voltage (greater than 33KV) power transfer applications. It used in power generation stations and Transmission substation. It has high insulation level.

Power Transformer

**Distribution Transformer**

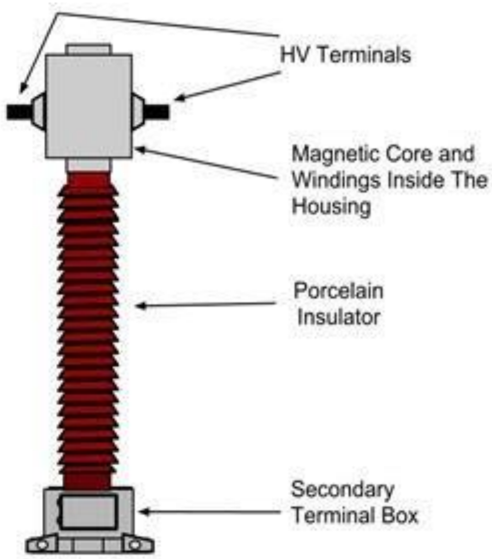
In order to distribute the power generated from the power generation plant to remote locations, these transformers are used. Basically, it is used for the distribution of electrical energy at low voltage is less than 33KV in industrial purpose and 440v-220v in domestic purpose.

- It works at low efficiency at 50-70%
- Small size
- Easy installation
- Low magnetic losses
- It is not always fully loaded

Distribution Transformer

**Measurement Transformer**

Used to measure the electrical quantity like voltage, current, power, etc. These are classified as potential transformers, current transformers etc.




Current Transformer

**Protection Transformers**

This type of transformers is used in component protection purpose. The major difference between measuring transformers and protection transformers is the accuracy that means that the protection transformers should be accurate as compared to measuring transformers.

**Transformers Based on the Place of Use**

These are classified as indoor and outdoor transformers. Indoor transformers are covered with a proper roof like as in the process industry. The outdoor transformers are nothing but distribution type transformers.


Indoor and Outdoor Transformers

**Transformers Based on Method of cooling we have:**

**1. Oil Filled Self-Cooled Type**

Oil filled self-cooled type uses small and medium-sized distribution transformers. The assembled windings and core of such transformers are mounted in a welded, oil-tight steel tanks provided with a steel cover. The tank is filled with purified, high quality insulating oil as soon as the core is put back at its proper place. The oil helps in transferring the heat from the core and the windings to the case from where it is radiated out to the surroundings.

For smaller sized transformers the tanks are usually smooth surfaced, but for large size transformers a greater heat radiation area is needed, and that too without disturbing the cubical capacity of the tank. This is achieved by frequently corrugating the cases. Still larger sizes are provided with radiation or pipes.

**2. Oil Filled Water Cooled Type**

This type is used for much more economic construction of large transformers, as the above-told self-cooled method is very expensive. The same method is used here as well- the windings and the core are immersed in the oil. The only difference is that a cooling coil is mounted near the surface of the oil, through which cold water keeps circulating. This water carries the heat from the device. This design is usually implemented on transformers that are used in high voltage transmission lines. The biggest advantage of such a design is that such transformers do not require housing other than their own. This reduces the costs by a huge amount. Another advantage is that the maintenance and inspection of this type is only needed once or twice in a year.

**3. Air Blast Type**

This type is used for transformers that use voltages below 25,000 volts. The transformer is housed in a thin sheet metal box open at both ends through which air is blown from the bottom to the top.

- Content/Topic 3: Emf equation of a transformer

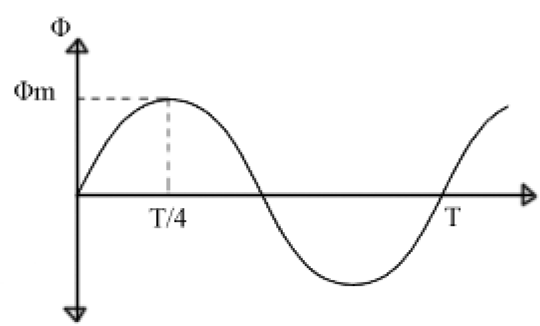
Let,

$N_1$ = Number of turns in primary winding

$N_2$ = Number of turns in secondary winding

$\Phi_m$ = Maximum flux in the core (in Wb) = ($B_m$ x A)

f = frequency of the AC supply (in Hz)




As, shown in the fig., the flux rises sinusoidally to its maximum value $\Phi_m$ from 0. It reaches to the maximum value in one quarter of the cycle i.e in T/4 sec (where, T is time period of the sin wave of the supply = 1/f).

Therefore,

average rate of change of flux = $\Phi_m / (T/4)$ = $\Phi_m / (1/4f)$

Therefore,

average rate of change of flux = 4f $\Phi_m$ ....... (Wb/s).

Now,

Induced emf per turn = rate of change of flux per turn

Therefore, average emf per turn = 4f $\Phi_m$ ..........(Volts).

Now, we know, Form factor = RMS value / average value
Therefore, RMS value of emf per turn = Form factor X average emf per turn.

As, the flux Φ varies sinusoidally, form factor of a sine wave is 1.11

Therefore, RMS value of emf per turn = 1.11 x 4f $\Phi_m$ = 4.44f $\Phi_m$.

RMS value of induced emf in whole primary winding ($E_1$) = RMS value of emf per turn X Number of turns in primary winding

$E_1 = 4.44f N_1 \Phi_m$ ............................ eq 1

Similarly, RMS induced emf in secondary winding ($E_2$) can be given as

$E_2 = 4.44f N_2 \Phi_m$. ........................... eq 2

from the above equations 1 and 2,

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44f\,\Phi m$$

This is called the **emf equation of transformer**, which shows, emf / number of turns is same for both primary and secondary winding.

For an **ideal transformer** on no load, $E_1 = V_1$ and $E_2 = V_2$ .
where, $V_1$ = supply voltage of primary winding
$V_2$ = terminal voltage of secondary winding

**Step-Up And Step-Down Transformer**

If $N1 > N2$, $V1 > V2$, that is the output voltage is less than the primary voltage. The transformer is said to be a ***step-down transformer*.**

If $N1 < N2$, $V1 < V2$, that is the output voltage is greater than the primary voltage. The transformer is said to be a ***step-up transformer*.**

For an ideal transformer, input VA = output VA

$V1I1=V2I2$

- Content/Topic 4: Transformer losses

Two types of losses occur in a transformer:

- Core loss or iron loss occurs in a transformer because it is subjected to an alternating flux.
- The windings carry current due to loading and hence copper losses occur.

**Core or Iron Loss**

The separation of core losses has already been introduced. The alternating flux gets set up in the core and it undergoes a cycle of magnetization and demagnetization. Therefore, loss of energy occurs in this process due to hysteresis. This loss is called *hysteresis loss* (**$P_h$**), which is expressed by

$$P_h = K_h B^{1.6} f V \text{ W}$$

where **$K_h$** is the hysteresis constant depending on the material, **$B_m$** is the maximum flux density, **$f$** is the frequency and **$V$** is the volume of the core.

The induced emf in the core sets up eddy current in the core, and hence eddy current loss (**$P_e$**) occurs, which is given by

$$P_e = K_e B_m^2 f^2 t^2 \text{ W}$$ per model W

where **$K_e$** is the eddy current constant and **$t$** is the thickness of the core.

Since the supply voltage **$V_1$** at rated frequency **$f$** is always constant, the flux in the core is almost constant. Therefore, flux density in the core remains constant. Hence, hysteresis and eddy current losses are constant at all loads. Thus, the core loss or iron loss is also known as constant loss. The iron loss is denoted by **$P_i$**.

Iron loss is reduced using high-grade core material such as silicon steel having very low hysteresis loop for reducing hysteresis loss and laminated core for reducing the eddy current loss.

**Copper Loss**

The loss of power in the form **$I^2R$** due to the resistances of the primary and secondary windings is known as ***copper losses***. The copper loss also depends on the magnitude of currents flowing through the windings. The total **Cu** loss is given by

$$P_{cu} = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} = I_2^2 R_{02}$$

Copper losses are determined on the basis of ***R*01** or ***R*02** which is determined from short circuit test. Since the standard operating temperature of electrical machine is taken as **75 °C**, it is then corrected to **75 °C**.

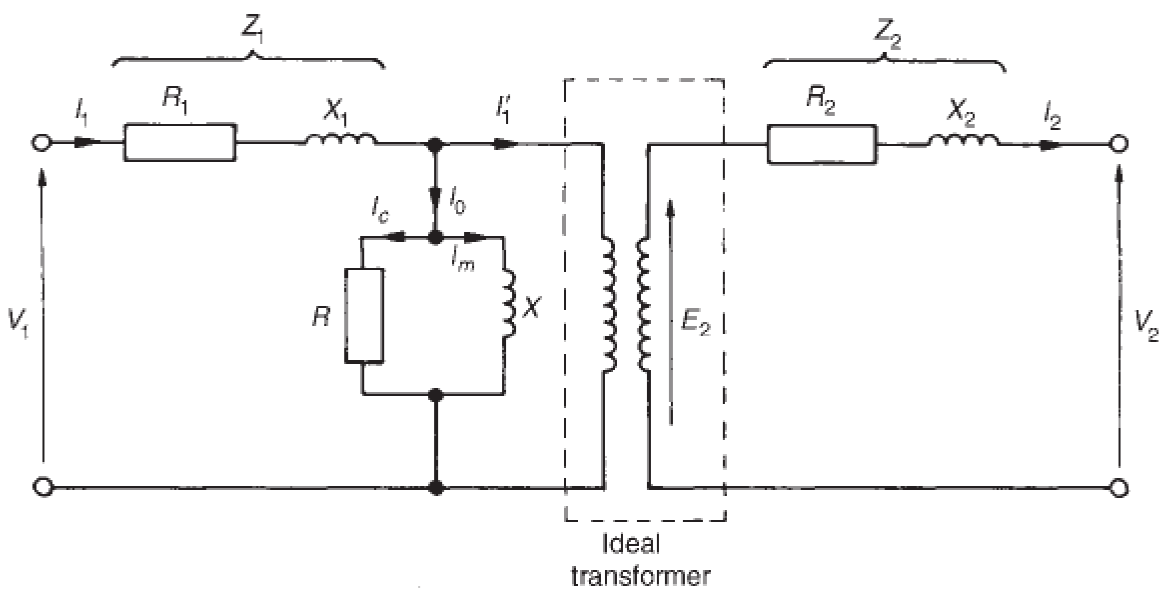
The copper loss due to full-load current is known as full-load Cu loss. If the load on the transformer is half, the Cu loss is known as
half-load Cu loss, which is less than the full-load Cu loss. The Cu loss is also known as ***variable loss***.

There are two other losses known as **stray loss** and **dielectric loss**. Since leakage field is present in a transformer, eddy currents are induced in the conductors, tanks walls and bolts etc. Stray losses occur due to this eddy currents. Dielectric loss occurs in insulating materials coil and solid insulation. These two losses are small and hence neglected.

Therefore, the total loss of the transformer = Iron loss + Cu loss = ***Pi* + *PCu***

***Equivalent circuit of a transformer***

Figure below shows an equivalent circuit of a transformer. R1 and R2 represent the resistances of the primary and secondary windings and X1 and X2 represent the reactances of the primary and secondary windings, due to leakage flux.



**Efficiency Of A Transformer**

Due to the losses in a transformer, its output power is less than the input power.

∴ Power output = Power input – Totallosses

∴ Power input = Power output + Total losses = Power output + $P_i$ + $P_{Cu}$

The ratio of power output to power input of any device is called its ***efficiency*** **($\eta$).**

$$\eta = \frac{Power\ output}{Power\ input} = \frac{Power\ output}{Power\ output + P_i + P_{cu}}$$

Output power of a transformer at full-load = $V_2 I_{2fl} \cos\vartheta$, where $\cos\vartheta$ is the power factor of the load, $I_{2fl}$ is the secondary current at full load and $V_2$ is the rated secondary voltage of the transformer.

Full-load copper loss of the transformer = $I^2_{2fl} R_{02}$

∴ Efficiency of the transformer at full load is given by

$$\eta_{fl} = \frac{V_2 I_{2fl} cos\theta}{V_2 I_{2fl} COS\theta + P_i + I^2_{2fl} R_{02}}$$

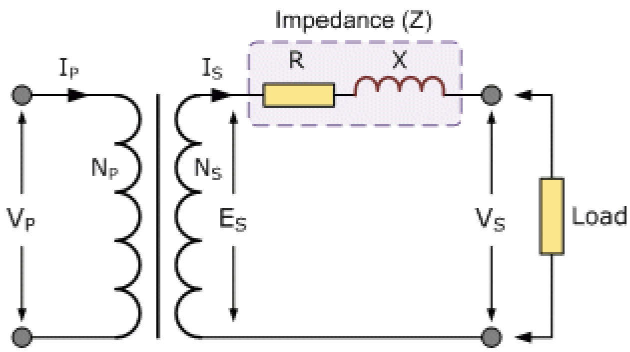
Now $V_2 I_{2fl}$ = $VA$ rating of the transformer.

$$\eta = \frac{(VA\ rating\ of\ the\ transformer) x cos\theta}{(VA\ rating\ of\ the\ transformer) x cos\theta + P_i + I^2_{2fl} R_{02}}\ p.u$$

$$\eta = \frac{(VA\ rating\ of\ the\ transformer) x cos\theta}{(VA\ rating\ of\ the\ transformer) x cos\theta + P_i + I^2_{2fl} R_{02}} x100$$

- <u>Content/Topic 5: Voltage regulation on a transformer</u>

Transformer voltage regulation is the ratio or percentage value by which a transformers output terminal voltage varies either up or down from its no-load value as a result of variations in the connected load current




Voltage Regulation of single-phase transformers is the percentage (or per unit value) change in its secondary terminal voltage compared to its original no-load voltage under varying secondary load conditions. In other words, regulation determines the variation in secondary terminal voltage which occurs inside the transformer as a result of variations in the transformers connected load thereby affecting its performance and efficiency if these losses are high and the secondary voltage becomes too low.

When there is no-load connected to the transformers secondary winding, that is its output terminals are open-circuited, there is no closed-loop condition, so there is no output load current (IL = 0) and the transformer acts as one single winding of high self-inductance. Note that the no-load secondary voltage is a result of the fixed primary voltage and the turns ratio of the transformer.

Loading the secondary winding with a simple load impedance causes a secondary current to flow, at any power factor, through the internal winding of the transformer. Thus voltage drops due to the windings internal resistance and its leakage reactance causes the output terminal voltage to change.

A transformers voltage regulation change between its secondary terminal voltage from a no-load condition when IL = 0, (open circuit) to a fully-loaded condition when IL = IMAX (maximum current) for a constant primary voltage is given as:

Transformer Voltage Regulation as a Fractional Change

$$\text{Regulation} = \frac{\text{Change in Output Voltage}}{\text{No-load Output Voltage}}$$

$$\therefore \text{Regulation} = \frac{V_{(\text{no-load})} - V_{(\text{full-load})}}{V_{(\text{no-load})}}$$

Note that this voltage regulation when expressed as a fraction or unit-change of the no-load terminal voltage can be defined in one of two ways, *voltage regulation-down*, (Regdown) and *voltage regulation-up*, (Regup). That is when the load is connected to the secondary output terminal, the terminal voltage goes down, or when the load is removed, the secondary terminal voltage goes up. Thus the regulation of the transformer will depend on which voltage value is used as the reference voltage, load or non-load value.

We can also express transformer voltage regulation as a percentage change between the no-load condition and the full-load conditions as follows:

Transformer Voltage Regulation as a Percentage Change

$$\%\text{Reg}_{(\text{down})} = \frac{V_{(\text{no-load})} - V_{(\text{full-load})}}{V_{(\text{no-load})}} \times 100\%$$

$$\%\text{Reg}_{(\text{up})} = \frac{V_{(\text{no-load})} - V_{(\text{full-load})}}{V_{(\text{full-load})}} \times 100\%$$

**Typical values of voltage regulation are about 3% in small transformers and about 1% in large transformers.**

So for example, if a single-phase transformer has an open-circuit no-load terminal voltage of 100 volts and the same terminal voltage drops to 95 volts on the application of a connected load, the transformers voltage regulation would therefore be 0.05 or 5%, ((100 – 95)/100)*100%). Therfore a transformers voltage regulation can be expressed as either a unit change value or as a percentage change value of the no-load voltage.

**Transformer Voltage Regulation Example**

The primary winding of a 500VA, 10:1 single-phase step-down transformer is fed from a constant 240Vrms supply. Calculate the percentage regulation of the transformer when connected to an impedance of 1.1Ω

Data given: VA = 500, TR = 10:1, VP = 240V, ZS = 1.1Ω, find %Reg.

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} \quad \therefore V_S = \frac{V_P\, N_S}{N_P} = \frac{240 \times 1}{10} = 24\,V$$

Therefore, VS(no-load) = 24 Volts

$$P = \frac{V_S^2}{Z} \quad \therefore V_S = \sqrt{P \times Z} = \sqrt{500 \times 1.1} = 23.45V$$

Therefore, VS(full-load) = 23.45 Volts

$$\%Reg. = \frac{V_{S(no\text{-}load)} - V_{S(full\text{-}load)}}{V_{S(no\text{-}load)}} \times 100\%$$

$$\%Reg. = \frac{24 - 23.45}{24} = 0.229 \times 100\% = 2.29\%$$

Then the percentage down regulation calculated for the transformer is given as: 2.29%, or 2.3% rounded-off

- <u>Content/Topic 5: Autotransformer</u>

Two-winding transformers have already been discussed. In such transformers, the two windings are electrically isolated and emf is induced in the secondary winding due to mutual induction. There also exists other types of transformers in which a part of the winding is common to both the primary and secondary circuits.
These transformers are termed as *autotransformers*. In an autotransformer, the two windings are electrically connected and it works on the principle of induction and conduction.

**Construction**

Figure below depicts an autotransformer in which only one winding is wound on a laminated magnetic core. It also shows that a single winding is used as primary and secondary and a part of the winding is common to both the primary and secondary. The autotransformers are also classified as step-up and step-down transformers because voltage can be stepped up and stepped down using these transformers.

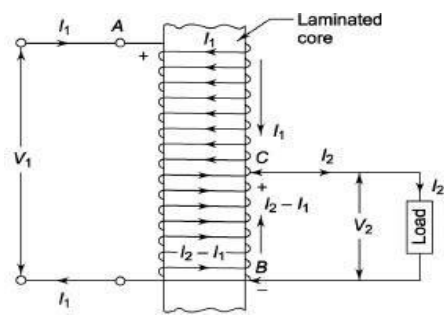



Figure (a) below shows a two-winding trans-former. Figure (b) shows an autotransformer, which steps down the voltage. In a step- up auto-transformer, the entire winding is used as a primary winding and the part of the winding is used as a secondary winding. Figure (c) shows an auto-transformer, which steps up the voltage.
The entire winding is used as a secondary winding.

From Figures (b) and (c) below, it seems that an autotransformer is similar to a resistance potential divider. An autotransformer can step up and step down the voltage. An autotransformer has less loss whereas more loss occurs in a potential divider. Therefore, the efficiency of an autotransformer is higher than that of a potential divider. In a potential divider, input current is more than the output current. In an autotransformer, if output voltage is less than the input voltage, the output current will be higher than the input current.

Let $I_1$ be the input current, $I_2$ be the output current, $V_1$ be the input voltage, $V_2$ be the output voltage, $\cos\theta_1$ be the input power factor and $\cos\theta_2$ be the output power factor.

If we neglect the losses

$$V_1 I_1 cos\theta_1 = V_2 I_2 cos\theta_2$$

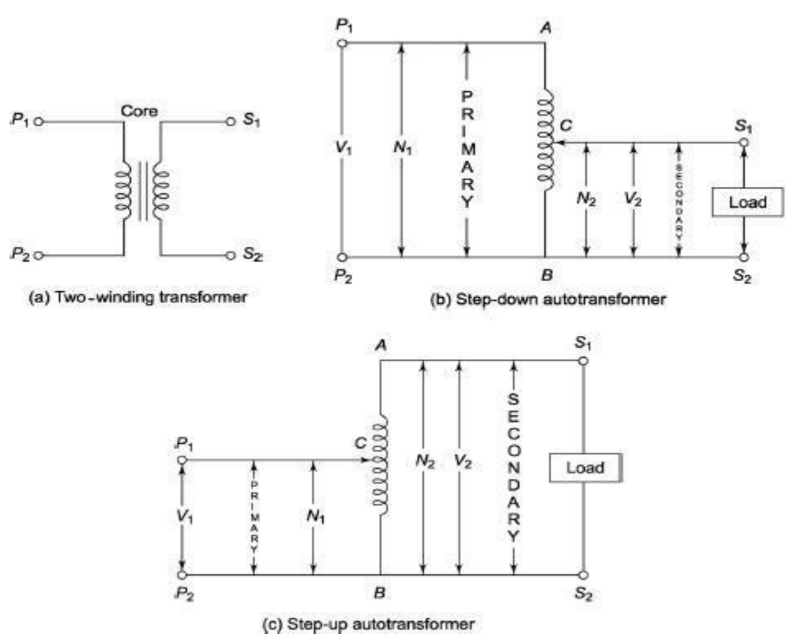
If we neglect the internal impedance drops and losses, the equation can be written as follows

$V_1 I_1 = V_2 I_2$

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{K}$$

where $N_1$ is the total number of turns in the primary and $N_2$ the total number of turns in the secondary.

Figure (d) shows a step-down autotransformer. Let the point $X$ be positive with respect to $Z$. At no load the exciting current flows from $X$ to $Z$ and it produces a working mmf vectorially downwards, i.e., from $X$ to $Z$. During the presence of load at the secondary, the current flows from $Z$ to $Y$ and it weakens the produced working magnetomotive force (mmf), and the transformer draws extra current from primary. Ultimately, it will maintain the same working mmf. In winding $XZ$, the current is $I_1$ (from $X$ to $Y$) whereas in winding $YZ$ the current is $I_2$ (from $Z$ to $Y$). $V_2 < V_1$, $I_2 > I_1$ and the net current through $YZ$ is $I_2 - I_1$ (from $Z$ to $Y$).



**mmf of winding**

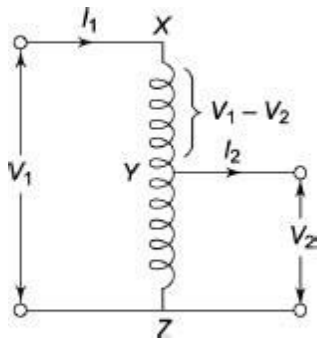



**Figure (b) of step-down Transformer**

$$XY = I_1(N_1 - N_2) = I_1N_1 - I_1N_2 = I_2N_2 - I_1N_2$$

$$[Since\ I_1N_1 = I_2N_2]$$

$$= (I_2 - I_1)N_2 = mmf\ of\ winding\ ZY$$

Voltamperes across winding *XY* are transferred by transformer action to load connected across winding *YZ*.

$$Transformed\ VA = V_{XY}I_{XY} = (V_1 - V_2)I_1$$

$$\frac{Transformer\ VA}{Input\ VA} = \frac{(V_1 - V_2)I_1}{V_1I_1} = \frac{V_1 - V_2}{V_1} = 1 - \frac{V_2}{V_1} = 1 - K$$

Total input *VA* is $V_1I_1$ and $(V_1 - V_2)I_1$ is transformed to the load by transformer action and the remaining *VA* is conducted directly.

Conducted *VA* = Total Input *VA*– Transformed *VA* = $V_1I_1 - (V_1 - V_2)I_1 = V_2I_1$

$$\frac{Conducted\ VA}{Input\ VA} = \frac{V_2I_1}{V_1I_1} = \frac{V_2}{V_1} = K$$

**Conversion of Two-winding Transformer into Single-phase Transformer**

Figure (a)below shows a two-winding transformer having primary and secondary voltages of 1,000 V and 400 V respectively. There are two ways to convert it into an autotransformer.

- ***Additive polarity**:* Figure (b)below shows an autotransformer obtained from a two-winding transformer by additive polarity and it results in a step-up transformer.
- ***Subtractive polarity**:* Figure (c) below shows an autotransformer obtained from a two-winding transformer by subtractive polarity and it results in a step-down transformer.

**Figure (a)**

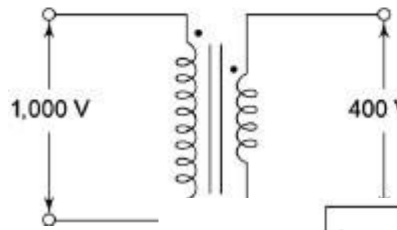



**Figure (b)**

(Additive Polarity)

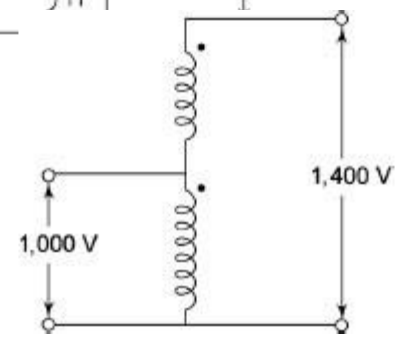
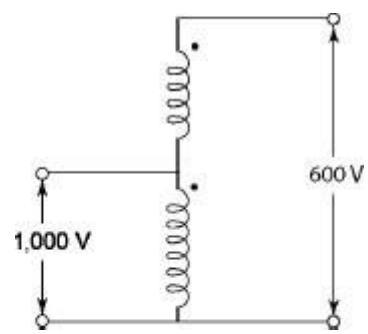



**Figure (c)**

(Subtractive Polarity)




**Advantages of Autotransformers**

Autotransformers have the following advantages:

- Less amount of copper is required.
- Due to smaller size, cost is less compared to two-winding transformer.
- The resistance and reactance are less compared to a two- winding transformer and hence it has superior voltage regulation.
- Copper (Cu) loss is less.
- Volt-ampere rating is more compared to a two-winding transformer.
- Since loss is less, efficiency is more.
- It is possible to get smooth and continuous variation of voltage.

**Disadvantages of Autotransformers**

In spite of various advantages of autotransformers, the following are the disadvantages of autotransformer:

- There is possibility of high short circuit currents for short circuits on the secondary side due to low impedance.
- The full primary current will appear across the secondary causing higher voltage on secondary resulting in danger of accidents if the common winding is open circuited.
- Since there is no electrical isolation between primary and secondary, risk factor appears at high voltage levels.
- It is economical only if the voltage ratio is less than 2.

**Applications of Autotransformers**

Autotransformers have the following applications:

- They are used as a starter for safely starting machines such as induction motors and synchronous motors.
- They are used as boosters to give a small boost to a distribution cable for compensating the voltage drop.
- They can be used as furnace transformers to supply power to the furnaces at the required supply voltage.
- They can be used as variac.

**Examples**

Problem 1. A transformer has 500 primary turns and 3000 secondary turns. If the primary voltage is 240 V, determine the secondary voltage, assuming an ideal transformer.

For an ideal transformer, voltage ratio = turns ratio i.e.,

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}, \text{ hence } \frac{240}{V_2} = \frac{500}{3000}$$

Thus secondary voltage $V_2 = \frac{(3000)(240)}{(500)} = \mathbf{1440\ V\ or\ 1.44\ kV}$

Problem 5. A 5 kVA single-phase transformer has a turns ratio of 10:1 and is fed from a 2.5 kV supply. Neglecting losses, determine (a) the full-load secondary current, (b) the minimum load resistance which can be connected across the secondary winding to give full load kVA, (c) the primary current at full load kVA.

(a) $\dfrac{N_1}{N_2} = \dfrac{10}{1}$ and $V_1 = 2.5 \text{ kV} = 2500 \text{ V}$

Since $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2}$, secondary voltage $V_2 = V_1\left(\dfrac{N_2}{N_1}\right)$

$$= 2500\left(\frac{1}{10}\right) = 250 \text{ V}$$

The transformer rating in volt-amperes $= V_2 I_2$ (at full load), i.e., $5000 = 250 I_2$

Hence full load secondary current $I_2 = \dfrac{5000}{250} = \mathbf{20\ A}$

(b) Minimum value of load resistance, $R_L = \dfrac{V_2}{I_2} = \dfrac{250}{20} = \mathbf{12.5\ \Omega}$

(c) $\dfrac{N_1}{N_2} = \dfrac{I_2}{I_1}$, from which primary current $I_1 = I_2\left(\dfrac{N_2}{N_1}\right)$

$$= 20\left(\frac{1}{10}\right) = \mathbf{2\ A}$$

Problem 6. A 2400 V/400 V single-phase transformer takes a no-load current of 0.5 A and the core loss is 400 W. Determine the values of the magnetizing and core loss components of the no-load current. Draw to scale the no-load phasor diagram for the transformer.

$V_1 = 2400$ V, $V_2 = 400$ V, $I_0 = 0.5$ A

Core loss (i.e. iron loss) $= 400 = V_1 I_0 \cos\phi_0$

$$\text{i.e. } 400 = (2400)(0.5)\cos\phi_0$$

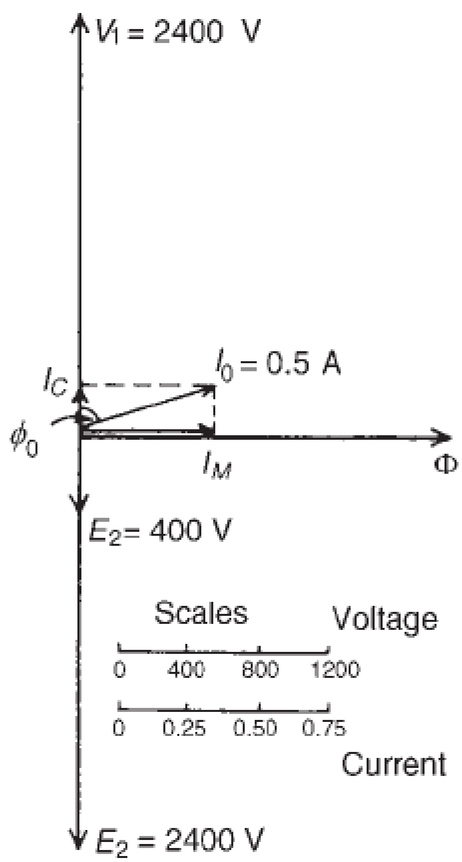
$$\text{Hence } \cos\phi_0 = \frac{400}{(2400)(0.5)} = 0.3333$$

$$\phi_0 = \arccos 0.3333 = 70.53^\circ$$

The no-load phasor diagram is shown in Figure 20.3.

Magnetizing component, $I_M = I_0 \sin\phi_0 = 0.5 \sin 70.53^\circ = \mathbf{0.471}$ **A**

Core loss component, $I_C = I_0 \cos\phi_0 = 0.5 \cos 70.53^\circ = \mathbf{0.167}$ **A**



Problem 7. A transformer takes a current of 0.8 A when its primary is connected to a 240 volt, 50 Hz supply, the secondary being on open circuit. If the power absorbed is 72 watts, determine (a) the iron loss current, (b) the power factor on no-load, and (c) the magnetizing current.

$I_0 = 0.8$ A, $V = 240$ V

(a) Power absorbed = total core loss = $72 = V_1 I_0 \cos\phi_0$

Hence $72 = 240\, I_0 \cos\phi_0$

and iron loss current, $I_c = I_0 \cos\phi_0 = \dfrac{72}{240} = \mathbf{0.30\ A}$

(b) Power factor at no load, $\cos\phi_0 = \dfrac{I_c}{I_0} = \dfrac{0.30}{0.80} = \mathbf{0.375}$

(c) From the right-angled triangle in Figure 20.2(b) and using Pythagoras' theorem, $I_0^2 = I_c^2 + I_M^2$

from which, magnetizing current, $I_M = \sqrt{(I_0^2 - I_c^2)}$

$= \sqrt{(0.80^2 - 0.30^2)}$

$= \mathbf{0.74\ A}$

Problem 8. A 100 kVA, 4000 V/200 V, 50 Hz single-phase transformer has 100 secondary turns. Determine (a) the primary and secondary current, (b) the number of primary turns, and (c) the maximum value of the flux.

$V_1 = 4000$ V, $V_2 = 200$ V, $f = 50$ Hz, $N_2 = 100$ turns

(a) Transformer rating $= V_1 I_1 = V_2 I_2 = 100\,000$ VA

Hence primary current, $I_1 = \dfrac{100\,000}{V_1} = \dfrac{100\,000}{4000} = \mathbf{25\ A}$

and secondary current, $I_2 = \dfrac{100\,000}{V_2} = \dfrac{100\,000}{200} = \mathbf{500\ A}$

(b) From equation (20.3), $\dfrac{V_1}{V_2} = \dfrac{N_1}{N_2}$

from which, primary turns, $N_1 = \left(\dfrac{V_1}{V_2}\right)(N_2) = \left(\dfrac{4000}{200}\right)(100)$

i.e., $N_1 = \mathbf{2000\ turns}$

(c) From equation (20.5), $E_2 = 4.44 f \Phi_m N_2$

from which, maximum flux $\Phi_m = \dfrac{E_2}{4.44 f N_2} = \dfrac{200}{4.44(50)(100)}$

(assuming $E_2 = V_2$)

$= \mathbf{9.01 \times 10^{-3}\ Wb\ or\ 9.01\ mWb}$

Problem 10. A single-phase 500 V/100 V, 50 Hz transformer has a maximum core flux density of 1.5 T and an effective core cross-sectional area of 50 $cm^2$. Determine the number of primary and secondary turns.

The e.m.f. equation for a transformer is $E = 4.44 f \Phi_m N$

and maximum flux, $\Phi_m = B \times A = (1.5)(50 \times 10^{-4}) = 75 \times 10^{-4}$ Wb

Since $E_1 = 4.44 f \Phi_m N_1$

$$\text{then primary turns, } N_1 = \frac{E_1}{4.44 f \Phi_m} = \frac{500}{4.44(50)(75 \times 10^{-4})}$$
$$= \mathbf{300\ turns}$$

Since $E_2 = 4.4 f \Phi_m N_2$

$$\text{then secondary turns, } N_2 = \frac{E_2}{4.4 f \Phi_m} = \frac{100}{4.44(50)(75 \times 10^{-4})}$$
$$= \mathbf{60\ turns}$$

Problem 11. A 4500 V/225 V, 50 Hz single-phase transformer is to have an approximate e.m.f. per turn of 15 V and operate with a maximum flux of 1.4 T. Calculate (a) the number of primary and secondary turns and (b) the cross-sectional area of the core.

(a) $$\text{E.m.f. per turn} = \frac{E_1}{N_1} = \frac{E_2}{N_2} = 15$$

$$\text{Hence primary turns, } N_1 = \frac{E_1}{15} = \frac{4500}{15} = \mathbf{300}$$

$$\text{and secondary turns, } N_2 = \frac{E_2}{15} = \frac{225}{15} = \mathbf{15}$$

(b) E.m.f. $E_1 = 4.44 f \Phi_m N_1$

from which, $\Phi_m = \dfrac{E_1}{4.44 f N_1} = \dfrac{4500}{4.44(50)(300)} = 0.0676$ Wb

Now flux $\Phi_m = B_m \times A$, where $A$ is the cross-sectional area of the core, hence

area $A = \dfrac{\Phi_m}{B_m} = \dfrac{0.0676}{1.4} = \mathbf{0.0483\ m^2}$ or $\mathbf{483\ cm^2}$

> Problem 14. A 5 kVA, 200 V/400 V, single-phase transformer has a secondary terminal voltage of 387.6 volts when loaded. Determine the regulation of the transformer.

$$\text{regulation} = \frac{\text{(No-load secondary voltage} - \text{terminal voltage on load)}}{\text{no-load secondary voltage}} \times 100\%$$

$$= \left[\frac{400 - 387.6}{400}\right] \times 100\%$$

$$= \left(\frac{12.4}{400}\right) \times 100\% = \mathbf{3.1\%}$$

> Problem 15. The open circuit voltage of a transformer is 240 V. A tap changing device is set to operate when the percentage regulation drops below 2.5%. Determine the load voltage at which the mechanism operates.

$$\text{Regulation} = \frac{(\text{no load voltage} - \text{terminal load voltage})}{\text{no load voltage}} \times 100\%$$

$$\text{Hence } 2.5 = \left[\frac{240 - V_2}{240}\right] 100\%$$

$$\text{Therefore } \frac{(2.5)(240)}{100} = 240 - V_2$$

i.e, $6 = 240 - V_2$

from which, **load voltage, $V_2 = 240 - 6 = 234$ volts**

Problem 16. A 200 kVA rated transformer has a full-load copper loss of 1.5 kW and an iron loss of 1 kW. Determine the transformer efficiency at full load and 0.85 power factor.

$$\text{Efficiency } \eta = \frac{\text{output power}}{\text{input power}} = \frac{\text{input power} - \text{losses}}{\text{input power}}$$

$$= 1 - \frac{\text{losses}}{\text{input power}}$$

Full-load output power $= VI \cos\phi = (200)(0.85) = 170$ kW

Total losses $= 1.5 + 1.0 = 2.5$ kW

Input power $=$ output power $+$ losses $= 170 + 2.5 = 172.5$ kW

Hence efficiency $= \left(1 - \dfrac{2.5}{172.5}\right) = 1 - 0.01449 = 0.9855$ or **98.55%**

Problem 17. Determine the efficiency of the transformer in Problem 16 at half full-load and 0.85 power factor.

Half full-load power output $= \frac{1}{2}(200)(0.85) = 85$ kW

Copper loss (or $I^2R$ loss) is proportional to current squared.

Hence the copper loss at half full-load is $\left(\frac{1}{2}\right)^2(1500) = 375$ W

Iron loss $= 1000$ W (constant)

Total losses $= 375 + 1000 = 1375$ W or 1.375 kW

Input power at half full-load = output power at half full-load + losses

$$= 85 + 1.375 = 86.375 \text{ kW}$$

$$\text{Hence efficiency} = \left(1 - \frac{\text{losses}}{\text{input power}}\right) = \left(1 - \frac{1.375}{86.375}\right)$$

$$= 1 - 0.01592 = 0.9841 \text{ or } \mathbf{98.41\%}$$

Problem 18. A 400 kVA transformer has a primary winding resistance of 0.5 Ω and a secondary winding resistance of 0.001 Ω. The iron loss is 2.5 kW and the primary and secondary voltages are 5 kV and 320 V respectively. If the power factor of the load is 0.85, determine the efficiency of the transformer (a) on full load, and (b) on half load.

(a) Rating $= 400$ kVA $= V_1 I_1 = V_2 I_2$

Hence primary current, $I_1 = \dfrac{400 \times 10^3}{V_1} = \dfrac{400 \times 10^3}{5000} = 80$ A

and secondary current, $I_2 = \dfrac{400 \times 10^3}{V_2} = \dfrac{400 \times 10^3}{320} = 1250$ A

Total copper loss $= I_1^2 R_1 + I_2^2 R_2$,

(where $R_1 = 0.5\ \Omega$ and $R_2 = 0.001\ \Omega$)

$$= (80)^2(0.5) + (1250)^2(0.001)$$

$$= 3200 + 1562.5 = 4762.5 \text{ watts}$$

On full load, total loss = copper loss + iron loss

$$= 4762.5 + 2500$$

$$= 7262.5 \text{ W} = 7.2625 \text{ kW}$$

Total output power on full load $= V_2 I_2 \cos\phi_2$

$$= (400 \times 10^3)(0.85)$$

$$= 340 \text{ kW}$$

Input power = output power + losses = 340 kW + 7.2625 kW

$$= 347.2625 \text{ kW}$$

$$\text{Efficiency, } \eta = \left[1 - \frac{\text{losses}}{\text{input power}}\right] \times 100\%$$

$$= \left[1 - \frac{7.2625}{347.2625}\right] \times 100\% = \mathbf{97.91\%}$$

(b) Since the copper loss varies as the square of the current, then total copper loss on half load $= 4762.5 \times \left(\frac{1}{2}\right)^2 = 1190.625$ W

Hence total loss on half load $= 1190.625 + 2500$

$= 3690.625$ W or 3.691 kW

Output power on half full load $= \frac{1}{2}(340) = 170$ kW

Input power on half full load = output power + losses

$= 170$ kW $+ 3.691$ kW

$= 173.691$ kW

Hence efficiency at half full load,

$$\eta = \left[1 - \frac{\text{losses}}{\text{input power}}\right] \times 100\%$$

$$= \left[1 - \frac{3.691}{173.691}\right] \times 100\% = \mathbf{97.87\%}$$

Problem 25. A single-phase auto transformer has a voltage ratio 320 V:250 V and supplies a load of 20 kVA at 250 V. Assuming an ideal transformer, determine the current in each section of the winding.

Rating = 20 kVA = $V_1 I_1 = V_2 I_2$

Hence primary current, $I_1 = \dfrac{20 \times 10^3}{V_1} = \dfrac{20 \times 10^3}{320} = \mathbf{62.5\ A}$

and secondary current, $I_2 = \dfrac{20 \times 10^3}{V_2} = \dfrac{20 \times 10^3}{250} = \mathbf{80\ A}$

Hence current in common part of the winding = 80 – 62.5 = **17.5 A**

Problem 26. Determine the saving in the volume of copper used in an auto transformer compared with a double-wound transformer for (a) a 200 V:150 V transformer, and (b) a 500 V:100 V transformer.

(a) For a 200 V:150 V transformer, $x = \dfrac{V_2}{V_1} = \dfrac{150}{200} = 0.75$

Hence from equation (20.12), (volume of copper in auto transformer)

= (1 – 0.75) (volume of copper in double-wound transformer)

= (0.25) (volume of copper in double-wound transformer)

= 25% of copper in a double-wound transformer

**Hence the saving is 75%**

(b) For a 500 V:100 V transformer, $x = \frac{V_2}{V_1} = \frac{100}{500} = 0.2$

Hence (volume of copper in auto transformer)

$= (1 - 0.2)$ (volume of copper in double-wound transformer)

$= (0.8)$ (volume in double-wound transformer)

$= 80\%$ of copper in a double-wound transformer

**Hence the saving is 20%**

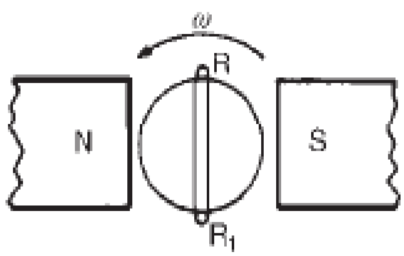
# Learning Unit 4 – Analyse 3-phase circuits

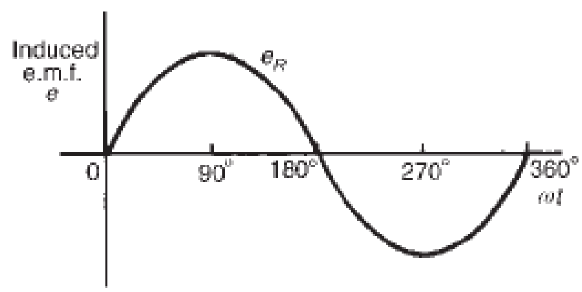
## LO 4.1 – Analyze AC 3 phase connection

- Content/Topic 1: Difference between single phase and three phase in AC

Generation, transmission and distribution of electricity via the National Grid system is accomplished by three-phase alternating currents. The voltage induced by a single coil when rotated in a uniform magnetic field is shown in Figures below and is known as a single-phase voltage. Most consumers are fed by means of a single-phase a.c. supply. Two wires are used, one called the live conductor (usually coloured red) and the other is called the neutral conductor

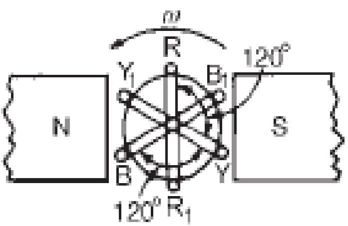
(usually coloured black). The neutral is usually connected via protective gear to earth, the earth wire being coloured green. The standard voltage for a single-phase a.c. supply is 240 V.



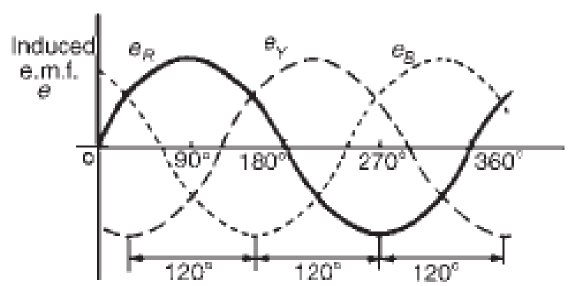




**Three-phase supply**

A three-phase supply is generated when three coils are placed 120° apart and the whole rotated in a uniform magnetic field as shown in Figure (a)below. The result is three independent supplies of equal voltages which are each displaced by 120° from each other as shown in Figure (b)below.




(a)




(b)

**Phase Sequence**

By phase sequence is meant the order in which the three phases voltage or current attain their peak or maximum values. For eg. If the phase sequence is said to be RYB, then first red phase attains maximum value and then with a phase difference of $120^0$ each, the yellow phase and blue phase attains their peak value.

**Numbering of Phases**

The three phases may be numbered 1, 2, 3 or a, b, c or as is customary, they may be given three colours. The colours used commercially are red, yellow (or sometimes white) and blue. In this case, the sequence is RYB. Obviously, in any three-phase system, there are two possible sequences in which the three coil or phase voltages may pass through their maximum values i.e. red → yellow → blue (RYB) or red → blue → yellow (RBY). By convention, sequence RYB is taken as positive and RBY as negative.

**Advantages of Three Phase System**

1. Power produced by three phase motor is high compared with that of same rating of single phase motor.
2. Three phase transmission is more efficiency and requires less copper for transmitting same power over the same distance.
3. Three phase motors are self starting while single phase motors are not.
4. Power factor of three phase motor is high.
5. Torque of three phase motor is uniform where as that of single phase motor is pulsating.
6. Size of three phase motor is smaller than single phase motor of same rating.

**Balanced Load**

If all the phase windings of three phase alternator are having equal impedance or phase angle, then it is called balance system. Similarly if each phase load of the three phase load connected to three phase supply is having equal impedance or phase angle then it is called balance load.

**Unbalance Load**

If three phase load having different value of load in each phase is connected to three phase supply, then it is called unbalance load.

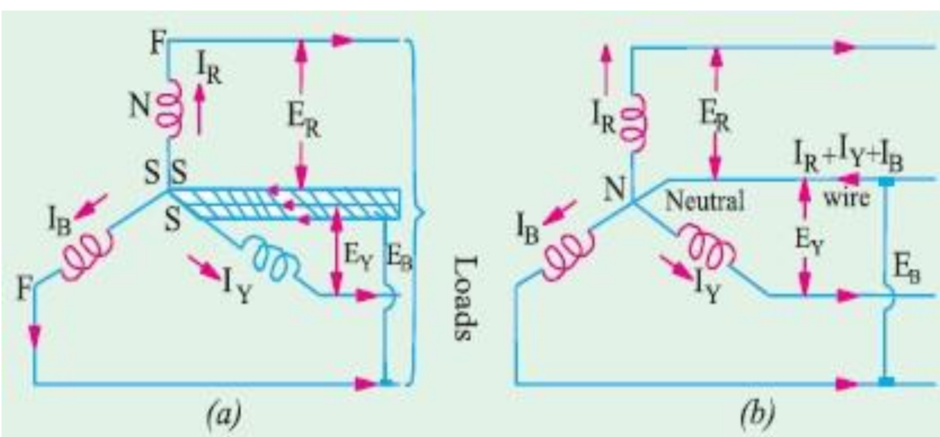
- Content/Topic 2: Connections in AC three phase

If the three armature coils of the 3-phase alternator are not interconnected but are kept separate, then each phase or circuit would need two conductors, the total number of conductors, in that case, being six. It means that each transmission cable would contain six conductors which will make the whole system complicated and expensive. Hence, the three phases are generally interconnected which results in substantial saving of copper. The general methods of interconnection are

(a) Star or Wye (Y) connection and
(b) Mesh or Delta ( Δ ) connection.

**Star or Wye (Y) Connection**

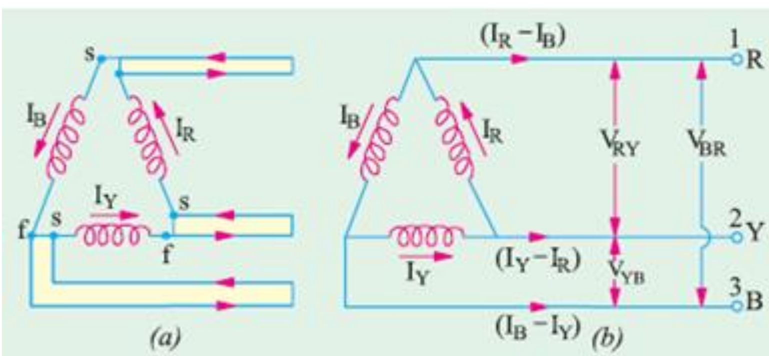
In this method of interconnection, the similar ends say, 'star' ends of three coils (it could be 'finishing' ends also) are joined together at point N as shown in Fig.(a) below.
The point N is known as star point or neutral point. The three conductors meeting at point N are replaced by a single conductor known as neutral conductor as shown in Fig.(b) below. Such an interconnected system is known as four-wire, 3-phase system and is diagrammatically shown in Fig.(b) below.
If this three-phase voltage system is applied across a balanced symmetrical load, the neutral wire will be carrying three currents which are exactly equal in magnitude but are 120° out of phase with each other. Hence, their vector sum is zero.
i.e. $I_R + I_Y + I_B = 0$ ... vectorially




The neutral wire, in that case, may be omitted although its retention is useful for supplying lighting loads at low voltages. The p.d. between any terminal (or line) and neutral (or star) point gives the phase or star voltage. But the p.d. between any two lines gives the line-to-line voltage or simply line voltage.

**Delta (Δ) or Mesh Connection**

In this form, of interconnection the dissimilar ends of the three phase winding are joined together i.e. the 'starting' end of one phase is joined to the 'finishing' end of the other phase and so on as showing in Fig. (a) below. In other words, the three windings are joined in series to form a closed mesh as shown in Fig. (b) below.
Three leads are taken out from the three junctions as shown as outward directions are taken as positive.
It might look as if this sort of interconnection results in shortcircuiting the three windings. However, if the system is balanced then sum of the three voltages round the closed mesh is zero, hence no current of fundamental frequency can flow around the mesh when the terminals are open. It should be clearly understood that at any instant, the e.m.f. in one phase is equal and opposite to the resultant of those in the other two phases.
This type of connection is also referred to as 3-phase, 3-wire system.




- Content/Topic 3: Three phase voltage and currents values

**Values of Phase Currents**

When considering the distribution of current in a 3-phase system, it is extremely important to bear in mind that:

(i) the arrow placed alongside the currents $I_R$ $I_Y$ and $I_B$ flowing in the three phases [Fig. (b) above] indicate the directions of currents when they are assumed to be positive and not the directions at a particular instant. It should be clearly understood that at no instant will all the three currents flow in the same direction either outwards or inwards. The three arrows indicate that first the current flows outwards in phase R, then after a phase-time of 120°, it will flow outwards from phase Y and after a further 120°, outwards from phase B.

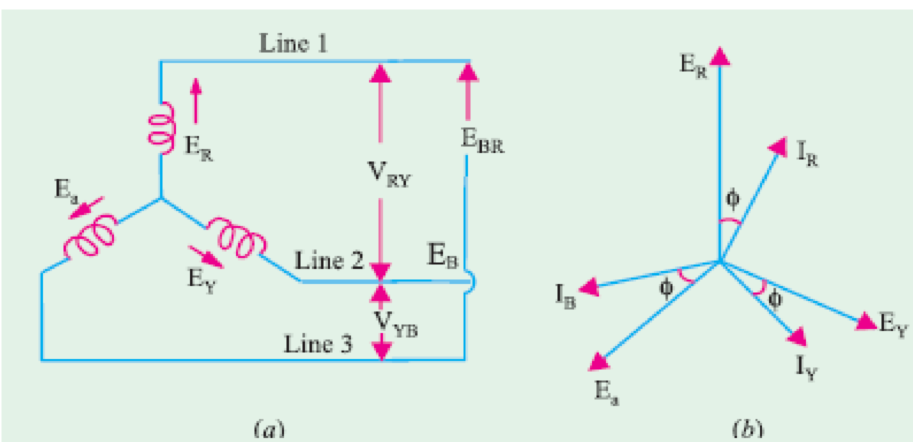
(ii) the current flowing outwards in one or two conductors is always equal to that flowing inwards in the remaining conductor or conductors. In other words, each conductor in turn, provides a return path for the currents of the other conductors.

## STAR OR WYE (Y) CONNECTION

### Voltages and Currents in Y-Connection

The voltage induced in each winding is called the **phase voltage** and current in each winding is likewise known as **phase current**. However, the voltage available between any pair of terminals (or outers) is called **line voltage ($V_L$)** and the current flowing in each line is called **line current ($I_L$)**. As seen from Fig.(a) below, in this form of interconnection, there are two phase windings between each pair of terminals but since their similar ends have been joined together, they are in opposition. Obviously, the instantaneous value of p.d. between any two terminals is the arithmetic difference of the two phase e.m.fs. concerned. However, the r.m.s. value of this p.d. is given by the vector difference of the two phase e.m.fs. The vector diagram for phase voltages and currents in a star connection is shown in Figure (b)below where a balanced system has been assumed. It means that $E_R = E_Y = E_{ph}$ (phase e.m.f.).

Line voltage $V_{RY}$ between line 1 and line 2 is the vector difference of $E_R$ and $E_Y$. Line voltage $V_{YB}$ between line 2 and line 3 is the vector difference of $E_Y$ and $E_B$. Line voltage $V_{BR}$ between line 3 and line 1 is the vector difference of $E_B$ and $E_R$.




### (a) Line Currents and Phase Currents

It is seen from Fig.(a) above that each line is in series with its individual phase winding, hence the line current in each line is the same as the current in the phase winding to which the line is connected.

Current in line 1 = $I_R$; Current in line 2 = $I_Y$; Current in line 3 = $I_B$ Since $I_R = I_Y = I_B$ = say, $I_{ph}$ → the phase current

∴ line current $I_L = I_{ph}$

**(b) Line Voltages and Phase Voltages**

The p.d. between line 1 and 2 is $V_{RY} = E_R - E_Y$ ... vector difference.
Hence, $V_{RY}$ is found by compounding $E_R$ and $E_Y$ reversed and its value is given by the diagonal of the parallelogram of Fig. below. Obviously, the angle between $E_R$ and $E_Y$ reversed is 60°.
Hence if $E_R = E_Y = E_B$ = say, $E_{ph}$ → the phase e.m.f., then

$$V_{RY} = 2 \times E_{ph} \times \cos(60°/2)$$

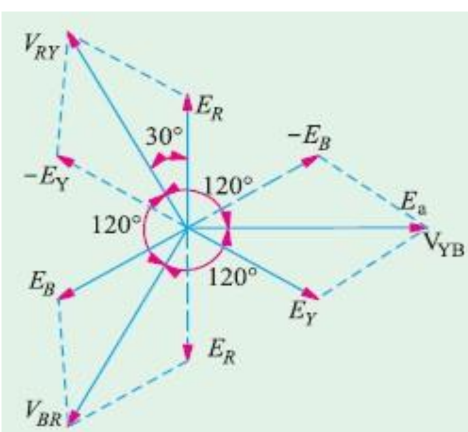
$$= 2 \times E_{ph} \times \cos 30° = 2 \times E_{ph} \times \frac{\sqrt{3}}{2} = \sqrt{3}\, E_{ph}$$

Similarly, $V_{YB} = E_Y - E_B = \sqrt{3} \cdot E_{ph}$ ...vector difference

and $V_{BR} = E_B - E_R = \sqrt{3} \cdot E_{ph}$

Now $V_{RY} = V_{YB} = Y_{BR} =$ line voltage, say $V_L$. Hence, in star connection $V_L = \sqrt{3} \cdot E_{ph}$

It will be noted from Fig. below that
1. Line voltages are 120° apart.
2. Line voltages are 30° ahead of their respective phase voltages.
3. The angle between the line currents and the corresponding line voltages is (30 + ϕ ) with current lagging.



**(c) Power**

The total active or true power in the circuit is the sum of the three phase powers. Hence, total active power = 3 × phase power or P = 3 × $V_{ph}$ $I_{ph}$ cosϕ

Now $V_{ph} = V_L / \sqrt{3}$ and $I_{ph} = I_L$

Hence, in terms of line values, the above expression becomes

$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos\phi \quad \text{or} \quad P = \sqrt{3}\, V_L\, I_L\, \cos\phi$$

It should be particularly noted that ϕ is the angle between phase voltage and phase current and not between the line voltage and line current.

Similarly, total reactive power is given by

$$Q = \sqrt{3}\, V_L\, I_L\, \sin\phi$$

By convention, reactive power of a coil is taken as positive and that of a capacitor as negative. The total apparent power of the three phases is

$S = \sqrt{3}\, V_L I_L$ Obviously, $S = \sqrt{P^2 + Q^2}$

**Delta (Δ) or Mesh Connection**

**(i) Line Voltages and Phase Voltages**

It is seen from Fig. (b) below that there is only one phase winding completely included between any pair of terminals. Hence, in Δ-connection, the voltage between any pair of lines is equal to the phase voltage of the phase winding connected between the two lines considered. Since phase sequence is R Y B, the voltage having its positive direction from R to Y leads by 120° on that having its positive direction from Y to B. Calling the voltage between lines 1 and 2 as $V_{RY}$ and that between lines 2 and 3 as $V_{YB}$, we find that $V_{RY}$ lead $V_{YB}$ by 120°. Similarly, $V_{YB}$ leads $V_{BR}$ by 120º. Let $V_{RY} = V_{YB} = V_{BR}$ = line voltage $V_L$. Then, it is seen that $V_L = V_{ph}$.

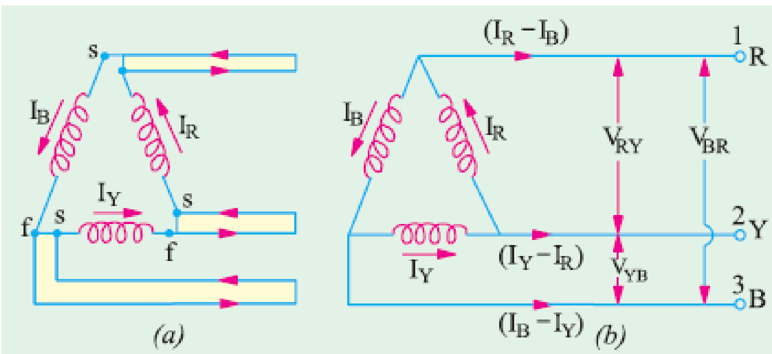
### (ii) Line Currents and Phase Currents

It will be seen from Fig. (b) below that current in each line is the vector difference of the two phase currents flowing through that line. For example

Current in line 1 is $I_1$ $I_R$ $I_B$

Current in line 2 is $I_2$ $I_Y$ $I_R$ vector difference

Current in line 3 is $I_3$ $I_B$ $I_Y$




Current in line No. 1 is found by compounding $I_R$ and $I_B$ reversed and its value is given by the diagonal of the parallelogram of Fig.below. The angle between $I_R$ and $I_B$ reversed (i.e. – $I_B$) is 60°. If $I_R = I_Y$ = phase current $I_{ph}$ (say), then Current in line No. 1 is

$$I_1 = 2 \times I_{ph} \times \cos(60°/2) = 2 \times I_{ph}\ \sqrt{3}/2 = \sqrt{3}\, I_{ph}$$

Current in line No. 2 is

$I_2 = I_B - I_Y$ ... vector difference = $\sqrt{3} I_{ph}$ and current

in line No. 3 is $I_3 = I_B - I_Y$ $\therefore$ Vector difference $= \sqrt{3} \cdot I_{ph}$

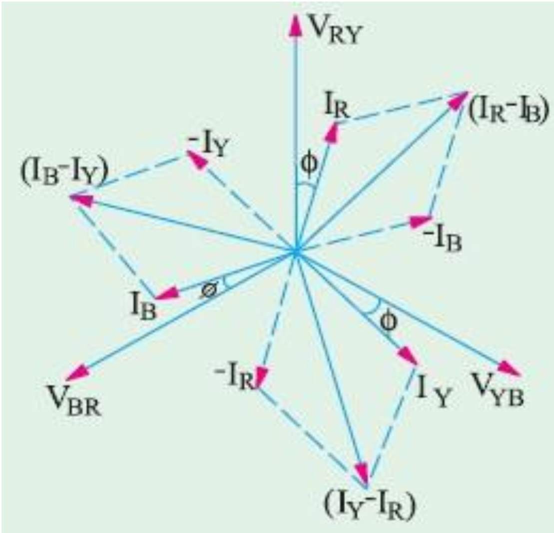
Since all the line currents are equal in magnitude *i.e.*

$$I_1 = I_2 - I_3 = I_L$$

$$\therefore\ I_L = \sqrt{3}\, I_{ph}$$

With reference to Fig. below, it should be noted that

1. line currents are 120º apart ;
2. line currents are 30º behind the respective phase currents ;
3. the angle between the line currents and the corresponding line voltages is (30 + ϕ ) with the current lagging.



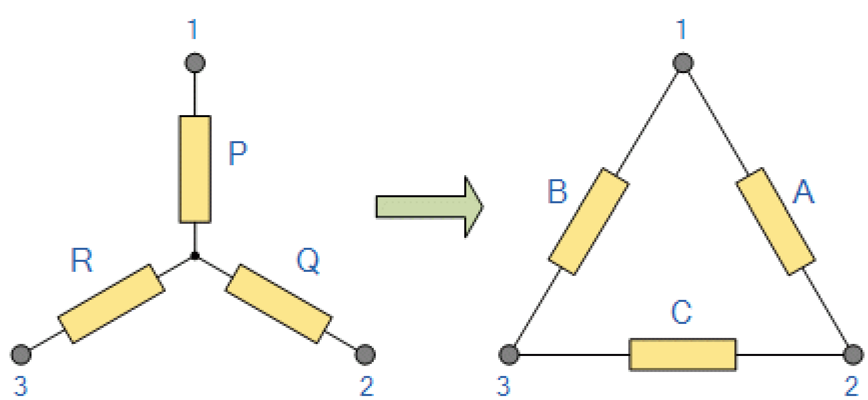

**(iii) Power**

Power/phase = $V_{ph}I_{ph}\cos\phi$ ; Total power = 3 × $V_{ph}I_{ph}\cos\phi$ . However, $V_{ph}$ = $V_L$ and $\sqrt{3}$ $I_{ph}$ = $I_L$/ Hence, in terms of line values, the above expression for power becomes

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi = \sqrt{3}\, V_L I_L \cos\phi$$

where ϕ is the phase power factor angle.

- Content/Topic 4: Star-delta / delta-star transformations



Star-Delta Transformations and Delta-Star Transformations allow us to convert impedances connected together in a 3-phase configuration from one type of connection to another

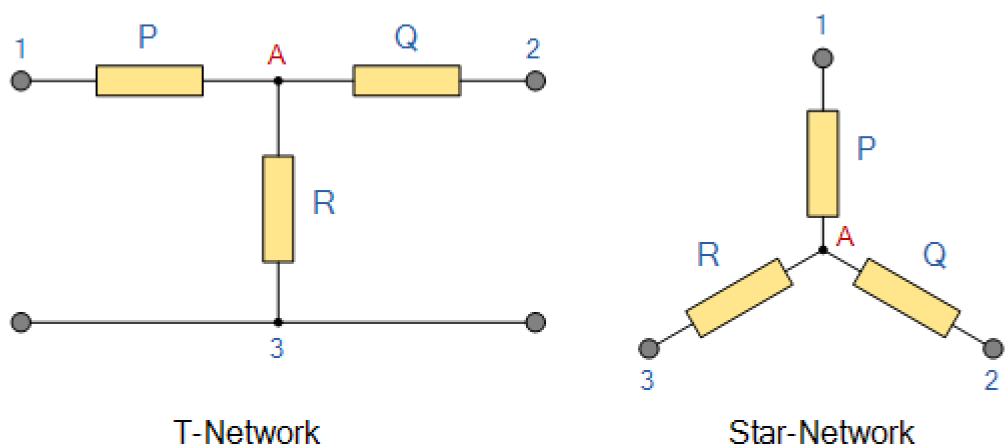
We can now solve simple series, parallel or bridge type resistive networks using **Kirchhoff´s Circuit Laws**, mesh current analysis or nodal voltage analysis techniques but in a balanced 3-phase circuit we can use different mathematical techniques to simplify the analysis of the circuit and thereby reduce the amount of math's involved which in itself is a good thing.

Standard 3-phase circuits or networks take on two major forms with names that represent the way in which the resistances are connected, a **Star** connected network which has the symbol of the letter, Υ (wye) and a **Delta** connected network which has the symbol of a triangle, Δ (delta).

If a 3-phase, 3-wire supply or even a 3-phase load is connected in one type of configuration, it can be easily transformed or changed it into an equivalent configuration of the other type by using either the **Star Delta Transformation** or **Delta Star Transformation** process.

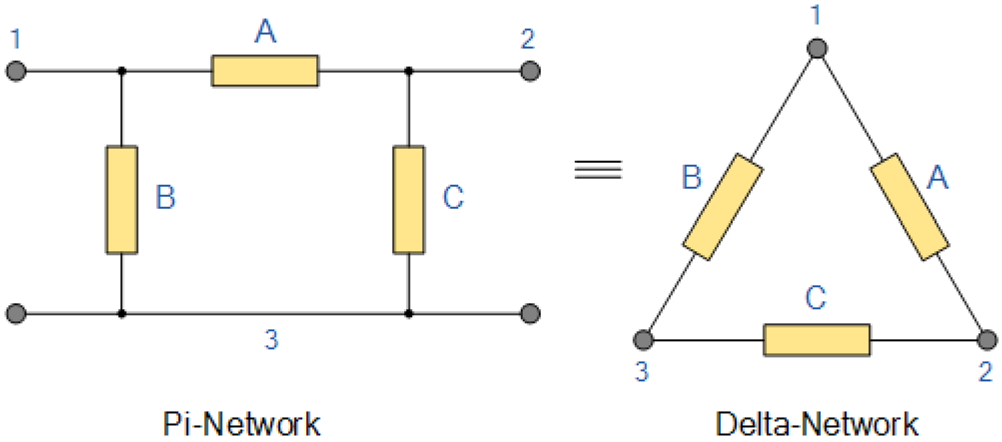
A resistive network consisting of three impedances can be connected together to form a T or "Tee" configuration but the network can also be redrawn to form a Star or Υ type network as shown below.

**T-connected and Equivalent Star Network**




As we have already seen, we can redraw the T resistor network above to produce an electrically equivalent **Star** or Υ type network. But we can also convert a Pi or π type resistor network into an electrically equivalent **Delta** or Δ type network as shown below.

**Pi-connected and Equivalent Delta Network**



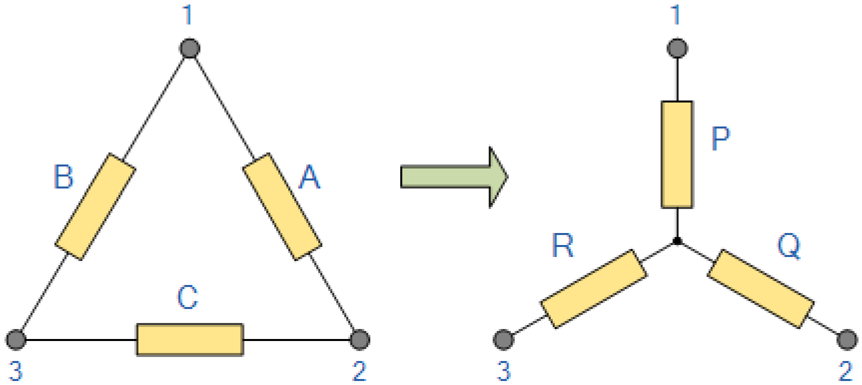

Having now defined exactly what is a **Star** and **Delta** connected network it is possible to transform the Y into an equivalent Δ circuit and also to convert a Δ into an equivalent Y circuit using a the transformation process.
This process allows us to produce a mathematical relationship between the various resistors giving us a **Star Delta Transformation** as well as a **Delta Star Transformation**.

These circuit transformations allow us to change the three connected resistances (or impedances) by their equivalents measured between the terminals 1-2, 1-3 or 2-3 for either a star or delta connected circuit. However, the resulting networks are only equivalent for voltages and currents external to the star or delta networks, as internally the voltages and currents are different but each network will consume the same amount of power and have the same power factor to each other.

**Delta Star Transformation**

To convert a delta network to an equivalent star network we need to derive a transformation formula for equating the various resistors to each other between the various terminals. Consider the circuit below.

**Delta to Star Network**




Compare the resistances between terminals 1 and 2.

$$P + Q = A \text{ in parallel with } (B + C)$$

$$P + Q = \frac{A(B + C)}{A + B + C} \quad \ldots EQ1$$

Resistance between the terminals 2 and 3.

$$Q + R = C \text{ in parallel with } (A + B)$$

$$Q + R = \frac{C(A + B)}{A + B + C} \quad \ldots EQ2$$

Resistance between the terminals 1 and 3.

$$P + R = B \text{ in parallel with } (A + C)$$

$$P + R = \frac{B(A + C)}{A + B + C} \quad \ldots EQ3$$

This now gives us three equations and taking equation 3 from equation 2 gives:

$$EQ3 - EQ2 = (P + R) - (Q + R)$$

$$P + R = \frac{B(A + C)}{A + B + C} \; - \; Q + R = \frac{C(A + B)}{A + B + C}$$

$$\therefore P - Q = \frac{BA + CB}{A + B + C} - \frac{CA + CB}{A + B + C}$$

$$\therefore \; P - Q = \frac{BA - CA}{A + B + C}$$

Then, re-writing Equation 1 will give us:

$$P + Q = \frac{AB + AC}{A + B + C}$$

Adding together equation 1 and the result above of equation 3 minus equation 2 gives:

$$(P - Q) + (P + Q)$$

$$= \frac{BA - CA}{A + B + C} + \frac{AB + AC}{A + B + C}$$

$$= 2P = \frac{2AB}{A + B + C}$$

From which gives us the final equation for resistor P as:

$$P = \frac{AB}{A + B + C}$$

Then to summarize a little about the above maths, we can now say that resistor P in a Star network can be found as Equation 1 plus (Equation 3 minus Equation 2) or Eq1 + (Eq3 – Eq2).
Similarly, to find resistor Q in a star network, is equation 2 plus the result of equation 1 minus equation 3 or Eq2 + (Eq1 – Eq3) and this gives us the transformation of Q as:

$$Q = \frac{AC}{A + B + C}$$

and again, to find resistor R in a Star network, is equation 3 plus the result of equation 2 minus equation 1 or Eq3 + (Eq2 – Eq1) and this gives us the transformation of R as:

$$R = \frac{BC}{A + B + C}$$

When converting a delta network into a star network the denominators of all of the transformation formulas are the same: A + B + C, and which is the sum of ALL the delta resistances. Then to convert any delta connected network to an equivalent star network we can summarized the above transformation equations as:

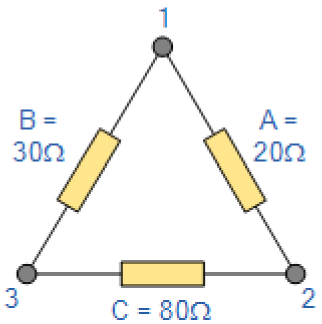
**Delta to Star Transformations Equations**

$$P = \frac{AB}{A + B + C} \quad Q = \frac{AC}{A + B + C} \quad R = \frac{BC}{A + B + C}$$

If the three resistors in the delta network are all equal in value then the resultant resistors in the equivalent star network will be equal to one third the value of the delta resistors. This gives each resistive branch in the star network a value of: RSTAR = 1/3*RDELTA which is the same as saying: (RDELTA)/3

**Delta – Star Example No1**

Convert the following Delta Resistive Network into an equivalent Star Network.




$$Q = \frac{AC}{A+B+C} = \frac{20 \times 80}{130} = 12.31\Omega$$

$$P = \frac{AB}{A+B+C} = \frac{20 \times 30}{130} = 4.61\Omega$$

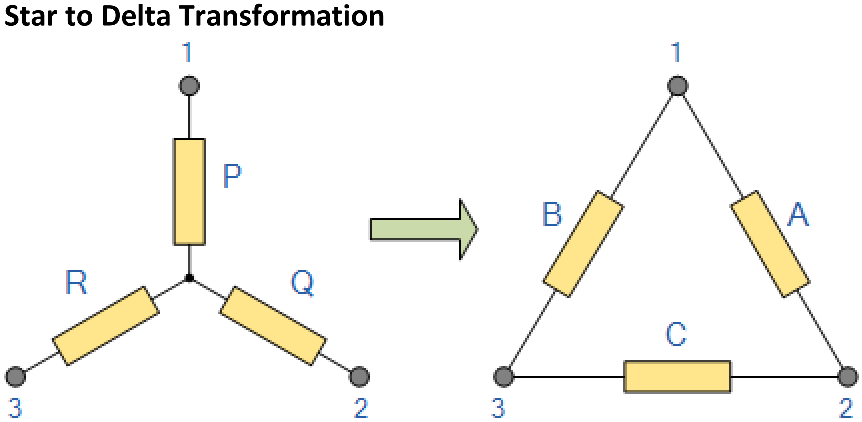
$$R = \frac{BC}{A+B+C} = \frac{30 \times 80}{130} = 18.46\Omega$$

**Star to Delta Transformation**

Star Delta transformation is simply the reverse of above. We have seen that when converting from a delta network to an equivalent star network that the resistor connected to one terminal is the product of the two delta resistances connected to the same terminal, for example resistor P is the product of resistors A and B connected to terminal 1.

By rewriting the previous formulas, a little we can also find the transformation formulas for converting a resistive star network to an equivalent delta network giving us a way of producing a star delta transformation as shown below.

**Star to Delta Transformation**



The value of the resistor on any one side of the delta, Δ network is the sum of all the two-product combinations of resistors in the star network divide by the star resistor located "directly opposite" the delta resistor being found. For example, resistor A is given as:

$$A = \frac{PQ + QR + RP}{R}$$

with respect to terminal 3 and resistor B is given as:

$$B = \frac{PQ + QR + RP}{Q}$$

with respect to terminal 2 with resistor C given as:

$$C = \frac{PQ + QR + RP}{P}$$

with respect to terminal 1.

By dividing out each equation by the value of the denominator we end up with three separate transformation formulas that can be used to convert any Delta resistive network into an equivalent star network as given below.

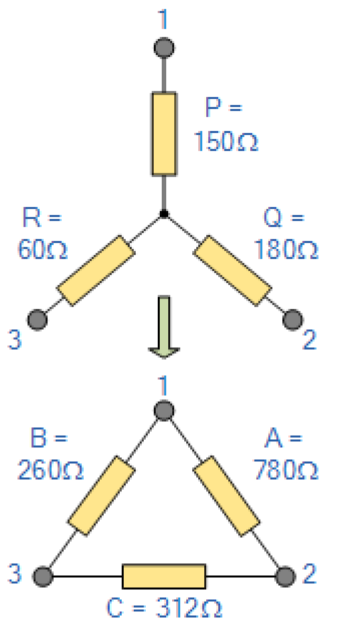
Star Delta Transformation Equations

$$A = \frac{PQ}{R} + Q + P \quad B = \frac{RP}{Q} + P + R \quad C = \frac{QR}{P} + Q + R$$

One final point about converting a star resistive network to an equivalent delta network. If all the resistors in the star network are all equal in value then the resultant resistors in the equivalent delta network will be three times the value of the star resistors and equal, giving: RDELTA = 3*RSTAR

**Star – Delta Example No2**

Convert the following Star Resistive Network into an equivalent Delta Network.




$$A = \frac{QP}{R} + Q + P = \frac{180 \times 150}{60} + 180 + 150 = 780\Omega$$

$$B = \frac{RP}{Q} + R + P = \frac{60 \times 150}{180} + 60 + 150 = 260\Omega$$

$$C = \frac{QR}{P} + Q + R = \frac{180 \times 60}{150} + 180 + 60 = 312\Omega$$

Both **Star Delta Transformation** and **Delta Star Transformation** allows us to convert one type of circuit connection into another type in order for us to easily analyse the circuit. These transformation techniques can be used to good effect for either star or delta circuits containing resistances or impedances.

- Content/Topic 5: Applications of Delta-Star/Star-Delta connection

**Applications of Delta-Star**

**Commonly used in a step-up transformer**
As for example, at the beginning of a HT transmission line. In this case neutral point is stable and will not float in case of unbalanced loading. There is no distortion of flux because existence of a Δ -connection allows a path for the third-harmonic components.
The line voltage ratio is √3 times of transformer turn-ratio and the secondary voltage leads the primary one by 30°. In recent years, this arrangement has become very popular for distribution system as it provides 3- Ø, 4-wire system.

**Commonly used in commercial, industrial, and high-density residential locations**
To supply three-phase distribution systems.
An example would be a distribution transformer with a delta primary, running on three 11kV phases with no neutral or earth required, and a star (or wye) secondary providing a 3-phase supply at 400 V, with the domestic voltage of 230 available between each phase and an earthed neutral point.

**Used as Generator Transformer**
The Δ-Y transformer connection is used universally for connecting generators to transmission systems because of two very important reasons.
First of all, generators are usually equipped with sensitive ground fault relay protection. The Δ-Y transformer is a source of ground currents for loads and faults on the transmission system, yet the generator ground fault protection is completely isolated from ground currents on the primary side of the transformer.

**Applications of Star-Delta Connection**

**It is commonly employed for power supply transformers.** This type of connection is commonly employed at the substation end of the transmission line. The main use with this connection is to step down the voltage. The neutral available on the primary side is grounded. It can be seen that there is phase difference of 30° between primary and secondary line voltages.

Commonly used in a **step-down transformer**, Y connection on the HV side reduces insulation costs the neutral point on the HV side can be grounded, stable with respect to unbalanced loads. As for example, at the end of a transmission line. The neutral of the primary winding is earthed.

In this system, line voltage ratio is 1/√3 Times of transformer turn-ratio and secondary voltage lags behind primary voltage by 30°. Also third harmonic currents flow in the to give a sinusoidal flux.

## LO 4.2 – Identify power in 3 phase AC circuits

- Content/Topic 1: Introduction of power in three phase AC circuits

**Three-phase electric power** is a common method of alternating current electric power generation, transmission, and distribution. It is a type of polyphase system and is the most common method used by electrical grids worldwide to transfer power. It is also used to power large motors and other heavy loads.

A three-**phase** circuit provides greater **power** density than a one-**phase** circuit at the same amperage, keeping wiring size and costs lower. In addition, three-**phase power** makes it easier to balance loads, minimizing harmonic currents and the need for large neutral wires.

- Content/Topic 2: Types of power in 3 phase AC circuits

The three-phase power is mainly used for generation, transmission and distribution of electrical power because of their superiority. It is more economical as compared to single-phase power and requires three live conductors for power supply. Power in a single-phase system or circuit is given by the relation shown below:

$$P = VI\cos\varphi$$

Where,

V is the voltage of single-phase, i.e. Vph
I is the current of single-phase, i.e. Iph and
Cosφ is the power factor of the circuit.

In 3 phase circuits (balanced load), the power is defined as the sum of various powers in a three-phase system. i.e.

$$P = 3V_{ph}I_{ph}\text{Cos}\varphi$$

**Power in star connections in a 3 phase circuits is given as**

$$P = 3\,\frac{V_L}{\sqrt{3}}\,I_L\text{Cos}\varphi \;\;\ldots\ldots..(1)$$

The phase voltage and line voltage in the star connection is represented as shown below:

$$V_{ph} = \frac{V_L}{\sqrt{3}} \text{ and } I_{ph} = I_L$$

Therefore, equation (1) can be written as:

$$P = \sqrt{3}\, V_L I_L \text{Cos}\varphi \quad \dots \dots .. (2)$$

**Power in delta connections in 3 phase circuits is given by the equation shown below:**

$$P = 3\, V_L \frac{I_L}{\sqrt{3}} \text{Cos}\varphi \quad \dots \dots .. (3)$$

In delta connections, the relation between phase and line voltage and phase and line current is given as:

$$V_{ph} = V_L \text{ and } I_{ph} = \frac{I_L}{\sqrt{3}}$$

Hence, equation (3) can be written as

$$P = 3\, V_L \frac{I_L}{\sqrt{3}} \text{Cos}\varphi \quad \dots \dots .. (3)$$

Thus, the total power in a 3 phase balanced load system, irrespective of their connections, whether the system is star connected or delta connected, the power is given by the relation:

**√3 VL IL Cosφ**

Its units are kilowatt (kW) or Watt (W).

**Apparent Power** is given as:

$$\sqrt{3} V_L I_L$$

The unit of apparent power is kilovolt-ampere (kVA) or volt-ampere (VA).

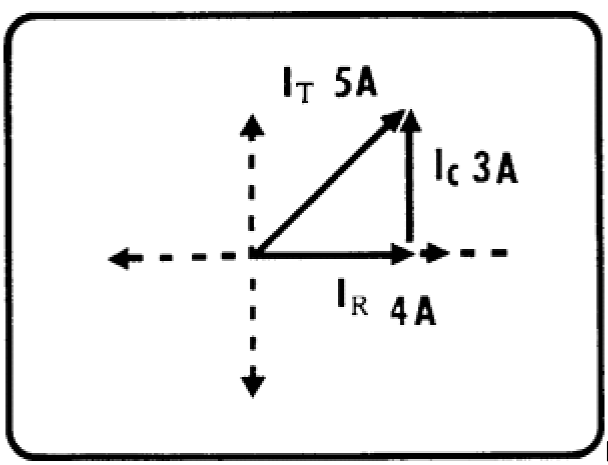
Similarly, **Reactive Power** is given by the equation:

$$\sqrt{3}\, V_L I_L \, \mathrm{Sin}\varphi$$

Its units are kilovolt-ampere reactive (kVAR) or volt-ampere reactive (VAR).

- Content/Topic 3: Phase angle (displacement)

The phase angle, theta can be recognized as the angle between the resistive current and the total current on the current phasor diagram in below  The value of IC=3 amperes, and IR =4 amperes. The phase angle is calculated as follow:



Phasor Diagram of Example Circuit for Calculating θ

$$\begin{aligned}\theta &= \arctan\left(\frac{I_C}{I_R}\right)\\ &= \arctan\left(\frac{3A}{4A}\right)\\ &= \arctan(0.75)\\ &= 37^\circ\end{aligned}$$

The phase angle is equal to the arctangent of the ratio of the capacitive current to the resistive current. Thus, the phase angle, θ, is equal to the arctangent of 3 amperes divided by 4 amperes which equals the arctangent 0.75. Using a calculator or trigonometric table, the angle whose arctangent is 0.75 is approximately 37 degrees. Thus, the phase angle of this parallel RC circuit is approximately 37 degrees.

- Content/Topic 4: Power measurement in three phase circuits

Following methods are available for measuring power in a 3-phase load.

**(a) Three Wattmeter Method**

In this method, three wattmeters are inserted one in each phase and the algebraic sum of their readings gives the total power consumed by the 3-phase load.

**(b) Two Wattmeter Method**

(i)This method gives true power in the 3-phase circuit without regard to balance or wave form provided in the case of Y-connected load. The neutral of the load is isolated from the neutral of the source of power. Or if there is a neutral connection, the neutral wire should not carry any current. This is possible only if the load is perfectly balanced and there are no harmonics present of triple frequency or any other multiples of that frequency.

(ii) This method can also be used for 3-phase, 4-wire system in which the neutral wire carries the neutral current. In this method, the current coils of the wattmeters are supplied from current transformers inserted in the principal line wires in order to get the correct magnitude and phase differences of the currents in the current coils of the wattmeter, because in the 3-phase, 4-wire system, the sum of the instantaneous currents in the principal line wires is not necessarily equal to zero as in 3-phase 3-wire system.

**(c) One Wattmeter Method**

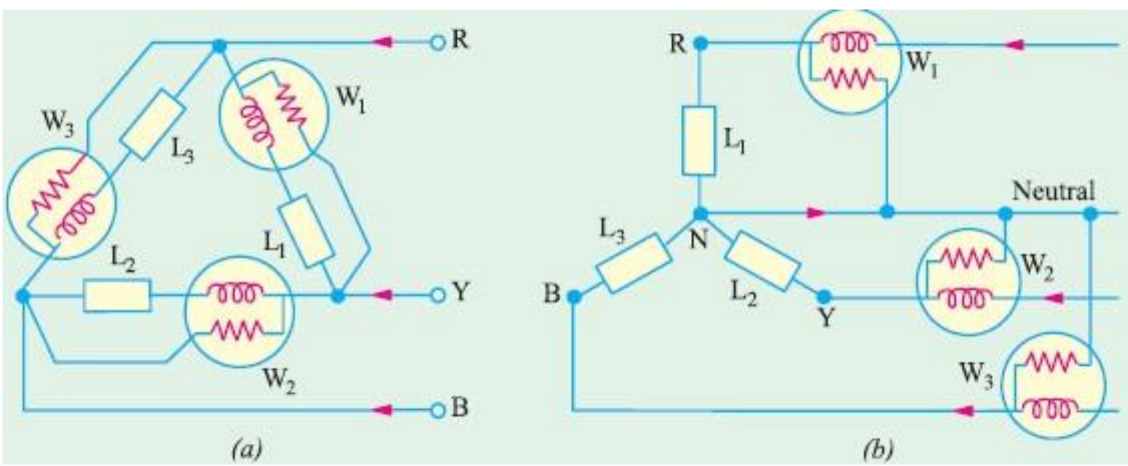
In this method, a single wattmeter is used to obtain the two readings which are obtained by two wattmeters by the two-wattmeter method. This method can, however, be used only when the load is balanced.

**Three Wattmeter Method**

A wattmeter consists of:
(i) a low resistance current coil which is inserted in series with the line carrying the current.
(ii) a high resistance pressure coil which is connected across the two points whose potential difference is to be measured.

A wattmeter shows a reading which is proportional to the product of the current through its current coil, the p.d. across its potential or pressure coil and cosine of the angle between this voltage and current.

As shown in Figures below in this method three wattmeters are inserted in each of the three phases of the load whether Δ-connected or Y-connected. The current coil of each wattmeter carries the current of one phase only and the pressure coil measures the phase-voltage of this phase.
Hence, each wattmeter measures the power in a single phase. The algebraic sum of the readings of three wattmeters must give the total power in the load.

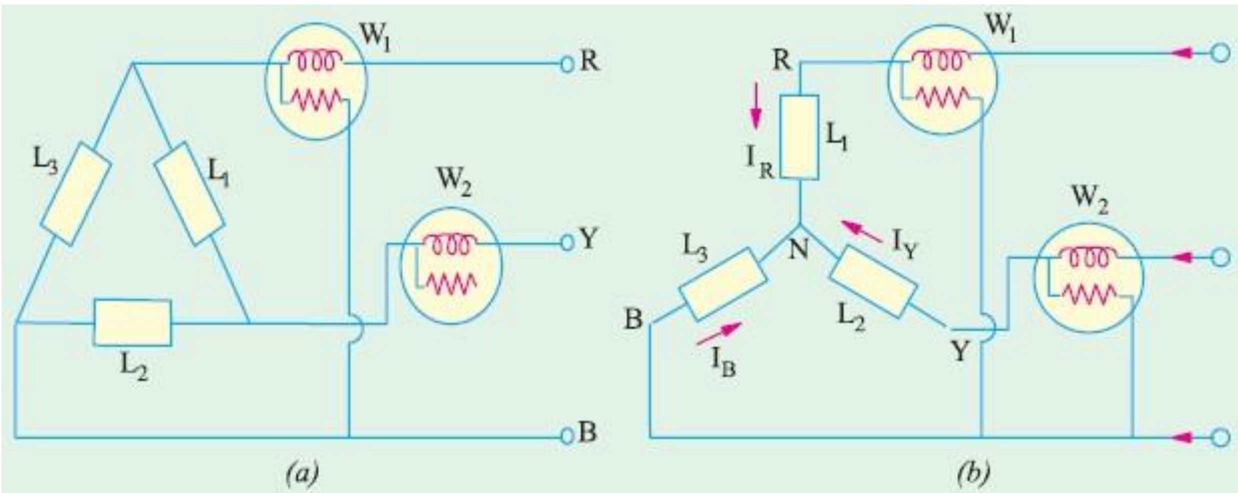



The difficulty with this method is that under ordinary conditions it is not generally feasible to break into the phases of a delta-connected load nor is it always possible, in the case of a Y-connected load, to get at the neutral point which is required for connections as shown in Fig. (b) below. However, it is not necessary to use three wattmeters to measure power, two wattmeters can be used for the purpose as shown below.

**Two Wattmeter Method-Balanced or Unbalanced Load**

The current coils of the two wattmeters are inserted in any two lines and the potential coil of each joined to the third line. It can be proved that the sum of the instantaneous powers indicated by $W_1$ and $W_2$ gives the instantaneous power absorbed by the three loads $L_1$, $L_2$ and $L_3$. A star-connected load is considered in the following discussion although it can be equally applied to Δ-connected loads because a Δ-connected load can always be replaced by an equivalent Y- connected load.

Now, before we consider the currents through and p.d. across each wattmeter, it may be pointed out that it is important to take the direction of the voltage through the circuit the same as that taken for the current when establishing the readings of the two wattmeters.




Instantaneous current through $W_1 = i_R$

p.d. across $W_1 = e_{RB} \qquad = e_R - e_B$

p.d. across power read by $W_1 = i_R (e_R - e_B)$

Instantaneous current through $W_2 = i_Y$

Instantaneous p.d. across $W_2 = e_{YB} = (e_Y - e_B)$

Instantaneous power read by $W_2 = i_Y (e_Y - e_B)$

$\therefore W_1 + W_2 = i_R (e_R - e_B) + i_Y (e_Y - e_B) = i_R e_R + i_Y e_Y - e_B (i_R + i_Y)$

Now, $i_R + i_Y + i_B = 0$ ... Kirchhoff's Current Law

$\therefore i_R + i_Y = -i_B$

or $W_1 + W_2 = i_R . e_R + i_Y . e_Y + i_B . e_B = p_1 + p_2 + p_3$

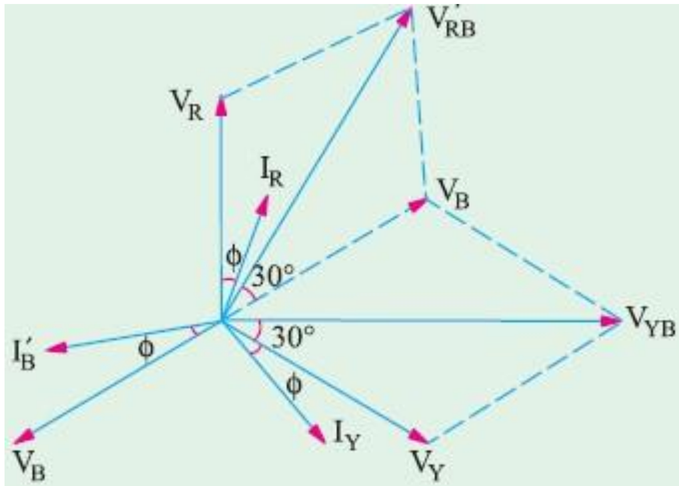
where $p_1$ is the power absorbed by load $L_1$, $p_2$ that absorbed by $L_2$ and $p_3$ that absorbed by $L_3$

$\therefore W_1 + W_2 =$ total power absorbed

**Two Wattmeter Method– Balanced Load**

If the load is balanced, then power factor of the load can also be found from the two wattmeter readings. The Y-connected load in

Fig. (b) above will be assumed inductive. The vector diagram for such a balanced Y-connected load is shown in Fig. below. We will now consider the problem in terms of r.m.s. values instead of instantaneous values.




Let $V_R$, $V_Y$ and $V_B$ be the r.m.s. values of the three phase voltages and $I_R$, $I_Y$ and $I_B$ the r.m.s. values of the currents. Since these voltages and currents are assumed sinusoidal, they can be represented by vectors, the currents lagging behind their respective phase voltages by $\phi$ .
Current through wattmeter $W_1$ [Fig.below] is = $I_R$.
P.D. across voltage coil of $W_1$ is
$V_{RB} = V_R - V_B$ ... vectorially
This $V_{RB}$ is foundby compounding $V_R$ and $V_B$ reversed. It is seen that phase difference between $V_{RB}$ and $I_R$ = (30° – $\phi$).

∴ Reading of W1 = $I_R V_{RB} \cos(30° - \phi)$
Similarly, as seen from Fig.below. Current through $W_2$ = $I_Y$
P.D. across $W_2 = V_{YB} = V_Y - V_B$ ... vectorially

Again, $V_{YB}$ is found by compounding $V_Y$ and $V_B$ reversed as shown in Fig. above. The angle between $I_Y$ and $V_{YB}$ is (30° + $\phi$). Reading of $W_2 = I_Y V_{YB} \cos(30° + \phi)$
Since load is balanced, $V_{RB} = V_{YB}$ = line voltage $V_L$; $I_Y = I_R$ = line current, $I_L$

∴ $W_1 = V_L I_L \cos(30° - \Phi)$ and $W_2 = V_L I_L \cos(30° + \phi)$
∴ $W_1 + W_2 = V_L I_L \cos(30° - \phi) + V_L I_L \cos(30° + \phi)$
$= V_L I_L [\cos 30° \cos\phi + \sin 30° \sin\phi + \cos 30° \cos\phi - \sin 30° \sin\phi]$
$= V_L I_L (2\cos 30° \cos\phi) = \sqrt{3}\, V_L I_L \cos\phi$ = total power in the 3-phase load

**Hence, the sum of the two wattmeter readings gives the total power consumption in the 3-phase load.**
It should be noted that phase sequence of RYB has been assumed in the above discussion. Reversal of phase sequence will interchange the readings of the two wattmeters.

**Variations in Wattmeter Readings**

It has been shown above that for a lagging power factor
$W_1 = V_L I_L \cos(30° - \phi)$ and $W_2 = V_L I_L \cos(30° + \phi)$

From this it is clear that individual readings of the wattmeters not only depend on the load but upon its power factor also. We will consider the following cases:
(a) When $\phi = 0$ i.e. power factor is unity (i.e. resistive load) then,
$W_1 = W_2 = V_L I_L \cos 30°$
Both wattmeters indicate equal and positive i.e. up-scale readings.

(b) When $\phi = 60°$ i.e. power factor = 0.5 (lagging)
Then $W_2 = V_L I_L \cos(30° + 60°) = 0$. Hence, the power is measured by $W_1$ alone.

(c) When $90° > \phi > 60°$ i.e. 0.5 > p.f. > 0, then $W_1$ is still positive but reading of $W_2$ is reversed because the phase angle between the current and voltage is more than 90°. **For getting the total power, the reading of $W_2$ is to be subtracted from that of $W_1$.**
Under this condition, $W_2$ will read 'down scale' i.e. backwards. Hence, to obtain a reading on $W_2$ it is necessary to reverse either its pressure coil or current coil, usually the

**All readings taken after reversal of pressure coil are to be taken as negative.**

(d) When $\phi = 90°$ (i.e. pure inductive or capacitive load), then
$W_1 = V_L I_L \cos(30° - 90°) = V_L I_L \sin 30°$;
$W_2 = V_L I_L \cos(30° + 90°) = -V_L I_L \sin 30°$

As seen, the two readings are equal but of opposite sign.
$\therefore W_1 + W_2 = 0$

The above facts have been summarised in the above table for a lagging power factor.

| $\phi$ | $0°$ | $60°$ | $90°$ |
|---|---|---|---|
| $\cos\phi$ | 1 | 0.5 | 0 |
| $W_1$ | +ve | +ve | +ve |
| $W_2$ | +ve $W_1 = W_2$ | 0 | − ve $W_1 = W_2$ |

**Power Factor–Balanced Load**

In case the load is balanced (and currents and voltages are sinusoidal) and for a lagging power factor:

$W_1 + W_2 = V_L I_L \cos(30° - \phi) + V_L I_L \cos(30° + \phi) = \sqrt{3}\, V_L I_L \cos\phi$ ... (i)

Similarly $W_1 - W_2 = V_L I_L \cos(30° - \phi) - V_L I_L \cos(30° + \phi)$

$= V_L I_L (2 \times \sin\phi \times 1/2) = V_L I_L \sin\phi$ ... (ii)

Dividing (ii) by (i), we have

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} \quad \text{... (iii)}$$

**Balanced Load – leading power factor**

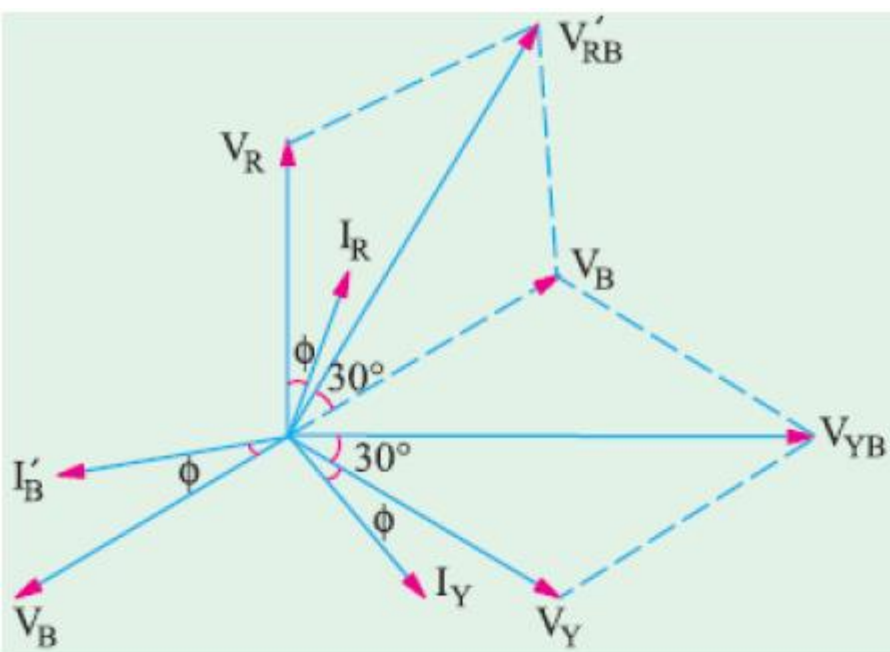
In this case, as seen from Fig.below

and $W_2 = V_L I_L \cos(30 - \phi)$

$\therefore W_1 + W_2 = 3 V_L I_L \cos\phi$ →as found above

$W_1 - W_2 = -V_L I_L \sin\phi$

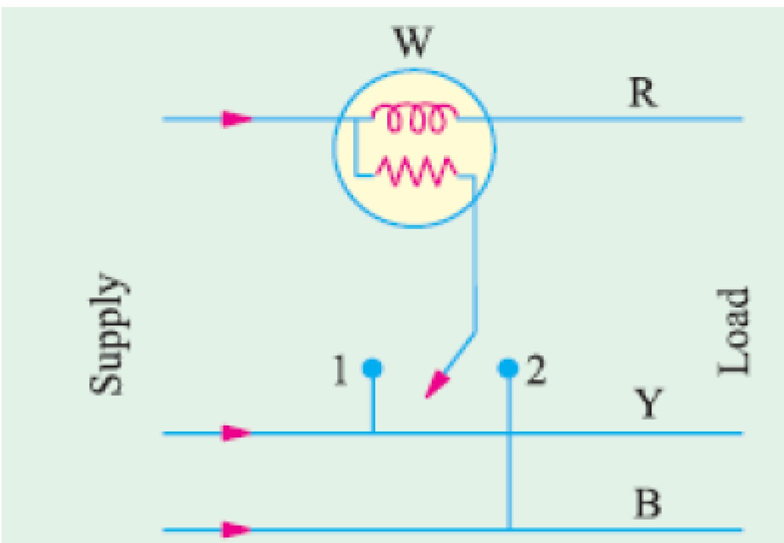
$$\tan\phi = -\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$



**One Wattmeter Method**

In this case, it is possible to apply two-wattmeter method by means of one wattmeter without breaking the circuit. The current coil is connected in any one line and the pressure coil is connected alternately between this and the other two lines (Fig.below). The two readings so obtained, for a balanced load, correspond to those obtained by normal two wattmeter method. It should be kept in mind that this method is not of as much universal application as the two wattmeter method because it is restricted to fairly balanced loads only.

However, it may be conveniently applied, for instance, when it is desired to find the power input to a factory motor in order to check the load upon the motor.
It may be pointed out here that the two wattmeters used in the two-wattmeter method are usually combined into a single instrument in the case of switchboard wattmeter which is then known as a polyphase wattmeter. The combination is affected by arranging the two sets of coils in such a way as to operate on a single moving system resulting in an indication of the total power on the scale.



**Examples:**

1.Three similar coils, each having a resistance of 20 ohms and an inductance of 0.05 H are connected in (i) star (ii) mesh to a 3-phase, 50-Hz supply with 400-V between lines. Calculate the total power absorbed and the line current in each case. Draw the vector diagram of current and voltages in each case.

**Solution.** $X_L \quad 2 \quad 50 \quad 0.05 \quad 15$ , $Z_{ph} \quad \sqrt{15^2 \quad 20^2} \quad$ **25Ω**

**(*i*) Star Connection.** [Fig. 19.29 (*a*)]

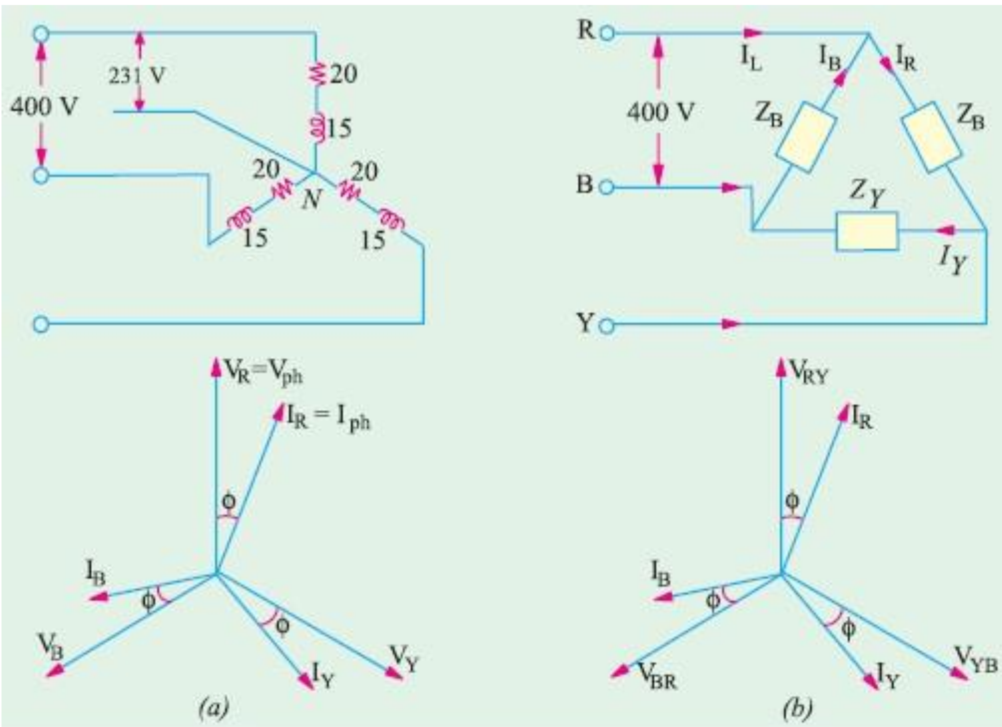



Fig. 19.29

$$V_{ph} = 400 / \sqrt{3} = 231 \text{ V}; \; I_{ph} = V_{ph} / Z_{ph} = 231/25 = \mathbf{9.24 \Omega}$$

$$I_L \quad I_{ph} \quad \mathbf{9.24\ A;}\ P \quad \sqrt{3} \quad 400 \quad 9.24 \quad (20/25) \quad \mathbf{5120\ W}$$

**(*ii*) Delta Connection** [Fig. 19.29 (*b*)]

$$V_{ph} \quad V_L \quad 400\,V; I_{ph} \quad 400/25 \quad 16\,A; I_L \quad \sqrt{3} I_{ph} \quad \sqrt{3} \quad 16 \quad \mathbf{27.7\ A}$$

$$P = \sqrt{3} \times 400 \times 27.7 \times (20/25) = \mathbf{15{,}360\ W}$$

**Note.** It may be noted that line current as well as power are three times the star values.

2. A Δ -connected balanced 3-phase load is supplied from a 3-phase, 400-V supply. The line current is 20 A and the power taken by the load is 10,000 W. Find (i) impedance in each branch (ii) the line current, power factor and power consumed if the same load is connected in star.

**Solution. (*i*) Delta Connection.**

$$V_{ph} = V_L = 400\ V;\ I_L = 20\ A;\ I_{ph} = 20/\sqrt{3}\ \text{A}$$

(*i*) $\therefore \quad Z_{ph} \quad \frac{400}{20/\sqrt{3}} \quad 20\sqrt{3} \quad \mathbf{34.64\ \Omega}$

Now $P = \sqrt{3}\ V_L I_L \cos\phi \quad \therefore \quad \cos\phi = 10{,}000/\sqrt{3} \times 400 \times 20 =$ **0.7217**

**(*ii*) Star Connection**

$$V_{ph} \quad \frac{400}{\sqrt{3}},\ I_{ph} \quad \frac{400/\sqrt{3}}{\sqrt{3}} \quad \frac{20}{3} A,\ I_L \quad I_{ph} \quad \frac{20}{3}\ \text{A}$$

Power factor remains the same since impedance is the same.

Power consumed $= \sqrt{3} \times 400 \times (20/3) + 0.7217 =$ **3,330 W**

**Note.** The power consumed is 1/3 of its value of Δ-connection.

3. Three similar resistors are connected in star across 400-V, 3-phase lines. The line current is 5 A. Calculate the value of each resistor. To what value should the line voltage be changed to obtain the same line current with the resistors delta-connected.

**Solution. Star Connection**

$I_L = I_{ph} = 5\ A;\ V_{ph} = 400/\sqrt{3} = 231\text{V} \ \therefore R_{ph} = 231/5 =$ **46.2 Ω**

**Delta Connection**

$I_L = 5$ A... (given); $I_{ph} = 5/\sqrt{3}$ A; $R_{ph} = \mathbf{46.2\ \Omega}$

... found above

$V_{ph} \quad I_{ph} \quad R_{ph} \quad 5 \quad 46.2/\sqrt{3} \quad \mathbf{133.3\ V}$

**Note.** Voltage needed is 1/3rd the star value.

4.Three equal star-connected inductors take 8 kW at a power factor 0.8 when connected across a 460 V, 3-phase, 3-phase, 3-wire supply. Find the circuit constants of the load per phase.

**Solution.** $P = \sqrt{3} V_L I_L \cos\phi$ or

$8000 = \sqrt{3} \times 460 \times I_L \times 0.8$

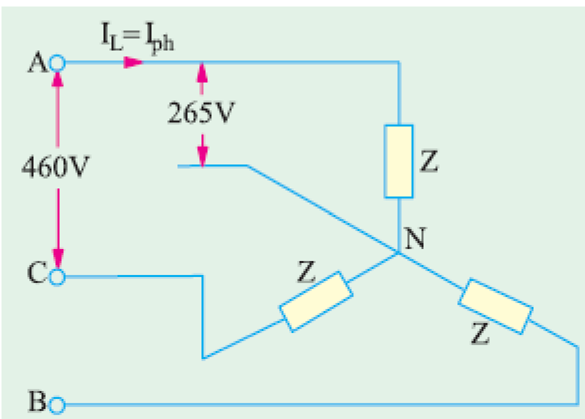
$\therefore \quad I_L = 12.55$ A $\therefore I_{ph} = 12.55\ A;$

$V_{ph} = V_L / \sqrt{3} \quad 460/\sqrt{3} \quad 265V$

$I_{ph} = V_{ph} / Z_{ph}; \therefore Z_{ph} = V_{ph} / I_{ph} = 265/12.55 = 21.1\Omega$

$R_{ph} = Z_{ph} \cos\phi = 21.1 \times 0.8 = 16.9\ \Omega$

$X_{ph} = Z_{ph} \sin\phi = 21.1 \times 0.6 = 12.66\,\Omega$




5.Given a balanced 3−ϕ, 3-wire system with Y-connected load for which line voltage is 230 V and impedance of each phase is (6 + J8) ohm. Find the line current and power absorbed by each phase.

**Solution.** $Z_{ph} = \sqrt{6^2 + 8^2} = 10\Omega;\ V_{ph} = V_L / \sqrt{3} = 230/\sqrt{3} = 133\text{V}$

$\cos\phi = R/Z = 6/10 = 0.6;\ I_{ph} = V_{ph} / Z_{ph} = 133/10 = 13.3\text{ A}$

$\therefore \quad I_L = I_{ph} =$ **13.3 A**

Power absorbed by each phase = $I_{ph}^{\ 2} R_{ph} = 13.3^2 \times 6 =$ **1061 W**

6. A balanced star-connected load of (8 + j6) Ω per phase is connected to a balanced 3-phase 400-V supply. Find the line current, power factor, power and total volt-amperes.

**Solution.** $Z_{ph} = \sqrt{8^2 + 6^2} = 10\,\Omega$

$V_{ph} = 400/\sqrt{3} = 231\text{ V}$

$I_{ph} = V_{ph}/Z_{ph} = 231/10 = 23.1\text{ A}$

(*i*) $I_L = I_{ph} =$ **23.1 A**

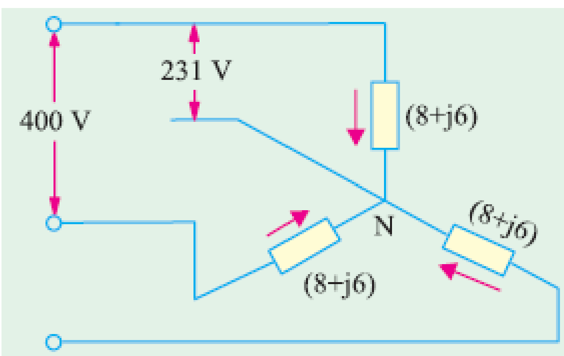
(*ii*) p.f. $= \cos\phi = R_{ph}/Z_{ph} = 8/10 =$ **0.8 (lag)**

7. (*iii*) Power $P$ $\sqrt{3}V_L I_L \cos$

$= \sqrt{3} \times 400 \times 23.1 \times 0.8 =$ **12,800 W** [Also, $P = 3I_{ph}^2 R_{ph} = 3(23.1)^2 \times 8 = 12{,}800$ *W*]

9. (*iv*) Total volt-amperes, $S = \sqrt{3}\, V_L I_L = \sqrt{3} \times 400 \times 23.1 =$ **16,000 VA**

10.




7. When the three identical star-connected coils are supplied with 440 V, 50 Hz, 3- ϕ supply, the 1- ϕ wattmeter whose current coil is connected in line R and pressure coil across the phase R and neutral reads 6 kW and the ammeter connected in R-phase reads 30 Amp.
Assuming RYB phase sequence find:
(i) resistance and reactance of the coil,
(ii) the power factor, of the load
(iii) reactive power of 3- ϕ load.

**Solution.** $V_{ph} = 440/\sqrt{3} = 254$ V; $I_{ph} = 30$A (Fig. 19.17.)

Now, $V_{ph} I_{ph} \cos\phi = 6000$ ; $254 \times 30 \times \cos\phi = 6000$

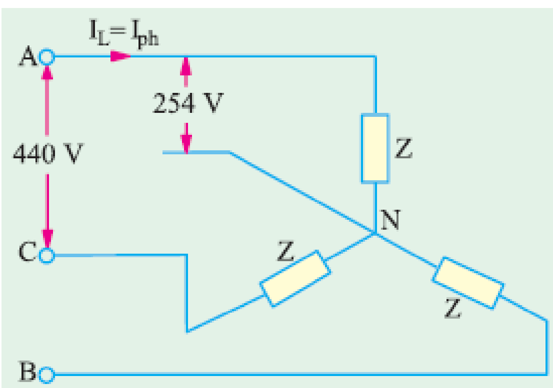
$\therefore \cos\phi = 0.787$ ; $\phi = 38.06°$ and $\sin\phi = 0.616°$; $Z_{ph} = V_{ph} / I_{ph} = 254/30 = 8.47\ \Omega$

(*i*) Coil resistance $R = Z_{ph} \cos\phi = 8.47 \times 0.787 = 6.66\ \Omega$

$X_L = Z_{ph} \sin\phi = 8.47 \times 0.616 =$ **5.22** $\Omega$

(*ii*) p.f. = cos $\phi$ = **0.787 (lag)**

(*iii*) Reactive power = $\sqrt{3}\ V_L I_L \sin \quad \sqrt{3} \quad 440 \quad 30 \quad 0.616 \quad 14{,}083\text{VA}$ **14.083kVA**




8. Calculate the active and reactive components in each phase of Y-connected 10,000 V, 3-phase alternator supplying 5,000 kW at 0.8 p.f. If the total current remains the same when the load p.f is raised to 0.9, find the new output.

**Solution.** $5000 \times 10^3 = \sqrt{3} \quad 10{,}000 \quad I_L \quad 0.8; I_L \quad I_{ph} \quad 361\text{A}$

active component= $I_L \cos\phi = 361 \times 0.8 =$ **288.8 A**

reactive component= $I_L \sin\phi = 361 \times 0.6 =$ **216.6 A**

New power $P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 10^4 \times 361 \times 0.9 =$ **5,625 kW**

[or new power = 5000 × 0.9/0.8 = 5625 kW]

9. A star-connected alternator supplies a delta connected load. The impedance of the load branch is (8 + j6) ohm/phase. The line voltage is 230 V. Determine
(a) current in the load branch,
(b) power consumed by the load,
(c) power factor of load, (d) reactive power of the load.

**Solution.** Considering the Δ-connected load, we have $Z_{ph} = \sqrt{8^2 + 6^2} = 10\,\Omega$; $V_{ph} = V_L =$ **230 V**

(*a*) $I_{ph} = V_{ph}/Z_{ph} = 230/10 = 23$ A

(*b*) $I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 23 = 39.8$ A; $P \quad \sqrt{3}\, V_L I_L \cos \quad \sqrt{3} \quad 230 \quad 39.8 \quad 0.8$ **12,684 W**

(*c*) p.f. $\cos\phi = R \mid Z = 8/10 =$ **0.8 (lag)**

(*d*) Reactive power $Q = \sqrt{3}\; V_L I_L \sin \quad \sqrt{3} \quad 230 \quad 39.8 \quad 0.6$ **9513 W**

10. A balanced delta connected load, consisting of three coils, draws $10\sqrt{3}$ A at 0.5 power factor from 100 V, 3-phase supply. If the coils are re-connected in star across the same supply, find the line current and total power consumed.

**Solution. Delta Connection**

$V_{ph} \quad V_L \quad 100V;\, I_L \quad 10\sqrt{3}\, A;\, I_{ph} \quad 10\sqrt{3}/\sqrt{3}$ **10 A**

$Z_{ph} = V_{ph}/I_{ph} = 100/10 = 10\,\Omega$; $\cos\phi = 0.5$ (given); $\sin\phi = 0.866$

$\therefore\; R_{ph} \quad Z_{ph}\cos \quad 10 \quad 0.5 \quad 5 \;;\, X_{ph} \quad Z_{ph}\sin \quad 10 \quad 0.866$ **8.66**

Incidentally, total power consumed = $\sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 100 \times 10\sqrt{3} \times 0.5 = 1500$ W

**Star Connection**

$V_{ph} \quad V_L/\sqrt{3} \quad 100/\sqrt{3};\, Z_{ph} \quad 10 \quad ;I_{ph} \quad V_{ph}/Z_{ph} \quad 100/\sqrt{3} \quad 10 \quad 10\sqrt{3}$ A

Total power absorbed = $\sqrt{3} \times 100 \times (10\sqrt{3}) \times 0.5 =$ **500 W**

It would be noted that the line current as well as the power absorbed are one-third of that in the delta connection.

11. Three 100Ω non-inductive resistances are connected in (a) star (b) delta across a 400-V, 50-Hz, 3-phase mains. Calculate the power taken from the supply system in each case. In the event of one of the three resistances getting open-circuited, what would be the value of total power taken from the mains in each of the two cases ?

**Solution. (*i*) Star Connection [Fig. 19.30 (*a*)]**

$V_{ph}$ $400/\sqrt{3}\ V$

$P = \sqrt{3}\ V_L I_L\ \cos\phi$

$= \sqrt{3} \times 400 \times 4 \times 1/\sqrt{3} =$ **1600 W**

**(*ii*) Delta Connection Fig. 19.30 (*b*)**

$V_{ph} = 400\text{V};\ R_{ph} = 100\,\Omega$

$I_{ph} = 400/100 = 4\text{ A}$

$I_L = 4 \times \sqrt{3}$ A

$P = \sqrt{3} \times 400 \times 4 \times \sqrt{3} \times 1$ = **4800 W**

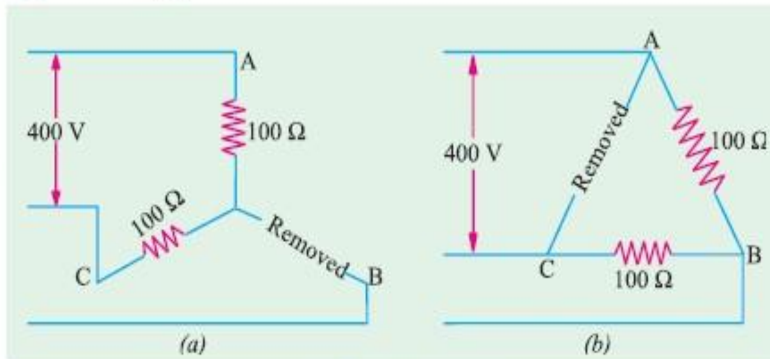



**Fig. 19.30**

When one of the resistors is disconnected

(i) Star Connection [Fig. 19.30 (a)]
The circuit no longer remains a 3-phase circuit but consists of two 100 Ω resistors in series across a 400-V supply. Current in lines A and C is = 400/200 = 2 A
Power absorbed in both = 400 × 2 = 800 W
Hence, by disconnecting one resistor, the power consumption is reduced by half.

(ii) Delta Connection [Fig. 19.30 (b)]
In this case, currents in A and C remain as usual 120° out of phase with each other.
Current in each phase = 400/100 = 4 A
Power consumption in both = 2 × 42 × 100 = 3200 W
(or P = 2 × 4 × 400 = 3200 W)

12. The load connected to a 3-phase supply comprises three similar coils connected in star. The line currents are 25 A and the kVA and kW inputs are 20 and 11 respectively.
Find the line and phase voltages, the kVAR input and the resistance and reactance of each coil.
If the coils are now connected in delta to the same three-phase supply, calculate the line currents and the power taken.

**Solution. Star Connection**

$\cos\phi$ k = W/kVA = 11/20  $I_L = 25$ A  $P = 11$ kW = 11,000 W

Now $P = \sqrt{3}\ V_L I_L\ \cos\phi$ $\therefore$ $11{,}000 = \sqrt{3} \times V_L \times 25 \times 11/20$

$\therefore\ V_L$ **462 V**;  $V_{ph} = 462/\sqrt{3} =$ **267 V**

kVAR $\sqrt{\text{kVA}^2 \quad \text{kW}^2}$ $\sqrt{20^2 \quad 11^2}$ **16.7**; $Z_{ph}$ $267/2$ $10.68$

$\therefore\ R_{ph} = Z_{ph} \times \cos\phi = 10.68 \times 11/20 =$ **5.87 Ω**

$\therefore\ X_{ph}$ $Z_{ph}$ sin 10.68 0.838 **897Ω**

**Delta Connection**

$V_{ph} = V_L = 462\ V$ and $Z_{ph} = 10.68\,\Omega$

$\therefore\ I_{ph}$ $462/10.68$ A, $I_L$ $\sqrt{3}$ $462/10.68$ 75 A

$P = \sqrt{3} \times 462 \times 75 \times 11/20 =$ **33,000 W**

13. A 3-phase, star-connected system with 230 V between each phase and neutral has resistances of 4, 5 and 6 Ω respectively in the three phases. Estimate the current flowing in each phase and the neutral current. Find the total power absorbed.

**Solution.** Here, $V_{ph} = 230$ V [Fig. 19.32 (*a*)]

Current in 4-Ω resistor = 230/4 = 57.5 A

Current in 5-Ω resistor = 230/5 = 46 A

Current in 6-Ω resistor = 230/6 = 38.3 A

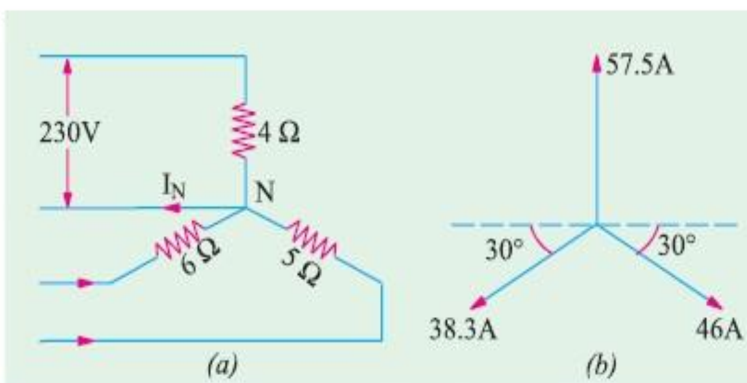



**Fig. 19.32**

These currents are mutually displaced by 120°. The neutral current $I_N$ is the vector sum* of these three currents. $I_N$ can be obtained by splitting up these three phase currents into their *X*-components and *Y*-components and then by combining them together, in diagram 19.32 (*b*).

*X*-component = 46 cos 30° – 38.3 cos 30° = **6.64 A**

*Y*-component = 57.5 – 46 sin 30° – 38.3 sin 30° = 15.3 A  $I_N \quad \sqrt{6.64^2 \quad 15.3^2} \quad \mathbf{16.71\,A}$

The power absorbed = 230 (57.5 + 46 + 38.3) = **32.610 W**

14. A 3-phase, 37.3 kW, 440-V, 50-Hz induction motor operates on full load with an efficiency of 89% and at a power factor of 0.85 lagging. Calculate the total kVA rating of capacitors required to raise the full-load power factor at 0.95 lagging. What will be the capacitance per phase if the capacitors are (a) delta-connected and (b) star-connected ?

**Solution.** It is helpful to approach such problems from the 'power triangle' rather than from vector diagram viewpoint.

Motor power input $P$ = 37.3/0.89 = 41.191 kW

**Power Factor 0.85 (lag)**

$$\cos\phi_1 = 0.85 : \phi_1 = \cos^{-1}(0.85) = 31.8°; \tan\phi_1 = \tan 31.8° = 0.62$$

$$\text{Motor kVAR}_1 = P \tan\phi_1 = 41.91 \times 0.62 = 25.98$$

**Power Factor 0.95 (lag)**

Motor power input $P$ = 41.91 kW ... as before

It is the same as before because capacitors are loss-free *i.e.* they do not absorb any power.

$$\cos\phi_2 = 0.95 \quad \therefore \phi_2 = 18.2°; \ \tan 18.2° = 0.3288$$

$$\text{Motor kVAR}_2 = P \tan\phi_2 = 41.91 \times 0.3288 = 13.79$$

The difference in the values of kVAR is due to the capacitors which supply ***leading*** kVAR to partially neutralize the ***lagging*** kVAR of the motor.

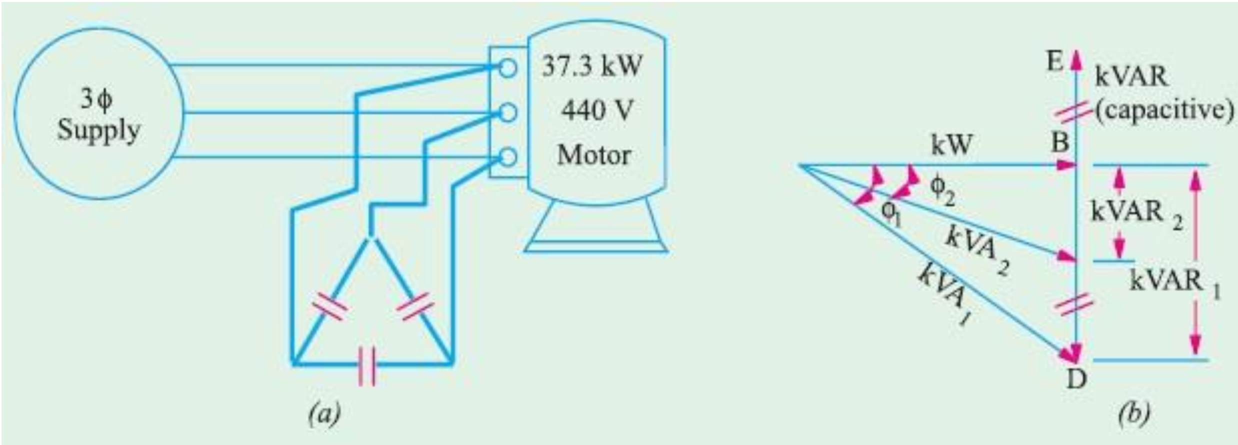



**Fig. 19.35**

∴ leading kVAR supplied by capacitors is

$= kVAR_1 - kVAR_2 = 25.98 - 13.79 =$ **12.19** ... *CD* in Fing. 19.35 (*b*)

Since capacitors are loss-free, their kVAR is the same as kVA

∴ kVA/capacitor = 12.19/3 = 4.063 ∴ VAR/capacitor = 4,063

**(*a*)** In Δ-connection, voltage across each capacitor is 440 V

Current drawn by each capacitor $I_C = 4063/440 = 9.23$ A

Now, $$I_c = \frac{V}{X_c} = \frac{V}{1/\omega C} = \omega VC$$

∴ $$C = I_c/\omega V = 9.23/2\pi \times 50 \times 440 = 66.8 \times 10^{-6}\text{F} = \mathbf{66.8\ \mu F}$$

**(*b*)** In star connection, voltage across each capacitor is = $440/\sqrt{3}$ volt

Current drawn by each capacitor, $I_c \quad \dfrac{4063}{440/\sqrt{3}} \quad 16.0$ A

$$I_c \quad \frac{V}{X_c} \quad VC \quad \text{or} \quad 16 = \frac{440}{\sqrt{3}} \times 2\pi \times 50 \times C$$

∴ $$C = 200.4 \times 10^{-6}\text{F} = \mathbf{200.4\ \mu F}$$

**Note.** Star value is three times the delta value.

15. Three impedance coils, each having a resistance of 20Ω and a reactance of 15Ω , are connected in star to a 400-V, 3- ϕ , 50-Hz supply. Calculate:
(i) the line current
(ii) power supplied
(iii) the power factor.
If three capacitors, each of the same capacitance, are connected in delta to the same supply so as to form parallel circuit with the above impedance coils, calculate the capacitance of each capacitor to obtain a resultant power factor of 0.95 lagging.

**Solution.** $V_{ph} \quad 100/\sqrt{3}V, Z_{ph} \quad \sqrt{20^2 \quad 15^2} \quad 25$

$\cos \ _1 \quad R_{ph}/Z_{ph} \quad 20/25 \quad 0.8 \text{lag}; \ _1 \quad 0.6 \text{ lag}$

where $\phi_1$ is the power factor angle of the coils.

**When capacitors are not connected**

(*i*) $I_{ph} = 400/25 \times \sqrt{3} = 9.24 \text{ A} \therefore I_L =$ **9.24 A**

(*ii*) $P = \sqrt{3}\ V_L I_L \cos\phi_1 = \sqrt{3} \times 400 \times 9.24 \times 0.8 =$ **5.120 W**

(*iii*) Power factor = **0.8 (lag)**

$\therefore$ Motor $VAR_1 = \sqrt{3} V_L I_L \sin\phi_1 = \sqrt{3} \times 400 \times 9.24 \times 0.6 = \mathbf{3,840}$

**When capacitors are connected**

Power factor, $\cos\phi_2 = 0.95, \phi_2 = 18.2°$ ; $\tan 18.2° = 0.3288$

Since capacitors themselves do not absorb any power, power remains the same *i.e.* 5,120 W even whin capacitors are connected. The only thing that changes is the VAR.

Now $VAR_2 = P \tan\phi_2 = 5120 \times 0.3288 = 1684$

*Leading* VAR supplied by the three capacitors is

$= VAR_1 - VAR_2 = 3840 - 1684 = 2156$ BD or CE in Fig 19.37 (*b*)

VAR/ Capacitor = 2156/3 = 719

For delta connection, voltage across each capacitor is 400 V $\therefore I_c = 719/400 = 1.798$ A

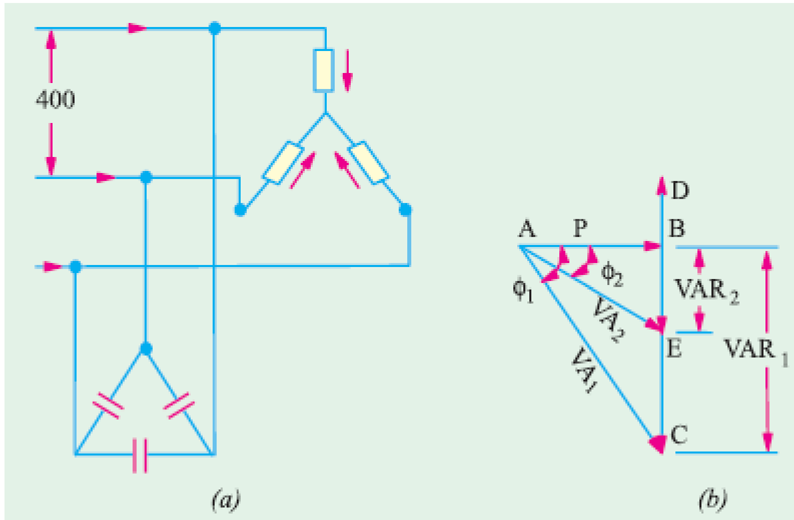
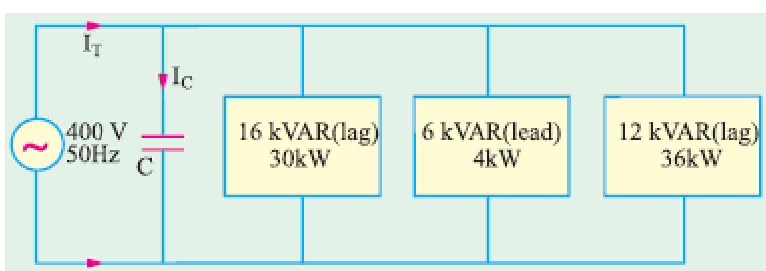



**Fig. 19.37**

Also $I_c = \dfrac{V}{I/\omega C} = \omega VC \therefore C = 1.798/\pi \times 50 \times 400 = 14.32 \times 10^{-6}\,\text{F} = \mathbf{14.32\ \mu F}$

16 For the power distribution system shown in Fig.below, find
(a) total apparent power, power factor and magnitude of the total current IT without the capacitor in the system
(b) the capacitive kVARs that must be supplied by C to raise the power factor of the system to unity ;
(c) the capacitance C necessary to achieve the power correction in part (b) above
(d) total apparent power and supply current IT after the power factor correction.



**Solution.** (*a*) We will take the inductive *i.e.* lagging kVARs as negative and capacitive *i.e.* leading kVARs as positive.

Total $Q = -16 + 6 - 12 = -22$ kVAR (lag); Total $P = 30 + 4 + 36 = 70$ kW

$\therefore$ apparent power $S = \sqrt{(-22)^2 + 70^2}$ = **73.4 kVA;** p.f. = cos $\phi$ = P/S = 70/73.4 = **0.95**

$$S = VI_T \text{ or } 73.4 \times 10^3 = 400 \times I_T \therefore I_T = \mathbf{183.5\ A}$$

(*b*) Since total lagging kVARs are **– 22**, hence, for making the power factor unity, 22 leading kVARs must be supplied by the capacitor to neutralize them. In that case, total $Q = 0$ and $S = P$ and p.f. is unity.

(*c*) If $I_C$ is the current drawn by the capacitor, then $22 \times 10^3 = 400 \times I_C$

Now, $I_C = V/X_C = V\omega C$

$= 400 \times 2\pi \times 50 \times C$

$\therefore 20 \quad 10^3 \quad 400 \quad (400 \quad 2 \quad 50 \quad C)$

$\therefore C =$ **483 μF**

(*d*) Since $Q = 0$,

hence, $S = \sqrt{10^2 + 70^2}$ = **70 kVA**

Now, $VI_T = 70 \times 10^3$;

$I_T = 70 \times 10^3 / 400 = 175$ A.

It would be seen that after the power correction, lesser amount of current is required to deliver the same amount of real power to the system.

## LO 4.3 – Apply power factor improvement techniques

- Content/Topic 1: Overview of power factor

Power factor is an expression of energy efficiency. It is usually expressed as a percentage—and the lower the percentage, the less efficient power usage is.

Power factor (PF) is the ratio of working power, measured in kilowatts (kW), to apparent power, measured in kilovolt amperes (kVA).

Apparent power, also known as demand, is the measure of the amount of power used to run machinery and equipment during a certain period. It is found by multiplying (kVA = V x A). The result is expressed as kVA units

The heating and lighting loads supplied from 3-phase supply have power factors, ranging from 0.95 to unity. But motor loads have usually low lagging power factors, ranging from 0.5 to 0.9. Single-phase motors may have as low power factor as 0.4 and electric wedding units have even lower power factors of 0.2 or 0.3.

The power factor is given by $\cos \quad \dfrac{\text{kW}}{\text{kVA}} \quad \text{or} \quad \text{kVA} \quad \dfrac{\text{kW}}{\cos}$

In the case of single-phase supply, $\text{kVA} = \dfrac{VI}{1000}$ or $I = \dfrac{1000\ kVA}{V} \qquad \therefore \quad I \propto \text{kVA}$

In the case of 3-phase supply $\text{kVA} = \dfrac{\sqrt{3}\, V_L I_L}{1000}$ or $I_L = \dfrac{1000\ \text{kVA}}{\sqrt{3} \times V_L} \qquad \therefore \quad I \propto \text{kVA}$

In each case, the kVA is directly proportional to current. The chief disadvantage of a low p.f. is that the current required for a given power, is very high. This fact leads to the following undesirable results.

**(i) Large kVA for given amount of power**

All electric machinery, like alternators, transformers, switchgears and cables are limited in their current-carrying capacity by the permissible temperature rise, which is proportional to $I^2$. Hence, they may all be fully loaded with respect to their rated kVA, without delivering their full power. Obviously, it is possible for an existing plant of a given kVA rating to increase its earning capacity
(which is proportional to the power supplied in kW) if the overall
power factor is improved i.e. raised.

**(ii) Poor voltage regulation**

When a load, having allow lagging power factor, is switched on, there is a large voltage drop in the supply voltage because of the increased voltage drop in the supply lines and transformers. This drop in voltage adversely affects the starting torques of motors and necessitates expensive voltage stabilizing equipment for keeping the consumer's voltage fluctuations within the statutory limits. Moreover, due to this excessive drop, heaters take longer time to provide the desired heat energy, fluorescent lights flicker and incandescent lamps are not as bright as they should be. Hence, all supply undertakings try to encourage consumers to have a high power factor.

- Content/Topic 2: Methods of improving power factor

The following equipment is generally used for improving or correcting the power factor :

**(i) Synchronous Motors (or capacitors)**

These machines draw leading kVAR when they are over-excited and, especially, when they are running idle. They are employed for correcting the power factor in bulk and have the special advantage that the amount of correction can be varied by changing their excitation.

**(ii) Static Capacitors**

They are installed to improve the power factor of a group of a.c. motors and are practically loss-free (i.e. they draw a current leading in phase by 90°). Since their capacitances are not variable, they tend to over-compensate on light loads, unless arrangements for automatic switching off the capacitor bank are made.

**(iii) Phase Advancers**

They are fitted with individual machines.

However, it may be noted that the economical degree of correction to be applied in each case, depends upon the tariff arrangement between the consumers and the supply authorities.

- Content/Topic 3: Benefits of improving power factor

Following are the merits and benefits of improvedPower factor;

1. Increase in efficiency of system and devices
2. Low Voltage Drop
3. Reduction in size of a conductor and cable which reduces cost of the Cooper
4. An Increase in available power
5. Line Losses (Copper Losses) I2R is reduced
6. Appropriate Size of Electrical Machines (Transformer, Generators etc)
7. Eliminate the penalty of low power factor from the Electric Supply Company
8. Low kWh (Kilo Watt per hour)
9. Saving in the power bill
10. Better usage of power system, lines and generators etc
11. Saving in energy as well as rating and the cost of the electrical devices and equipment is reduced

- Content/Topic 4: Drawbacks of low power factor

1. Large kVA rating and size of Electrical equipments
2. Greater conductor size and cost of transmission line
3. High Transmission loss hence poor efficiency
4. Poor Voltage regulation
5. Penalty from power supply company

**Causes of Low Power Factor**

The main causes for the low power factor are because of the inductive load. In the case of inductive load, the current lag behind the voltage. Therefore power factor becomes lagging nature.
Following are the main Causes of Low power factor.

**1. Inductive Load**

90% of the industrial load consists of induction motors (1-φ and 3-φ). Such machines draw magnetizing current to set up the magnetic field for its proper working and hence work at a low power factor
For induction motors, the power factor is usually extremely low (0.2 - 0.3) at light loading conditions and rises to 0.8 to 0.9 at full load.
The current drawn by inductive loads are lagging which results in poor power factor.
Other inductive machines such as transformers, generators, arc lamps, electric heating furnaces, electric discharge lamps, etc also work at low power factor.

**2. Variations in power system loading**

A modern power system is the interconnected power system. So according to the different session and time, the load on the power system is not always constant. It varies during the entire day. It is more during the morning and evening (Peak load) but less during the rest period of time.

When the system is loaded lightly, voltage increases which increase the magnetization current demand of the machines. This results in Poor power factor.

**3. Harmonic Current**

The presence of harmonic current in the system also reduces the power factor of the system.

In some cases, due to improper wiring or electrical accidents in which 3- phase power imbalance occurs. This results in low power factor too.

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