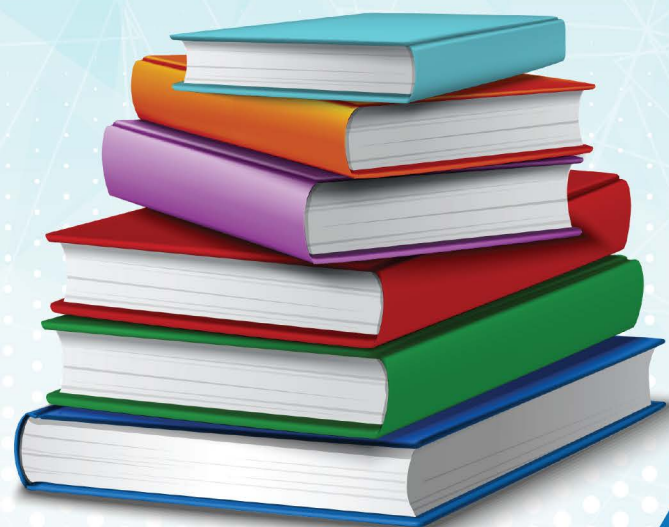




TVET LEVEL II



BASIC EDUCATION

Math

TRAINEE MANUAL



Approved by:  Workforce
Development
Authority



USAID
FROM THE AMERICAN PEOPLE



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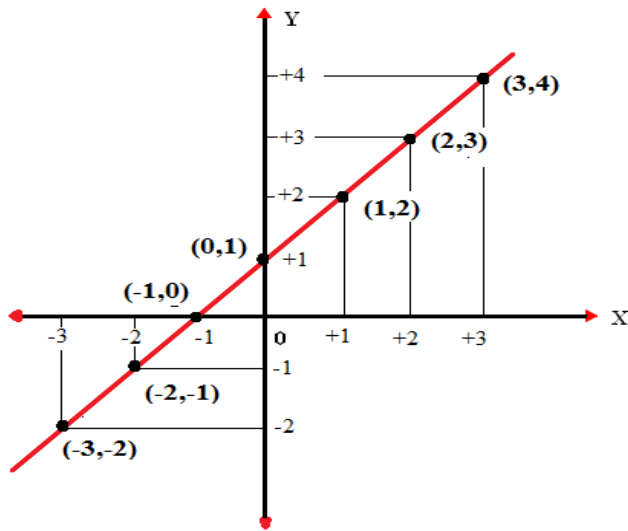
BASIC MATHEMATICS AT THE WORKPLACE

Unit 1: Algebraic and set concepts

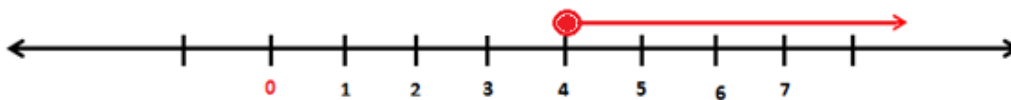
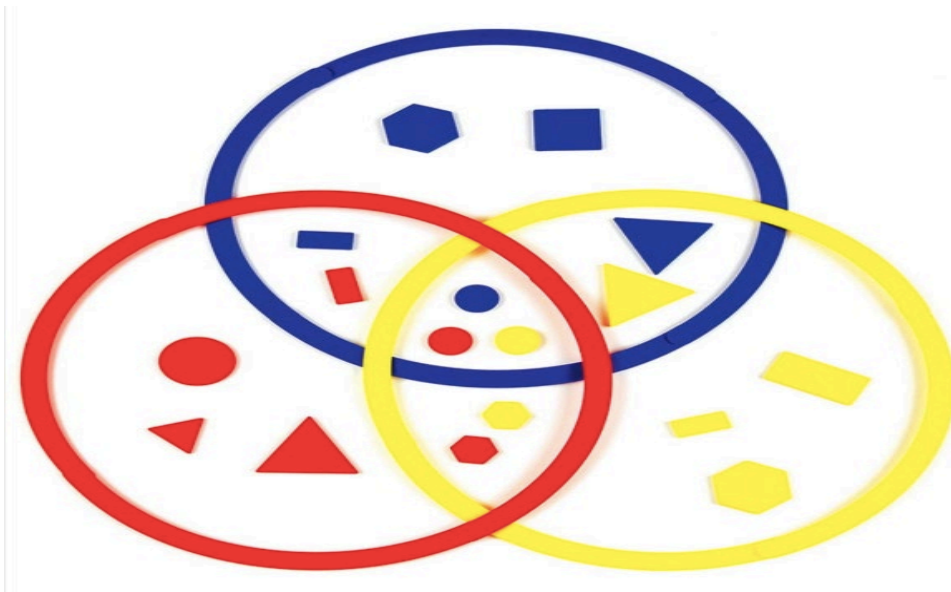
Unit 2: Geometric shapes and angles

Unit 3: Statistical concepts and calculations

Unit 1: Apply algebraic and set concepts



$$y = x + 1$$



Topics

- 1.1** Use Venn diagrams to represent set operations and properties
- 1.2** Calculate proportions, ratios, percentages, and mixtures
- 1.3** Use percentages to calculate discount, commission, profit, loss, interest, and taxes
- 1.4** Manipulate and solve simple algebraic equations and inequalities
- 1.5** Plot and interpret graphs of linear and quadratic functions

Unit Summary:

In everyday we use algebra, groupings, pictures and graphs to understand different types of situations.

The purpose of this unit is to:

- ✓ Introduce set language, notation, and visual representations to enable counting and calculations, so that set problems can be solved.
- ✓ Use the language of sets for understanding the relationships between objects or numbers of different types through ratios and proportions.
- ✓ Solve everyday life problems involving buying and selling activities where the concepts of percentage, discount, commission, profit and loss, interest, and tax will be explored in context.
- ✓ Apply algebraic concepts to form and solve both equations and inequalities.
- ✓ Plot and interpret simple graphs of lines and parabolas.

Self-Assessment: Unit 1

1. Look at the illustration. What is happening? What do you think this unit will be about? What topics might be covered? Write three or four sentences in your notebook that describe what you expect to learn based on your observations.
2. Fill in the self-assessment below. There is no right or wrong way to answer this survey. It is for your own use during this unit. The trainer will read a skill that is listed in the left column. Think about yourself: do you think you can do this? How well? Read the statements across the top. Put a check in column that best represents your situation.

At the end of this unit, we'll take this survey again to see how you have improved.

My experience	I have no experience doing this topic.	I know a little about this topic.	I have some experience doing this topic.	I have a lot of experience with this topic	I am confident in this topic.
Knowledge, skills and attitudes					
Use Venn diagrams and accurately determine union, intersection, and complement of 2 or 3 sets					
Express percentages as fractions and ratios					
Explain direct and indirect proportion using practical examples					
Solve everyday problems involving direct or indirect proportion					
Find the quantity and the price of mixtures					
Increase and decrease quantities by percentages					
Solve problems of discount, commission, profit/loss, interest, and tax using percentages					
Plot and interpret graphs of linear and quadratic functions					
Apply algebraic concepts to form/solve equations and inequalities					

Topic 1.1: Venn diagram representations to carry out set operations (union, intersection, complement) and solve problems

Key Competencies:

1. Describe the purpose of representing of 2 or 3 sets in a Venn diagram	1. Create a Venn diagram to represent 2 or 3 finite sets then use the diagram to solve related problems	1. Creative
2. Explain Union and Intersections of sets	2. Represent and determine union of 2 or 3 sets	2. Innovative
3. Describe the complement of a set.	3. Represent and determine intersection of 2 or 3 sets	3. Accurate



Getting Started: What do we know and where are we going?



Tasks:

Topic 1.1 Task 1

1. Copy the following words into your notebook and write a short definition beside each word.

Set, Element, Sub-set, Intersection, Union, Complement

Topic 1.1 Task 2

1. You will be assigned a number and form sets with the other trainees based on the properties of that number and the instruction given by the trainer.
 - a. All odd numbers go to the front of the class and all even numbers go to the back of the class
 - b. All numbers less than 10 go to the back of the room and all even number go to the front of the room.
 - c. All the numbers that are multiples of 4 go to the back of the room and all the numbers that are 2 go to the front of the room.
 - d. Trainer will provide other sets – or think of one to share with the class!



Problem Solving Activity



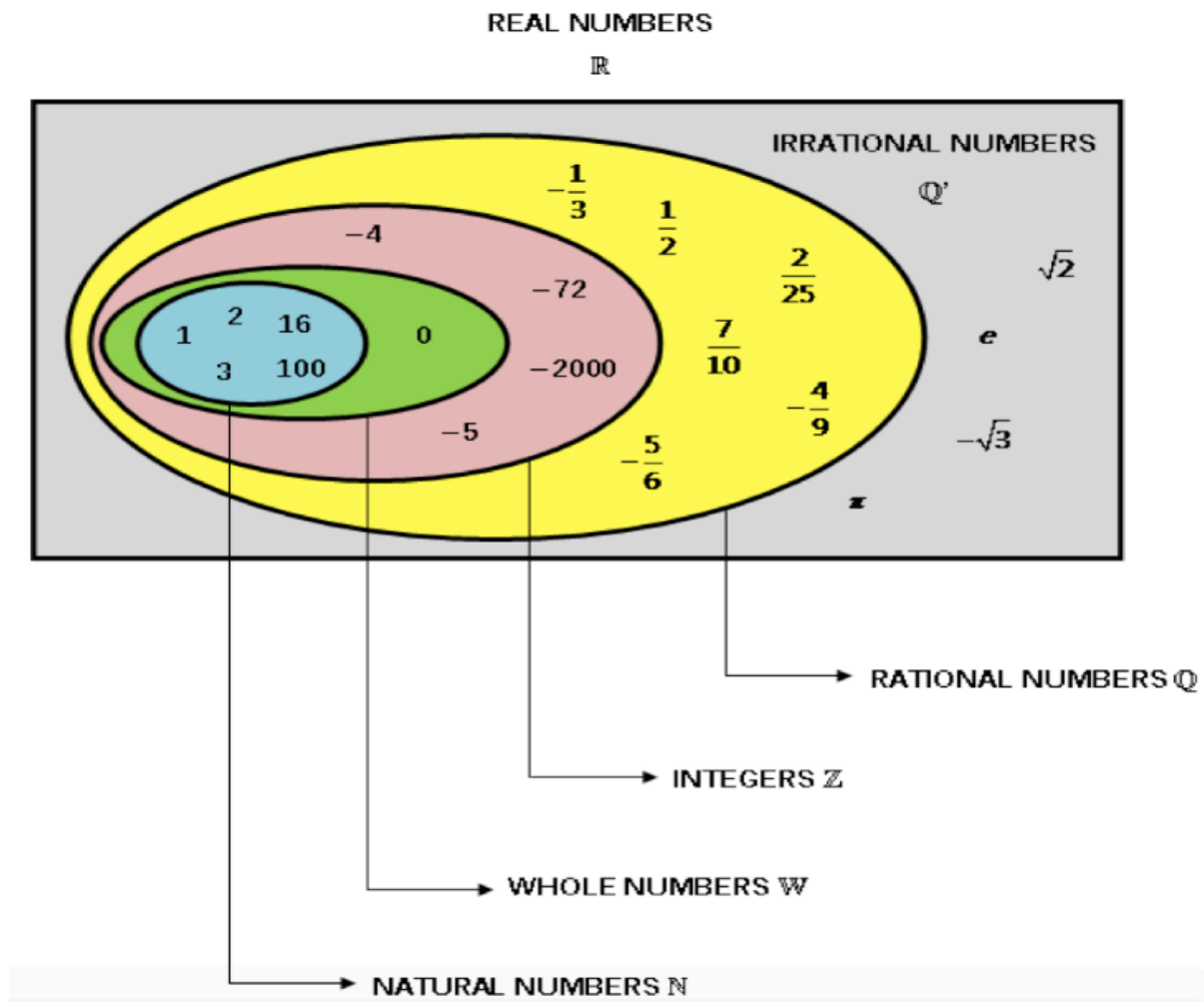
Tasks:

Topic 1.1 Task 3

1. Draw a number line along the top of your notebook page.
2. Label the number 0 in the centre of their number line, then label also the numbers -3, -2, -1, 0, 1, 2, 3, 4, and 5 on their number line.

Topic 1.1 Task 4

- Describe the different sets shown in the diagram below and identify which are sub-sets of another set.



1

Topic 1.1 Task 5

- Review the illustration from Task 4 to assist in responding to the questions below.
 - Give an element of the Integers that is NOT an element of the Whole numbers.
 - Give an element of the Rational numbers that is also an element of the Natural numbers.
 - Give an element of the Whole numbers that is NOT an element of the Natural numbers.
 - Give an element of the rational numbers that is also an irrational number.
 - Which sets does the number 0.75 belong to?
 - Which sets does the number -31 belong to?
 - Give an example of a number that is an integer but not a whole number.
 - Give an element of the Rational numbers that is also an integer

¹ Ndyabasa, E. et al (2016). Mathematics for Rwandan Schools Senior 1, Student's Book. Kigali, Rwanda: Longhorn Publishers Limited.



Activity 3: Guided Practice



Tasks:

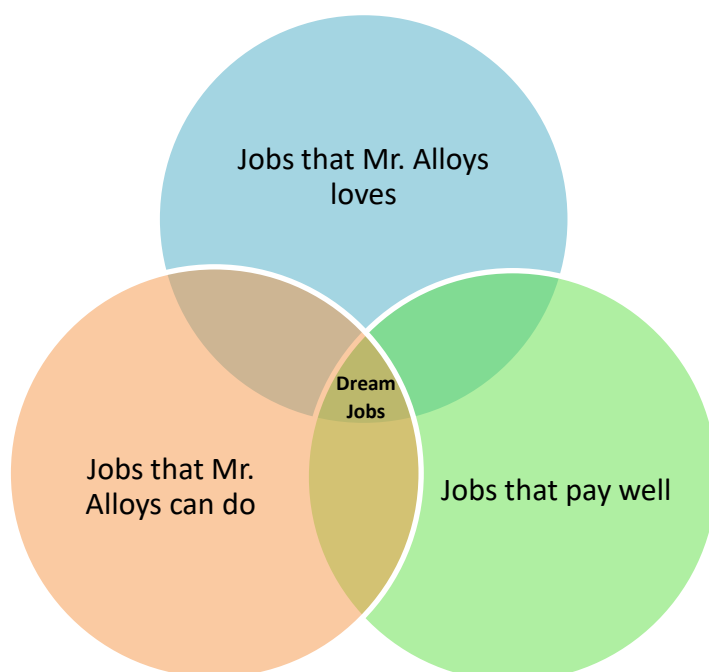
Topic 1.1 Task 6

1. Read the following:

Choosing a Dream Job: Mr. Alloys is a level 2 student. At the end of his studies he wonders what might be his dream job. There are 3 factors which matter to him when choosing a job:

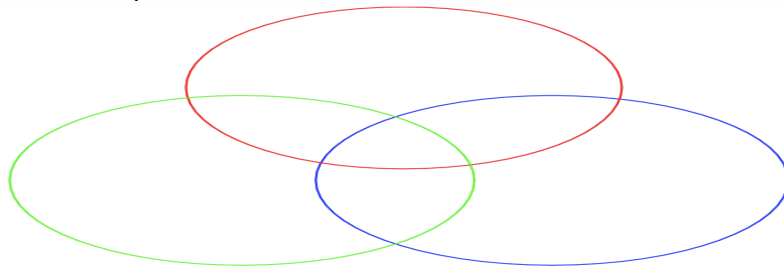
- Jobs that Mr. Alloys loves to do,
- Jobs that Mr. Alloys is good at,
- Jobs that will pay good money to Mr. Alloys.

Mr. Alloys thinks about potential jobs and puts them into the proper spaces inside the following Venn diagram:



- Describe each different part of the Venn diagram—and give examples of different jobs that may fit into the three sections. Write the answers in your notebook.
- Decide if you agree with the following statements (or not) and justify your feeling:
 - Jobs that Mr. Alloys is good at would be the most practical choice.
 - Jobs that Mr. Alloys loves to do would be the most practical choice.
 - Jobs with good earning potential would be the most practical choice for Mr. Alloys.
 - A job which includes all of these three criteria would be the dream job for Mr. Alloys.
- Identify all jobs that satisfy the following criteria FOR YOU! List all jobs for each category and then think if there are any jobs that you have listed which fit into more than one of the categories and then write them inside the spaces on a Venn diagram.
 - Set A contains “jobs that you think you are (or will be) good at”, $A = \{\dots\dots\dots\}$
 - Set B contains “jobs that you would love to do”, $B = \{\dots\dots\dots\}$
 - Set C contains “jobs that you think will pay well”, $C = \{\dots\dots\dots\}$

- d. After you have identified jobs, already organized into 3 sets, create a Venn diagram as was used for Mr. Alloys. Note: each job should only be written once—if it is found in two sets then it is placed in the overlapping area, if it is all three sets put it in the very middle where all three circles overlap.



Topic 1.1 Task 7

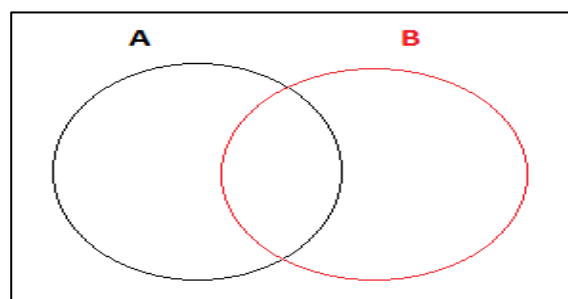
Using Venn diagrams to solve problems

1. Consider all the integers (counting numbers) from 0 to 10. Because this is the complete set of elements considered, we call this our **universal set**.
2. Label the **Universal Set** using the letter E: $E = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
Inside E there are two smaller sets: set A = {1, 3, 5, 7, 9} and set B = {1, 2, 3, 4, 5}
Note that both A and B are **subsets** of set E so we could write: $A \subset E$ and $B \subset E$.
3. Answer the following questions regarding the sets E, A and B:
 - a. Which elements are listed in both sets A and B? These are the common elements.
 - b. Find all the elements of set A which are NOT in set B:
 - c. Find all the elements of set B which are NOT in set A:
 - d. List the elements that are found in A as well as those that are found in B: (only list each element one time)
 - e. Try to represent this situation using a Venn diagram: (hint: pay attention to shared elements)

Topic 1.1 Task 8 Cities that people have visited

Travel agents surveyed 100 people to find out how many of them had visited the cities of Kibuye and Huye. They were told 31 people visited the city of Kibuye, 26 people visited Huye, and 12 people had visited both cities. Some people had not visited either city.

- a. Let set A represent the people who visited Kibuye, and set B the people who visited Huye. Fill in the Venn diagram below with the number of people who visited each of the cities. Take care with the number that visited both cities, and the number that visited neither city.



- b. Based on the information in your Venn diagram, find the number of people who had visited:
 - i. The city Kibuye or Huye but not both.
 - ii. The city Huye but not Kibuye.
 - iii. Either no cities or both cities.
 - iv. The number who have not visited Huye.
- c. Identify which of the parts of their Venn diagrams represent the following:
 - i. Intersection of A and B
 - ii. Union of A and B
 - iii. The complement of set B

Topic 1.1 Task 9

There are 17 trainers in a school. 10 teach Economics and 9 teach Mathematics. There are some trainers that teach no classes this term, and there are some trainers that teach both Mathematics and Economics. Use X to represent the number of trainers that are NOT teaching. There are two times as many trainers that teach both subjects as there are that teach none, so use $2x$ for the number of teachers teaching both.

Make a Venn diagram to represent this situation, then figure out the actual values for each area in the Venn diagram and answer the following questions:

How many trainers teach:

- a. Both subjects?
- b. None of the subjects?
- c. Only one subject?

1.1 Key Facts

Real Numbers and subsets of the Real Numbers:

- **Natural numbers:** are counting numbers not including zero. **N**
- **Integers:** Are whole counting numbers, negative, positive, and zero. **Z**
- **Rational numbers:** are numbers that can be expressed as a fraction. **Q**
Note: remember that whole numbers can be written as fractions: $4=4/1$, and so can decimals.
- **Irrational numbers:** are numbers that cannot be written as a fraction. **I**
Examples: pi, square root of 2, other non-repeating, infinite decimal numbers
- **Real numbers:** are numbers that can be found on a number line. **R**
 Real numbers include both rational and irrational numbers.

Venn diagrams

- A **Venn diagram** is a way to represent sets and their relationships. Circles represent the sets of interest. The circles are usually drawn inside a rectangle. The rectangle represents the **universal set** which is ALL elements considered.

- The region outside a circle represents the **complement** of the set—which is the elements in the universal set that are NOT in the set itself.
- The overlapping region of two sets represents the **intersection** of the two sets—that is the elements that are shared by both sets.
- Two sets together—all elements inside both circles-- represent the **union** of the sets.



Application Activity

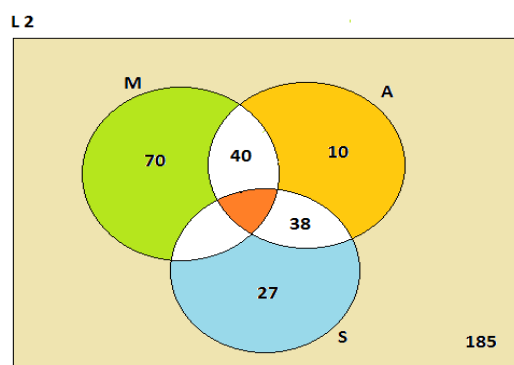


Tasks:

Topic 1.1 Task 10

A school in Gisenyi has 185 trainees in level two. 110 trainees study the Mathematics module, 88 study the Agriculture module, and 65 study the Science module. 40 trainees study both Mathematics and Agriculture modules, 38 study Agriculture and Science modules.²

Use the given venn diagram to answer the following questions regarding numbers of students in group:

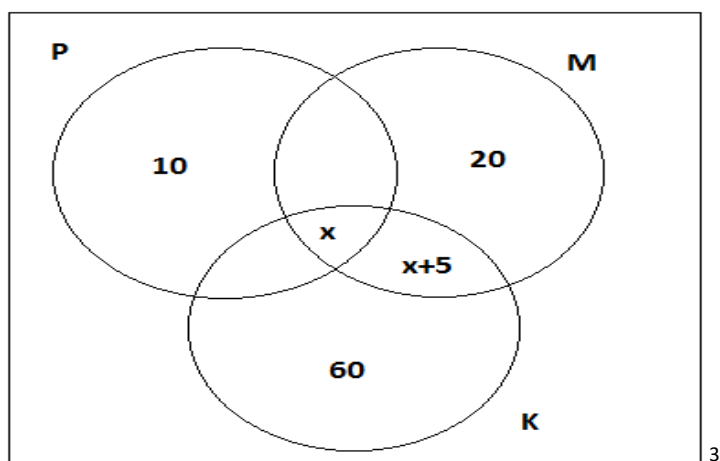


- Find the number of trainees who study Science module only. (S)
- Find the number of trainees who study Mathematics module only. (M)
- Find the number of trainees who study Agriculture module only. (A)
- Find the total number of trainees who study Mathematics.
- Find the total number of trainees who study Agriculture.
- Find the number of trainees who study Mathematics and Agriculture. ($M \cap A$)
- Find the number of trainees who study Science and Agriculture. ($A \cap S$)
- Find the total number of trainees who study Science.
- Find the number of trainees who study Science and Mathematics. ($M \cap S$)
- Find the total number of trainees who study all three. ($S \cap M \cap A$)

Topic 1.1 Task 11

The Venn diagram below shows the number of Senior 1 students who like Mathematics (M), Physics (P) and Kinyarwanda (K). Some like more than one subject. In total 55 students like Mathematics.

² Adapted from Ndyabasa, E. et al (2016). Mathematics for Rwandan Schools Senior 1, Student's Book. Kigali, Rwanda: Longhorn Publishers Limited.



- How many trainees like the three subjects?
- How many trainees like Physics?
- How many trainees like Kinyarwanda?
- How many trainees like both Kinyarwanda and Physics?
- Find the total number of Senior 1 trainees in the school.



Points to Remember

- In Mathematics there are different types of numbers which we can consider as sub-sets of the Real Numbers. These are: Natural numbers, Integers, Rational numbers, Irrational numbers. All sub-sets of Real numbers can be found on a number line.
- Venn diagrams are used to illustrate and solve real life problems involving relationships between sets.



Formative Assessment

- Consider the following sets:

The universal set being considered is $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ Inside E we have:

$A = \{6, 7, 8, 9, 10\}$ $B = \{1, 2, 3, 4, 5, 6, 7\}$ $C = \{2, 4, 6, 8, 10, 12\}$ and $D = \{1, 3, 5, 7, 9\}$

List out the elements that are found in the following:

- The INTERSECTION of A and B , that is $A \cap B$
 - The UNION of B and C , that is $B \cup C$
 - The INTERSECTION of C and D , that is $C \cap D$
 - The complement of set C , that is C' and
 - The INTERSECTION of sets A , B , and C , that is $A \cap B \cap C$
- Given sets $M = \{1, 3, 5, 7, 9\}$ and $N = \{2, 5, 6, 9\}$;

³ Adapted from Ndyabasa, E. et al (2016). Mathematics for Rwandan Schools Senior 1, Student's Book. Kigali, Rwanda: Longhorn Publishers Limited.

Represent set M and N on a Venn diagram then list out the elements in $M \cap N$

3. A survey was carried out. 50 people were asked about where they had eaten lunch over the past month. Among the Hilltop, Serena, and Lemigo hotels, it was found out that 15 people had eaten at Hilltop, 30 people had eaten at Serena, and 19 people had eaten at Lemigo, 8 people had eaten at both Hilltop and Serena, 12 people had eaten at both Hilltop and Lemigo, and 7 people had eaten at Serena and Lemigo. Just 5 people had eaten at all three: Hilltop, Serena, and Lemigo.
- Represent the information on a Venn diagram.
 - How many people ate only at Hilltop?
 - How many ate at Hilltop and Serena but not at Lemigo?
 - How many people did not eat from any of these three hotels?
4. A store surveyed all their customers during one day—there were 79 customers total. The survey asked how many of those customers bought bread, milk, both, or neither. Let x be the number of customers who bought both bread and milk; $52-x$ is the number of customers who bought milk only; $32-x$ is the number of customers who bought bread only. 15 customers bought neither bread nor milk.

Draw a Venn diagram to represent this situation—then replace the value of x with the actual number that is found from the information given.

Topic 1.2: Proportions, ratios, percentages, and mixtures

Key Competencies:

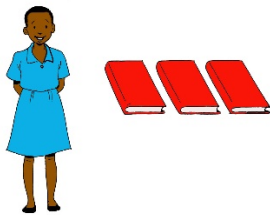
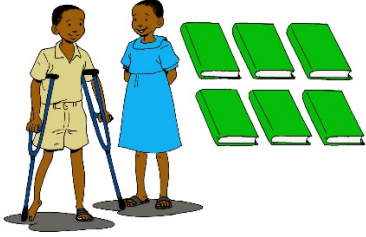
1. Explain decimals, fractions, and percentages and how to use them in life contexts	1. Convert decimals into fractions and fractions into percentage and verse versa	1. Persistent
2. Describe ratios and proportions, both direct and indirect in practical and everyday contexts.	2. Calculate percentage increase and decrease in quantities, and find percentage profit or loss	2. Creative
3. Understand the concept of calculating mixtures and explain it to others	3. Solve every day problems related to percentages, ratios, proportions and mixtures	3. Accurate

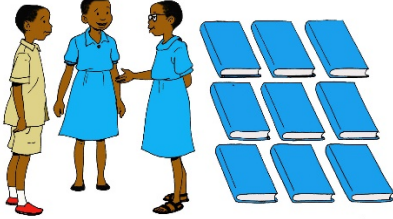
Getting Started: What do we know and where are we going?



Tasks:

Topic 1.2 Task 1

	Mary has 3 books.	Mary has: 3 of 18 books
	Jane and Peter have 6 books.	Jane and Peter have: 6 of 18 books

	<p>Sarah, Steve and Ruth have 9 books.</p>	<p>Sarah, Steve and Ruth have:</p> <p>9 out of 18 books</p>
---	--	---

- What is the ratio of red books to the total number of books?
- What is the ratio of blue books to red books?
- What is the ratio of green books to blue books?
- What is the ratio of blue books to green books?

Topic 1.2 Task 2

1. Observe the table showing the number of pens owned by students in a middle school class:

Number of students	Number of pens
1	5
2	10
3	15
4	(a)
5	(b)
10	(c)
(d)	85

2. Complete the table by filling in the spaces labelled by (a), (b) and (c). Then explain a pattern you see which will help you solve these problems.

Brainstorm: What is a percentage? What are examples where percentages are used in life?

Topic 1.2 Task 3

Convert the following decimal numbers and percentages into fractions and explain how.

- 0.5
- 0.25
- 0.95
- 85%
- 30%
- 6%



Problem Solving Activity



Tasks:

Topic 1.2 Task 4

In a school, there are 56 female trainers out of a total of 70 trainers.

- What is the ratio of female trainers to the total number of trainers?
- What is the ratio of male trainers to the total number of trainers?
- What is the ratio of female to male trainers?
- What percentage of the trainers are female?
- What percentage of the trainers are male?
- What is the relationship between the two percentages we found in (a) and (b)?



Guided Practice Activity

1.2 Key Facts

- In math, we use **ratios** to compare quantities of different items.
- Ratios are usually expressed in their lowest form and have no units.
- In general, if **a** and **b** are two quantities of the same units, then the ratio of **a** to **b** is written as **a : b** and can be expressed as a fraction as **a / b**.
 - For example, 1 boy has 2 books. A boy and books are different items. In this case, 1:2 is the ratio of boys to books, also written as $\frac{1}{2}$.
- Two ratios that are equal to each other are called a **proportion**. A proportion is a relationship between four numbers or quantities in which the ratio of the first pair equals the ratio of the second pair, and is written as $a:b = c:d$ and is read as “a is to b as c is to d.” We can use proportions to solve math problems involving ratios.
- A direct proportion is when we have a situation such that:
 - If one quantity increases, the second quantity increases in a similar way, or
 - If one quantity decreases, the second quantity decreases in a similar way.
- An indirect proportion is when we have a situation such that:
 - If one quantity increases, the second quantity decreases, or
 - If one quantity decreases, the second quantity increases.

Topic 1.2 Task 5

In a classroom of 50 trainees, there are 30 girls and 20 boys.

- Give a ratio for girls in the class, and for boys in the class using fraction notation.
- You are asked to represent the number of boys out of 100, how should you proceed?
- What is another word for the number of boys out of 100?
- One day, 5 trainees are absent. What percentage are absent from the class?

Topic 1.2 Task 6

Study the following table showing the number of trainers to trainees in a school in Burundi. Fill in the missing numbers. State the ratio used and explain the pattern used to find that ratio.

Number of trainers	1	2	3	4	5	-----	7
Number of pupils	30	60	----	-----	150	180	-----

Topic 1.2 Task 7

Find the price of 8 pens given the price of 10 pens is 1 500 Frw.

Topic 1.2 Task 8

At a special school of 300 trainees, all trainees perform well in national examinations. This makes the school more popular so they must register more trainees the next year. The number of existing trainees was increased by 40%. Find out the number of newly registered trainees.

Topic 1.2 Task 9

A business man mixed 200 kg of one kind of soya beans which cost 600 Frw per kg with 300 kg of another type of soya beans which cost 800 Frw per kg, the reason being that the soya beans costing 800 Frw were not preferred by the people because of the high price. What is the average price of 1kg of the mixed soya beans?



Application Activity



Tasks:

Topic 1.2 Task 10

4 dresses can be made from 6 meters of cloth.

- How many meters of cloth must I buy for 6 dresses?
- If I have 15 m of cloth, how many dresses can I make?

Topic 1.2 Task 11

On a scale drawing, 1.5 cm represents 20 km of road.
What length of road is represented by 6 cm?

Topic 1.2 Task 12

A cooperative named “ABOROZI B’INKOKO” sold 500 eggs a week. On Monday, they sold 25% of the eggs, on Tuesday they sold 30% of the eggs, and on Friday, they sold 5% of the eggs. On Saturday, they sold the rest of the eggs.

- What percentage of eggs is left to be sold on Saturday?
- How many eggs were sold on:
Monday? On Tuesday? On Friday? On Saturday?

Topic 1.2 Task 13

For purposes of sales promotion, the price of a book has been reduced by 20% to a sale price of 3 600 Frw. What was the price before the reduction?

Topic 1.2 Task 14

A mixture of two types of juice is made from 120 l of 1st quality juice costing 150 Frw per litre and 80 l of a 2nd quality type (diluted).

Find the price of 1 litre of the second type if 1 litre of the mixture was 138 Frw per litre.



Points to Remember

- A ratio is a mathematical statement, which shows how two quantities compare.

- In simplifying ratios, the two quantities of a ratio may be multiplied or divided by the same number without changing the value of the ratio
- When using percentages, you can calculate either using a fraction or a decimal that is equivalent to the percentage. For example: $75\% = 0.75 = 75/100 = 3/4$



Formative Assessment

1. A company produced 23 000 shirts in September. This was 8% less than the August production. How many shirts did the company produce in August?
2. A farmer bought a cow at 55 000 Frw and sold it a year later for 42 900 Frw. Find her percentage loss when she sold her cow.
3. The price of 8 small tables for nursery schools is 36 000 Frw. How many tables are you going to produce if you are paid 99 000 Frw by the school administration?
4. A business woman mixed 250 kg of fine rice which cost 660 Frw per kg with 125 kg of broken-up rice which cost 360 Frw per kg, the reason was that the business woman wanted to lower the total price so many people could still eat rice. What is the price of 1kg of the mixed rice?
5. A man's daily wage was increased by 25% to 5 000 Frw. Find his salary before the increase.

Topic 1.3: Calculating discount, commission, profit, loss, interest, taxes

Key Competencies:

1. Explain discounts, profits and losses and how to calculate them	1. Solve everyday problems involving discounts, profits and losses	1. Persistent
2. Describe times when a commission might be provided, or a tax applied and explain how to calculate both	2. Given percentage rates, calculate commissions and calculate the taxes applied	2. Creative
3. Understand the concept of interest and explain how to calculate it	3. Solve every day problems of simple interest	3. Accurate



Getting Started: What do we know and where are we going?



Task:

Topic 1.3 Task 1

Read the following paragraph. In your own works, write a definition for each underlined word. You may use examples from your own life to help explain particular concepts.

There is a new shopkeeper named Mary in the neighbourhood. She sells different types of grains and beans, including: red beans, green peas, sorghum, maize, soya beans, and rice. The Rwandan government charges a tax on imported grains which make the cost of those grains more expensive, but some people still prefer those items. Mary also sells locally grown grains. To make her business more productive, Mary is advised to use different agents and pay the agents a commission when they make a sale for her. Mary will determine which grains and beans make the most profit and also calculate if she is taking a loss on the sale of any of her products. If some of her grains are not selling well Mary will consider introducing a discount for some items.



Problem Solving Activity



Tasks:

Topic 1.3 Task 2

You are starting a printing business and you need to buy paper from a local supplier. You talk to representatives from three businesses in town. The following are prices and conditions they quote, review the three quotes then answer the following questions.

Business 1:

The standard price for a ream of paper is 7 500 Frw

BUT if you buy 10 or more reams, they will give you a 2% discount.

Business 2:

The price for a ream of paper is the basic cost of 6 700 Frw BUT because they import the paper from Tanzania, they need to add the cost of import tax which is 11%.

Business 3:

The standard price for a ream of paper is 7 450 Frw. No discounts or taxes.

- For each of the business offers above, calculate the cost of 1 ream, 5 reams and 10 reams of paper. Keep in mind the percentage problems that we did during the last Topic 1.2.
- For each of the business offers above, list at least one reason why it would be a good choice—and one reason why it may not be a good choice.

Topic 1.3 Task 3

Consider the same printing business. Assume that you have chosen Business 1 and have bought 12 reams of paper. The cost of colour printing on that paper adds 6 250 per ream.

The reams are used to make monthly picture calendars. 150 calendars have been printed total.

- Calculate the cost of each calendar.
- If you want to make 50 Frw profit on each calendar you sell, what will the selling or LIST PRICE be for each of your calendars?



Guided Practice Activity



Tasks:

Topic 1.3 Task 4

Mary is offered a part-time job selling restaurant equipment. She will be paid Frw100 000 per month plus a 6% commission on her sales. The sales manager tells Mary she can expect to sell between Frw 800 000 and Frw 1 500 000 worth of equipment per month. To help her decide whether to accept the job, Mary does a few calculations.

Based on the sales manager's estimate, what monthly income can Mary expect from this job? What annual salary would that provide? Compute monthly and annual incomes for each sales total shown in the table. The first two examples of monthly income have been done for you. Discuss if this is a good job for Mary to take.

Sales Amount	Calculation: 6% of Sales (6%=0.06)	Mary's Monthly Income	Annual Income (monthly x 12)
500 000	$500\,000 \times 0.06 = 30\,000$	$100\,000 + 30\,000 = 130\,000$	
800 000	$800\,000 \times 0.06 = 48\,000$	$100\,000 + 48\,000 = 148\,000$	
1,000,000	$1\,000\,000 \times 0.06 = 60\,000$		
1,500,000			
2,000,000			
3,000,000			

Topic 1.3 Task 5

A company's sales representative sold goods worth 6 760 000 Frw in a certain month. The representative earns a base salary of 150 240 Frw and gets a commission of 10% on the sales that are above 5 200 000 Frw. Calculate how much the sales representative earned that month.

1.3 Key Facts

- **Commission** is the money paid to sales representatives for sales made. It is usually calculated as a percentage of the value of the sales on a monthly or annual basis. The more the sales, the more the commission to encourage the representative to get sales.
- The **list price**, is the price initially quoted by the seller to the buyer. Sometimes the seller may lower a price below the list price. The amount reduced is known as the **discount**. The exact price for which an item is sold after a discount is known as the **sale price**.
- **Discount = marked price – sale price.**
- **Profit** is the extra amount gained after selling a commodity at a price higher than the cost. **Profit = selling price – cost**
- **Loss** refers to the amount of money lost when a commodity is sold below the cost. **Loss = cost – selling price**
- **Simple interest** is an amount charged when borrowing money or taking out a loan. Interest is usually a percentage of the amount that is borrowed (called the principal), and is usually charged on a yearly (annual) basis.
- **Interest = % Rate x Principal x time (in years)**
Note: simple interest is paid INTO a bank account by the bank when you save money too.

Topic 1.3 Task 6

Robert borrowed 200 000 Frw from a bank for 2 years at a rate of 8% per year. How much interest did he pay for the service of the loan?

Topic 1.3 Task 7

How much simple interest will a savings account with 50 000 Frw earn at a rate of 4% per year in 5 years?



Application Activity



Tasks:

Topic 1.3 Task 8

Calculate the percentage loss for an item bought at 2 000 Frw and sold at a price of 1 850 Frw.

Topic 1.3 Task 9

Find the sale price of a watch whose marked price is 30 000 FRW if 20% discount is given.

Topic 1.3 Task 10

A trader bought a pair of shoes at 3 600 Frw and sold them for 4 500 Frw.

Find her profit or loss and then find the percentage profit or loss.

Topic 1.3 Task 11

The percentage profit on each cell phone bought at an original cost of 48 000 Frw is 20%. At what price does the business person sell each cell phone to make that profit?

Topic 1.3 Task 12

A saleslady received a commission of 5% for the first sale of 80 000 Frw and 6% for sales above 80 000 Frw. In one month she made sales amounting to 168 000 Frw. Find her total commission that month.

Topic 1.3 Task 13

A company makes a product at a cost of 120 000 Frw which is sold at a list price of 130 000 Frw. The Value Added Tax (VAT) is a tax on manufactured goods. The tax is calculated ONLY on the amount of profit added to the cost when list price is determined. Currently the standard rate of VAT in Rwanda is 18%. This extra 18% will be paid to the tax authority after the sale of the product. Find the amount of tax that the company must pay to the government.



Points to Remember

- **Percentages** are used frequently in every-day life, including discounts, profits/losses, commissions, interest rates, and taxes. It is often easiest to calculate percentages if they are turned into decimal numbers which can be calculated directly.
- **Interest** is the amount **charged for borrowing** money—and Interest is also the amount that is **earned for saving** money in bank accounts.
You **get** money for saving but you **pay** money to take out a loan.



Formative Assessment

1. Find the simple interest earned from 3 400 Frw saved in a bank account for 3 years at the rate of 10% per year. How much will the entire account have in it at the end of this period?
2. Peter borrowed 100 000 Frw from a bank to start a business making greeting cards. The bank charged him an interest rate of 12% per year. How much interest did he pay after 2 years?

3. A company of weavers produces baskets at a cost of 20 000 Frw each and sells each basket for the price of 28 000 Frw. Calculate the Value Added Tax (VAT) of 18% that the company will pay to the tax authority after the sale of each product, keeping in mind that the VAT is calculated only on the amount of the profit that is added to the cost of the basket.
4. Find the sale price of a watch whose marked price is 30 000 Frw if 10% discount is given.
5. An auto parts sales person received a commission of 4% for sales up to 50 000 Frw and 5% for sales above 50 000 Frw. One month her sales amounted to 88 000 Frw. Find her commission.

Topic 1.4: Solving simple algebraic equations and inequalities

Key Competencies:

1. Describe the concept of simple algebraic equations with one unknown	1. Form and solve equations with variables	1. Persistent
2. Explain the concept of simple algebraic inequalities in one unknown	2. Form and solve inequalities with variables	2. Proactive
		3. Accurate

Getting Started: What do we know and where are we going?



Tasks:

Topic 1.4 Task 1

Solve the following given the values for the different variables:

- If $a = 4$, $b = 8$ and $c = 2$, find the value of $a + b + c$
- If $x = 3$ and $y = 4$, find the value of $(2 + x)(5 + y)$
- If $x = z$, $y = -8$ and $z = -2$, find the value of the product of the three terms: xyz

Topic 1.4 Task 2

Use a mathematical sentence (inequality) AND a number line to show and write the following numbers: (hint: pay close attention to the difference between $<$ and \leq or $>$ and \geq note the circle on the number line is either filled in or left empty!)

- All numbers greater than 2
- All numbers greater or equal to 2.
- All numbers less than 2.
- All numbers less or equal to 2.



Problem Solving Activity



Tasks:

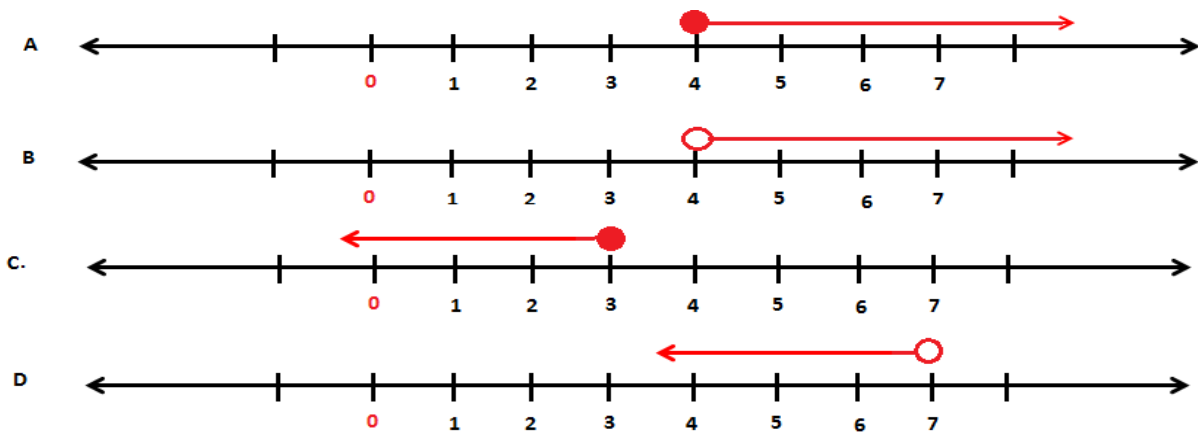
Topic 1.4 Task 3

Trainer will read each sentence aloud. Trainee writes it using math symbols and solves:

- I have a number. If I subtract 20 from my number, my answer is 6. What is my number?
- Think of a number, double it and subtract 6 to get 10. What is the number?

Topic 1.4 Task 4

Observe the following number lines and write the algebraic inequalities they represent:



Guided Practice Activity



Tasks:

Topic 1.4 Task 5

For each of the following inequalities, draw number lines to illustrate the solution set:

(a) $x > 3$
(b) $x \geq 3$
(c) $x < -2$
(d) $x \leq -2$

Topic 1.4 Task 6

Solve for x :

- $x + 2 = 10$
- $x - 4 = 7$
- $5x = 25$
- $5(x + 3) - 3(x + 4) = 9$

Topic 1.4 Task 7

Solve the following inequalities and represent your solution on a number line.

- $2(x - 1) \geq x + 2$
- $x - 1 \geq 3x + 2$

1.4 Key Facts

- A letter which represents an unknown value is called a **variable**.

- A mathematical sentence with the symbol $=$ is called an **equation**. Such a statement expresses the two sides of the equal sign have the same value.
- Mathematical statements containing these symbols (greater than: $>$, less than: $<$, greater than or equal to: \geq , less than or equal to: \leq) are called **inequalities**. With an inequality, the two sides do not necessarily have the same value, but can still be solved.
- A statement such as $x > 2$ means 'all numbers that are greater than 2', which is a range of values so there are many answers that are correct. Solving an inequality means obtaining all the possible values of the variable which make the statement true. This is often done with the solution on a number line to show a range (many) values.
- A **linear equation** can be written in the form of $y = mx + b$. For example: $17 = x + 3$, or $25 = 2m - 5$, or it may look like this: $n - 3 = 8$. These are all linear equations.
- **Different steps used to solve equations are:**
Adding the same number to both sides or Subtracting the same number from both sides.
Multiplying both sides by the same number or Dividing both sides by the same number.
- **We use the same steps to solve inequalities EXCEPT** the direction of the inequality changes if we multiply or divide both sides by a negative number.



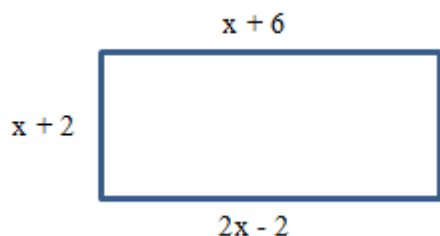
Application Activity



Tasks:

Topic 1.4 Task 8

A rectangular plot of land has dimensions in meters. Observe the diagram below:



- The 2 parallel sides of a rectangle are equal. Use that property to find the value of x .
- Find the length and the width of the plot of land.
- Calculate the perimeter of the plot of land in meters.
- Calculate the area of the rectangular plot of land in meter square.

Topic 1.4 Task 9

Solve the following inequalities and represent the solutions on number lines.

- | | |
|-------------------------|-----------------------|
| 1. (a) $x + 4 > 11$ | (b) $x - 6 \leq 5$ |
| 2. (a) $2x - 8 \leq 4$ | (b) $3x + 4 > 19$ |
| 3. (a) $3 > 4x - 2$ | (b) $7 \leq 5x + 12$ |
| 4. (a) $3 - 2x < 5$ | (b) $4 - 5x \geq -11$ |
| 5. (a) $13 - x - 3 > 4$ | (b) $15 - x + 2 < 1$ |



Points to Remember

A letter such as x used above is called a **variable**. Variables can be used in both **equations** and in **inequalities** to represent an unknown value (or many unknown values) which we find by using ALGEBRA and ARITHMETIC.

We use arithmetic steps to solve equations. We can:

Adding the same number to both sides or Subtracting the same number from both sides.

Multiplying both sides by the same number or Dividing both sides by the same number.

We use the same steps to solve inequalities EXCEPT watch out for negative numbers!

The direction of the inequality symbol changes when we multiply or divide an inequality by a negative number!



Formative Assessment

1. Solve the following equations:

- | | |
|-------------------|-------------------|
| (a) $x + 6 = 20$ | (e) $x + 9 = 23$ |
| (b) $16 + x = 25$ | (f) $17 + x = 25$ |
| (c) $x - 3 = 12$ | (g) $3x - 5 = 1$ |
| (d) $12 - x = 10$ | (h) $14 - x = 8$ |

2. Solve the following inequalities and represent the solutions on number lines:

- (a) $2(1 + x) + 3(x - 2) \geq 25$
(b) $3(4 - 3x) - (5x - 3) \leq 2$

Topic 1.5: Plotting and interpreting graphs of linear and quadratic functions

Key Competencies:

Knowledge	Skills	Attitudes
1. Explain the difference between a linear and a quadratic function	1. Plot and name points on a Cartesian plane	1. Accurate
2. Describe a Cartesian plane (x-axis and y-axis with ordered pairs as Points)	2. Plot linear and quadratic functions and interpret the graphs of those functions	2. Innovative
3. Explain how to use the plot of quadratic or linear functions to provide solutions.	3. Solve linear and quadratic functions using a graph	3. Persistent

Getting Started: What do we know and where are we going?



Task:

Class activity:

Draw a Cartesian plane in your notebook. Plot the points: (3, 2), (5, 2) (-2, 5) (-3, -2) (2, -3)

Topic 1.5 Task 1

Plot the following points on your Cartesian plane. Label each point with a letter and the ordered pair – the coordinates—on the grid:

a) (2, 3) b) (4, 1) c) (-2, 3) d) (-2, -3) e) (4, -1) f) (0, 5) g) (3, 0)



Problem Solving Activity



Task:

Topic 1.5 Task 2

- Plot the coordinates (-3, -2); (-2, -1); (-1, 0); (0, +1); (+1, +2); (+2, +3) and (+3, +4) on a plane drawn in your copy book then join all the points by drawing one straight line.
- Find a relationship between the x and the y values for the points given in (a)—that is, what math is done to the x value to equal the y value?



Guided Practice Activity



Tasks:

Topic 1.5 Task 3

Complete the following tables, list out the points given, then make a sketch of the graphs.

a.

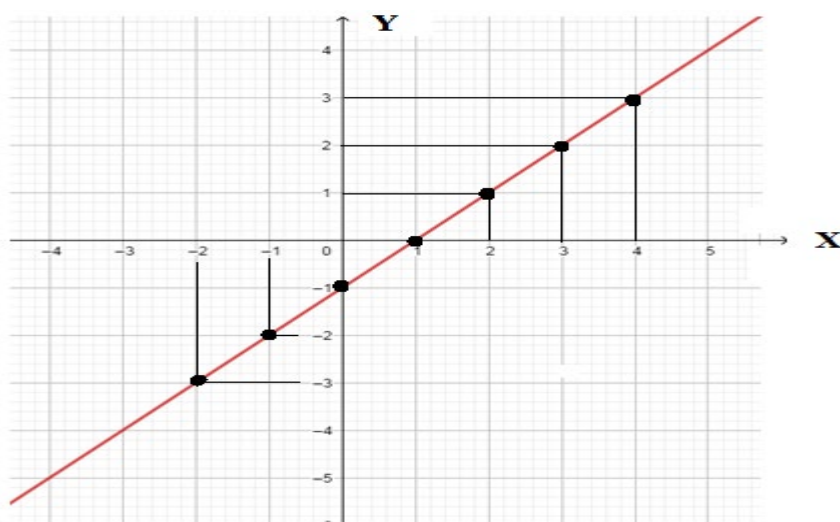
x	-3	-2	-1	0	1	2	3
$Y = 2x + 2$							

b.

x	-1	0	1	2	3	4	5
$Y = x^2 - 4x + 3$							

Topic 1.5 Task 4

Given the following graph, write out coordinates for at least 5 points then try to find an equation that lets you find y for any given value of x.



Topic 1.5 Task 5

Consider the function: $y = 2x^2 + 3x - 2$

a. Use the equation given to complete the following table of values:

x	-3	-2	-1	0	1	2
$Y = 2x^2 + 3x - 2$	7	0	-3	-2	3	12

b. List the coordinates in ordered pair format (x, y) for all the values that you have found.

c. Plot the graphs on Cartesian plane.

- d. List the points where the graph crosses the x-axis. These points are called the X-INTERCEPTS.
- e. Give the coordinates for the Y-INTERCEPT is where the graph crosses the y axis.
- f. Describe the shape of the curve you find.

Topic 1.5 Task 6

- a. Find a few points on the given function's graph. State if the function is a line or a curve. Sketch the graph for the function. And find the x- and y-intercepts.
 $y = 3x + 6$
- b. $y = x^2 - x - 2$
- c. $y = x - 4x + 9$
- d. $y = x^2 - 9$

Topic 1.5 Task 7

The local football team is trying to make money to cover costs of a trip to Goma to participate in an international tournament. They are selling flags. For each flag they sell they make 500Frw profit, each team member needs to make 8 500 Frw to cover the cost of their trip. If they sell 0 flags, they make 0 profit. If they sell one flag, they make 500 Frw profit and so forth. This gives the points (0, 0) and (1, 500). Find a few more points so you can plot the graph.

- a. Plot the graph which shows the number of flags sold as the x value and the total profit as they value.
 NOW use your graph to determine:
- b. how many flags they need to sell to make a profit of 8 500 Frw
- c. how much profit is made if they sell 7 flags

1.5 Key Facts

- An **ordered pair** describes the positions of a point on a Cartesian plane. An ordered pair is made up of two coordinates: the first number is called the **x-coordinate** giving the distance from the origin in the direction of the x-axis, the second number is called the **y-coordinate** giving the distance from the origin in the direction of the y-axis.
- A point where a graph crosses the x axis is called an **x-intercept**. The y coordinate is 0.
- A point where a graph crosses the y axis is called a **y-intercept**. The x coordinate is 0.
- The graph of any equation in the form $y = mx + c$ will be a straight line. Such equations are known as **linear functions**. C is the y-coordinate of the y intercept (where the graph crosses the y axis) and m is the slope.
- When drawing the graph of a linear equation, it is sufficient to plot only two points. BUT it is wise to plot three points to check whether there could be an error in your calculation. To draw the graph for a curve, you will need to plot more than 3 points.

- The equation $y = ax^2 + bx + c$, where a , b and c are numbers, is called a **quadratic function** of the second degree (highest power of x is two).
- Any quadratic function has a curve for a graph called a parabola—it is a symmetrical shape with an axis of symmetry which is parallel to the y -axis with the equation $x = -b/2a$.
- The vertex is the point where the graph of the function crosses the line of symmetry. Consider the quadratic equation $ax^2 + bx + c = 0$. The quadratic formula
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
 can be used to solve any quadratic equation provided there is a solution (there is not always).



Application Activity



Tasks:

Topic 1.5 Task 8

Mary is offered a part-time job selling restaurant equipment. She will be paid Frw100 000 per month plus a 6% commission on her sales. The sales manager tells Mary she can expect to sell about Frw 800 000 worth of equipment per month. Draw a graph to represent this situation to help Mary understand her potential income for different levels of sales, so x will be sales and y will be income for your points. The values are given in the table below.

Use your graph to determine how much Mary will have to sell to make 200 000 Frw.

Monthly incomes for each sales total.

Sales (x -value)	6% of Commission	Mary's Monthly Income (y -value)
500000	$\frac{6 \times 500000}{100} = 30000$	$100000 + 30000 = 130000$
800000	$\frac{6 \times 800000}{100} = 48000$	$100000 + 48000 = 148000$
1,000,000	$\frac{6 \times 1000000}{100} = 60000$	$100000 + 60000 = 160000$
1,200,000	$\frac{6 \times 1200000}{100} = 72000$	$100000 + 72000 = 172000$

Topic 1.5 Task 9

Graph the quadratic equation given below.

$$x^2 + 7x + 6 = y$$

Topic 1.5 Task 10

Given function: $y = x^2 + 9x + 14$

Find coordinates for points by putting values for x into the equations to find related y values. Plot the graph of the function.

Find the solutions to the equation $0 = x^2 + 9x + 14$, that is, find the x intercepts of the graph.



Points to Remember

- Coordinates that define points on a Cartesian plane are always in the order of (x, y) so
 - x -intercepts will always have a zero in the y position.
Ex.: The x intercept for the graph of the equation $y=3x - 6$ will be $(2, 0)$
 - And y intercepts will always have a zero in the x position
Ex.: The y intercept for the graph of the equation $y=-2x + 5$ will be $(0, 5)$
- Graphing a function can be done by plotting points on a grid—the more points that you plot the more accurate the sketch of your graph will be.
- A linear equation will give the graph of a straight line—a quadratic function (with a variable that is squared) will give the graph of a parabola which is a curve.



Formative Assessment

1. Find coordinates of x and Y for the given functions, sketch the graph, and give coordinates for the x - and the y -intercepts.
 - a. $y = 2x - 1$
 - b. $y = x^2 - 1$



Self-Reflection

1. Draw ☹, ☺, or 😊 to indicate how well you feel you have understood the Unit 1 concepts.
2. Take a few minutes to RE-TAKE the self-assessment at the beginning of Unit 1. Now that you have completed the unit of study, use an X to indicate how well you have understood the concepts listed on the left side and see which areas you have improved.

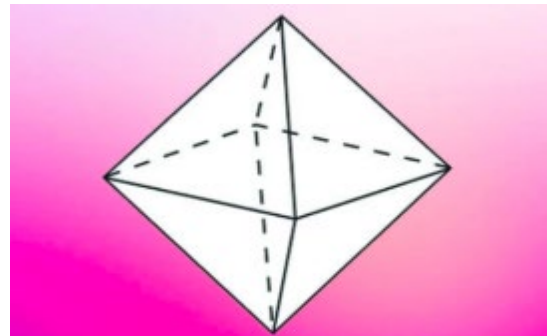
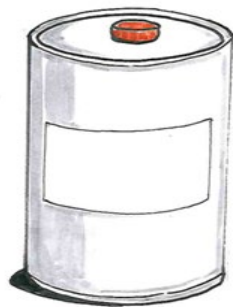
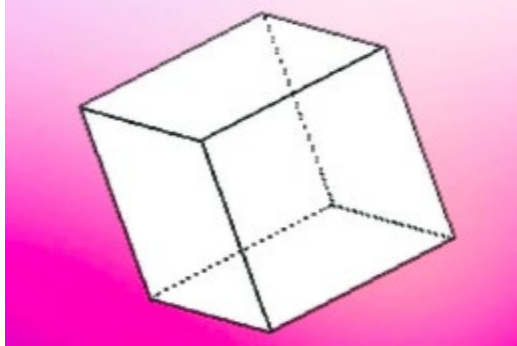
Note: this self-assessment is designed to help you identify your strengths and the areas where you need improvement, and to enable the trainer to focus support to where you most need it.

My experience	I have no experience doing this topic.	I know a little about this topic.	I have some experience doing this topic.	I have a lot of experience with this topic	I am confident in this topic.
Knowledge, skills and attitudes					
Use Venn diagrams and accurately determine union, intersection, and complement of 2 or 3 sets					
Express percentages as fractions and ratios					
Explain direct and indirect proportion using practical examples					
Solve everyday problems involving direct or indirect proportion					
Find the quantity and the price of mixtures					
Increase and decrease quantities by percentages					
Solve problems of discount, commission, profit/loss, interest, and tax using percentages					
Plot and interpret graphs of linear and quadratic functions					
Apply algebraic concepts to form/solve equations and inequalities					

3. Complete the following table to summarize and focus learning intentions for the future.

Areas of strength	Areas for improvement	Actions to be taken to improve
1.	1.	1.
2.	2.	2.

Unit 2: Geometric Shapes and Angle Properties



Topics

2.1 Sketch shapes and angles to illustrate relevant relationships and properties

- 2.2** Draw regular and irregular shapes and angles to represent properties of the given figures
- 2.3** Measure lengths and calculate areas of shapes using formulas and Thales theorem
- 2.4** Construct mathematical arguments using angle properties of parallel lines
- 2.5** Calculate, using given formulas, the surface area and volume of common geometric solids

Unit Summary:

People use geometrical concepts and properties in order to make their work more precise, more beautiful, and more efficient. Geometrical shapes, solids and angle properties are used in different area like painting, carpentry, construction, agriculture, art, ... The purpose of this unit is to:

- ✓ Investigate properties of different geometric shapes (triangle, square, rectangle, circle, trapeziums, as well as irregular polygons) and solids (cube, cylinder, prism, cone, and sphere), as well as irregular solids
- ✓ Solve everyday problems involving geometric shapes and solids where the concepts perimeter, area, surface area, and volume will be explored in context.

Self-Assessment: Unit 2

1. Look at the illustration at the beginning of this unit. Describe what you see. What do you think this unit will be about? What topics might be covered? Write three or four sentences in your notebook that describe what you expect to learn based on your observations.
2. Fill in the self-assessment below. There is no right or wrong way to answer this survey. It is for your own use to monitor your own learning. The trainer will read a skill that is listed in the left column. Think about yourself: do you think you can do this? How well? Read the statements across the top. Put a check in column that best represents your situation.
3. At the end of this unit, we'll take this survey again to see how you have improved.

My experience	I don't have any experience doing this topic.	I know a little about this topic.	I have some experience doing this topic.	I have a lot of experience with this topic.	I am confident in my ability to do this topic.
Knowledge, skills and attitudes					
Sketch triangle, square, rectangle, hexagon, and pentagon in accordance with their properties					
Find the sum of interior angle measures of any regular polygon (triangle, square, pentagon, and hexagon, etc.)					
Calculate side lengths and areas of similar shapes					
Draw two parallel lines and a transversal, identify all angles that are equal, vertically opposite, corresponding, alternate, or supplementary					
Solve missing angle problems using the angle properties of parallel lines having constructed mathematical arguments					
Calculate the surface area and volume of a cube, rectangular and triangular prisms, pyramid, cylinder, cone and sphere					

Topic 2.1: Sketch shapes and angles to illustrate relevant relationships and properties

Key Competencies:

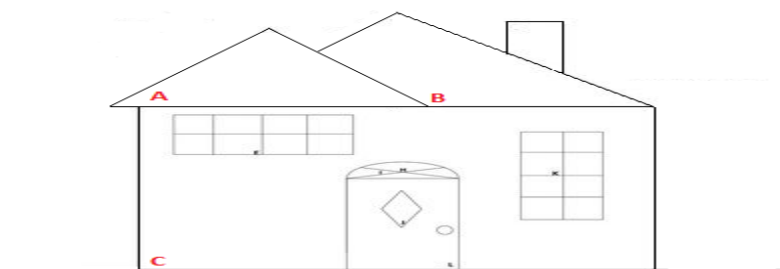
Knowledge	Skills	Attitudes
1. Classify shapes and angles based on their properties (triangle, square, rectangle, circle)	1. Sketch 2-d shapes, measure sides and angles, and calculate area and perimeter	1. Persistent
2. Based on parallel line properties, identify equal angles, vertically opposite angles, corresponding angles, alternate, and supplementary angles	2. Analyse properties of parallel lines and the angles	2. Attentive to details

🔄 Getting Started: What do we know and where are we going?



Task:

Topic 2.1 Task 1:



- Observe the picture carefully. List out all the shapes you see in the picture.
- Look at the different angles formed in the picture: Identify examples of:
 - right angles (90 degrees—the intersecting lines are perpendicular)
 - acute angles (less than 90 degrees)
 - obtuse angles (greater than 90 degrees)
 - supplementary angles (angles that add up to be 180 degrees—or a straight line)
- Label angles A, B and C as either right, acute, or obtuse.
- Explain the use of the following geometry tools: Protractor, Ruler, T-square, Compass



Problem Solving Activity



Task:

Topic 2.1 Task 2

Each trainee works on one talk-and-task then shares answers within their group:

Talk and Task A:	
<p>Talk A: What do you know about the square and rectangle? How are they the same? How are they different? What is a parallelogram?</p>	<p>Task A: On a piece of paper, accurately draw: A square with perimeter 16 cm A rectangle with an area of 20 cm²</p>
Talk and Task B	
<p>Talk B: What do you know about triangle properties? What is an isosceles triangle? What is a right triangle? What is an equilateral triangle?</p>	<p>Task B: Aloys has been asked to re-draw the isosceles triangle below on a plain sheet of paper using only a protractor and a ruler. Create a step by step plan to show how to do this.</p> <div data-bbox="612 1050 976 1296" data-label="Image"> <p>The diagram shows an isosceles triangle with a dashed base labeled '7 cm' and two dashed sides. The base angles are each labeled '40°'.</p> </div> <p>Now draw a right-angled triangle with a height of 8 cm and a base of 6 cm.</p>
Talk and Task C	
<p>Talk C: What are perpendicular lines? What are parallel lines? What angle is formed when two lines have a perpendicular intersection? What is the radius of a circle? What is the diameter of a circle? What is the circumference of a circle?</p>	<p>Task C: Observe the picture and find out the sum of the angles in red.</p> <div data-bbox="596 1592 1062 1951" data-label="Image"> <p>The diagram shows two perpendicular lines intersecting. At the intersection, there is a red square. To the right of the intersection, a right angle is labeled '90°'.</p> </div> <p>Now draw a circle with radius 2 cm.</p>

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2.1 Key Facts

A **polygon** is a closed shape formed by three or more straight sides.

A **regular polygon** has all sides the same length and all angles equal.

An **irregular polygon** is a shape with different length sides and different sized angles.

A **convex polygon** has interior angles all less than 180° (first example below).

A concave polygon has at least one interior angle greater than 180° (second example).

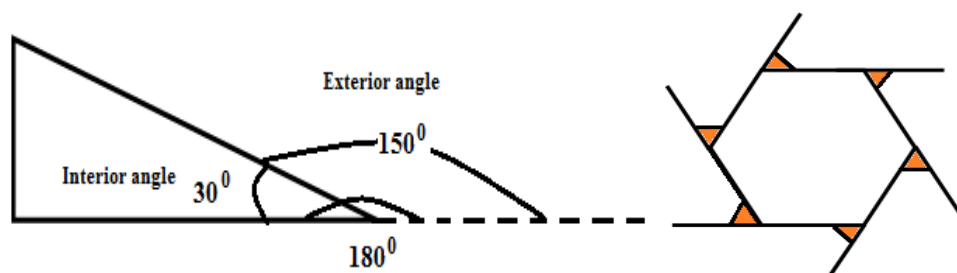
Concave Polygon:



Convex Polygon:



The **exterior angle** is the angle between any side of a shape and a line extended from the next side. Examples below show exterior angles for a triangle and for a hexagon.



Count one exterior angle at each vertex of a polygon and the sum is always 360° .

A **square** has 4 equal sides, 4 right (90°) angles, and 2 equal diagonals that intersect at right angles.






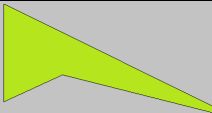

A **rectangle** has 4 sides with opposite sides are equal. All angles are right angles (90°). Diagonals are equal but may not meet at right angles.

A **parallelogram** has 4 sides. Opposite sides and opposite angles are equal.

Right triangle is a triangle that has a right angle (90°). **Isosceles triangles** have two equal sides and two equal angles. **Equilateral triangles** have all equal sides and all equal angles.

A **circle** is a curved shape such that all points on the circle are equidistant (the same distance) from the centre. **Radius** is the distance from the centre to the edge of a circle. **Diameter** is two times the radius—or the distance from one edge, passing through the centre, straight to the other edge. **Circumference** is the distance around the circle.

A **Quadrilateral** is a 4-sided polygon.

Name of the shape:	Picture of Quadrilateral	Properties of the Quadrilateral
Square		All sides are equal. Opposite sides are parallel. All angles are equal and measure 90° .
Rectangle		Opposite sides are parallel. Opposite sides are equal. All angles are equal and measure 90° .
Parallelogram		Opposite sides are parallel. Opposite sides are equal. Opposite angles are equal.
Rhombus		All sides are equal. Opposite angles are equal.
Trapezoid or Trapezium		Top and bottom are parallel.
Any quadrilateral		4 sides which may not be equal 4 angles which may not equal
Circle		A closed curve where all points are equidistant from the centre.
NOTE: the only regular polygon shown above is a square. Explain why!		



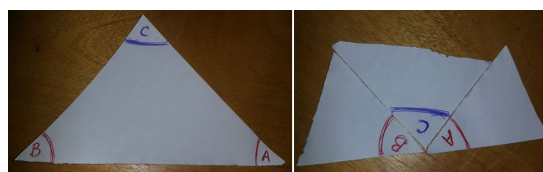
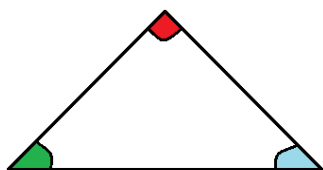
Guided Practice Activity



Tasks:

Topic 2.1 Task 3

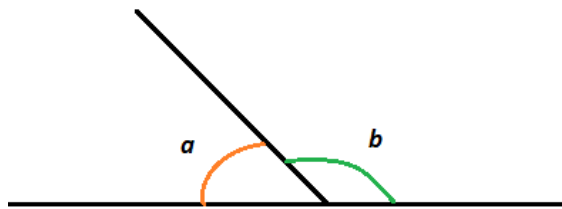
Use a ruler to draw a triangle on the page of their notebook. Carefully cut out the triangle. THEN rip off the corners (which are called vertices). Place all vertices together so angles are adjacent (next to one another) but not overlapping as shown as is shown in the figures below:



- What do you observe?
- Notice that each trainee made a slightly different triangle, have they found the same thing?
- What does this tell us about ALL triangles?

Topic 2.1 Task 4

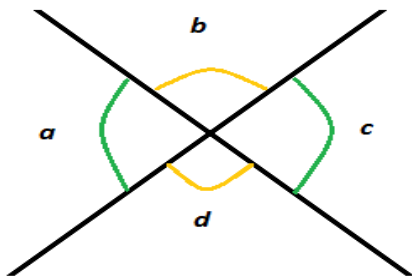
Observe the following diagram carefully.



- We can see that angles a and b , when taken together, form a straight line. What do we call two such angles?
- What is the angle measure of $a + b$?
- What angle will we get if we take $180^\circ - a$?

Topic 2.1 Task 5

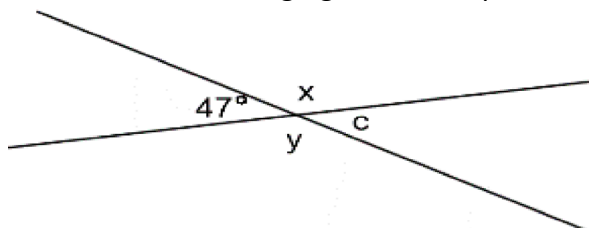
Observe the following figure and respond to the questions below.



- What do you notice about angles b and d ?
- What do you notice about angles a and c ?
- Is this always the case?
- What do you notice about angles a and b ?

Topic 2.1 Task 6

Observe the following figure and respond to the questions below.



- Which angle is opposite to angle x ?
- Which angles are supplementary to angle x ?
- Find the angle measures for angle c , angle x , and angle y .



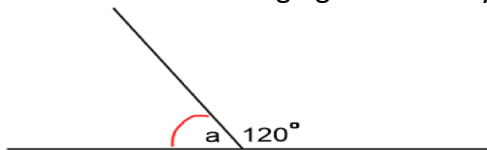
Application Activity



Tasks:

Topic 2.1 Task 7

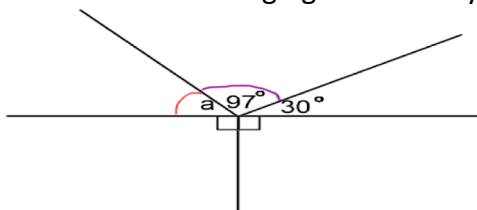
Observe the following figure carefully and find the measure of angle a.



- What do we call angle a and angle 120 together?
- Find the angle measure of angle a.

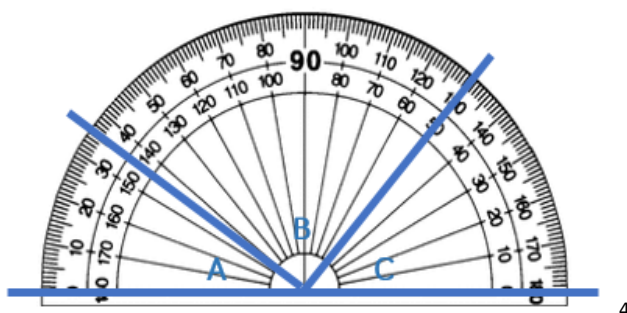
Topic 2.1 Task 8

Observe the following figure carefully and find the measure of angle a.



Points to Remember

- Two lines are **parallel** if they have no point of intersection.
- When two lines intersect at a point, four angles are formed.
- Two lines are **perpendicular** if their intersection forms right angles.
- To bisect an angle means divide the angle into two equal (congruent) parts.
- When measuring angles, make sure that:
 - ▶ The centre of the protractor is placed on the vertex of the angle.
 - ▶ The zero of the protractor is placed on one side of the angle.



4

⁴ Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher’s Manual and Learner’s Book*. Waltham: Education Development Center) Education Development Center.

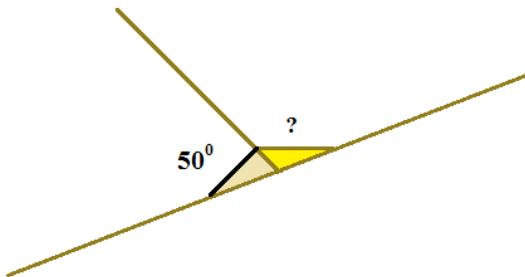


Formative Assessment

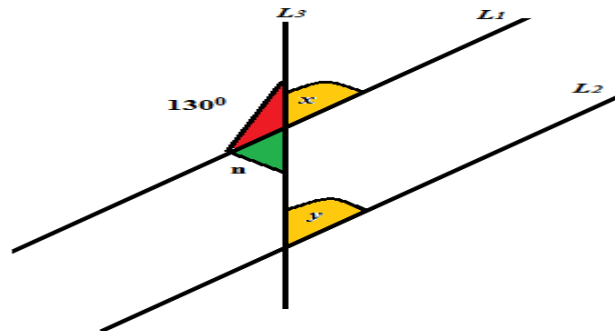
1.

Observe the figures, explain how the missing angle can be calculated, and find the angle measure for the following figures:

a)



b)



2.

List quadrilaterals you know and their properties. Can you describe how properties can help you to sketch square, parallelogram, trapezium, rectangle or any other quadrilateral?

Topic 2.2: Draw regular and irregular shapes as well as angles to represent properties of the given figures

Key Competencies:

Knowledge	Skills	Attitudes
1. State the properties of polygons	1. Draw regular and irregular polygons based on their properties and measures	1. Attentive to detail
2. Explain how to find the sum of interior angles of a regular polygon	2. Calculate the sum of interior angles of any regular polygon	2. Innovative
3. Explain how to find perimeter and area of regular and irregular shapes	3. Determine perimeter and area of a regular and irregular shapes	3. Accurate



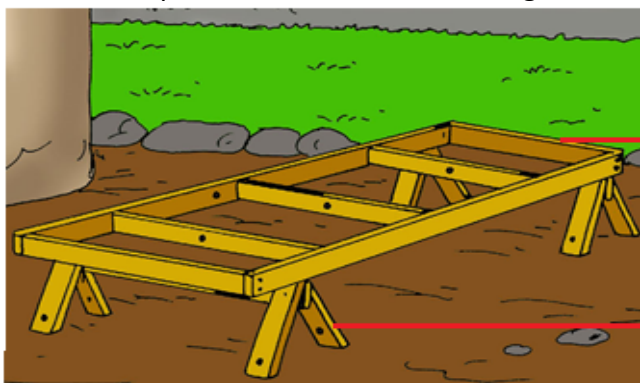
Getting Started: What do we know and where are we going?



Tasks:

Topic 2.2 Task 1

Observe the picture below. Name the angles and shapes used by a carpenter to make this table:

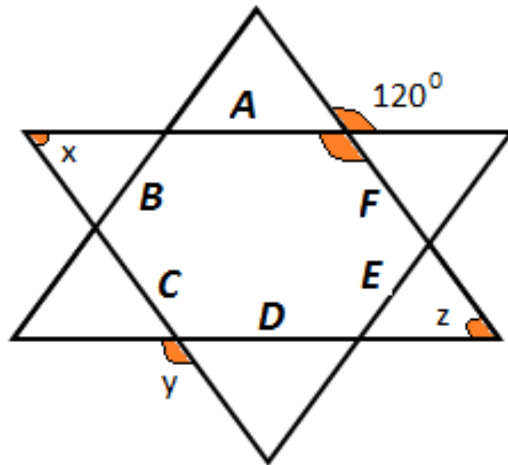


This part is made following rectangle properties

This part is made following triangle properties

Topic 2.2 Task 2

Observe the following illustration. Notice that the star is made up of three different sets of parallel lines.



- Extend the sides of the star to show better which pairs of lines are parallel.
- Look carefully at the figure and see if you can find a parallelogram inside this figure. Outline that parallelogram with a thicker line.
- Now that we have several sets of parallel lines and also a parallelogram, and the measure of one angle, find the angle measures for the angles x , y , and z . Be sure to explain your logic.



Problem Solving Activity



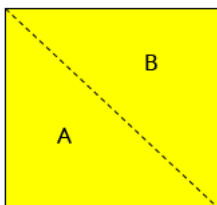
Tasks:

Topic 2.2, Task 3

Review the activity in the last Topic: you drew a triangle and cut it out then placed the vertices together, finding there are 180 degrees total for the angles inside a triangle.

Keep that information in mind: **The angles inside a triangle add to a total of 180 degrees.**

Draw a square. Split the square into 2 triangles with a diagonal drawn between opposite vertices per the picture:







- What do the angles of triangle A add up to?
- What do the angles of triangle B add up to?
- Given that the angles inside the square equal the sum of the angles from the two triangles, make a guess what the angles inside a square add up to.
- Think about the properties of a square: we know a square has 4 right angles. Add up the total of 4 right angles to confirm your answer from (c).

Topic 2.2 Task 4

Think about patterns made by dividing polygons into triangles.
We can use such a pattern to figure out the interior angle sum for any regular polygon.

- a. Review the first two rows of the following table and then, complete the missing information by filling in the remaining information on the last three rows in the table below.

Shape	Number of sides	Number of triangles	$180 \times \text{number of triangles}$	Sum of internal angles
 Square	4	2	180×2	360°
 Pentagon	5	3	180×3	540°
 Hexagon	6			
 Heptagon	7			
n sides Polygon	n sides			

- b. How can we use this to work out the interior angles of regular polygons? Can we spot a pattern in the table? What predictions can we make?



Guided Practice Activity

2.2 Key Facts

A **polygon** is a closed shape formed by three or more straight sides. Included among polygons are triangles, squares, rectangles, trapezoid, parallelogram, hexagon, etc.

The **properties of polygons** include number of sides, number of vertices, length of sides, size of angles, parallel lines, diagonals, and lines of symmetry.

The sum of the measures of interior angles of a polygon with n-sides is always $(n-2) \times 180^\circ$

The **perimeter** of any regular or irregular polygon is the sum of its sides.

Area of a parallelogram (square, rectangle, rhombus) is calculated by base \times height ($b \times h$)

Area of a triangle is calculated by $\frac{1}{2} \times \text{base} \times \text{height}$ ($\frac{1}{2} \times b \times h$).

Area of an irregular polygon is calculated after dividing it into already known shapes like triangles and parallelograms. Calculate the area for known shapes then find the sum.

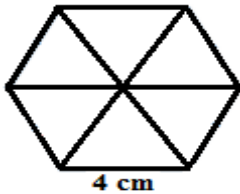
In a right triangle we can use the **Pythagorean Theorem** to find the length of all sides. Given a triangle with sides a , b , and c , with c being the hypotenuse we have $a^2 + b^2 = c^2$



Tasks:

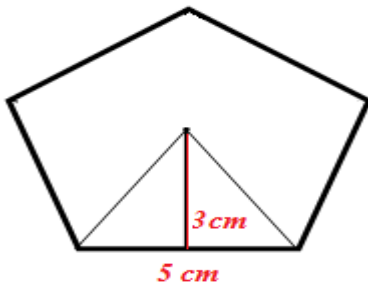
Topic 2.2 Task 5

Observe the hexagon below. Each side measures 4 cm. Calculate the perimeter of the hexagon.



Topic 2.2 Task 6

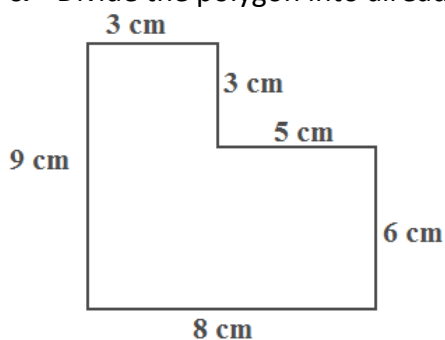
Observe the following regular polygon. Find the perimeter and calculate the area if the side is 5 cm and the height of the given triangle is 3 cm.



Topic 2.2 Task 7

Observe the irregular polygon below.

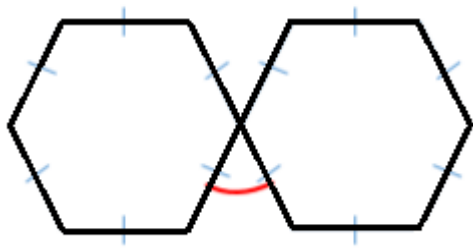
- How many sides does the figure have?
- Find the perimeter.
- Divide the polygon into already known shapes and calculate the area.



Topic 2.2 Task 8

Observe the two regular hexagons below. Each hexagon has 6 sides ($n=6$) and each side is 5 cm. We know that the sum of the interior angles of a hexagon is 720° . $(n-2) \times 180 = 4 \times 180 = 720^\circ$.

- Find the missing angle marked between the two hexagons below.
- Find the perimeter and area of the hexagon if each of the interior triangles has height of 3 cm.



Application Activity

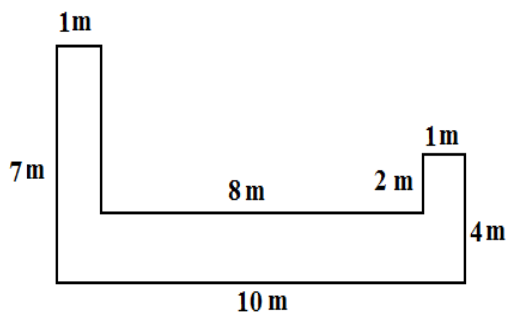


Tasks:

Topic 2.2 Task 9

A plot of land has an irregular form with shape and dimensions shown below.

- The farmer wants to put up a fence area. How many meters of fencing is needed?
- The farmer knows that sheep should each have 3m^2 of pasture to graze well. How many sheep can the farmer put inside this enclosed land if they are to eat well?



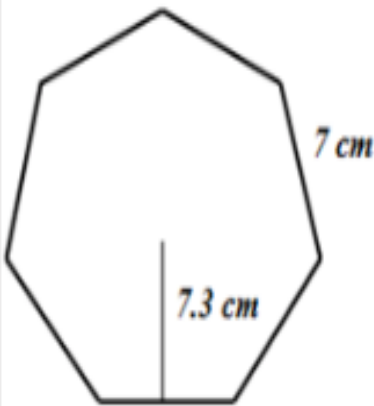
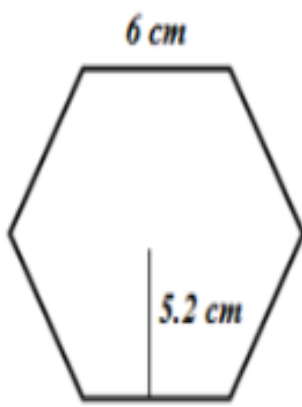
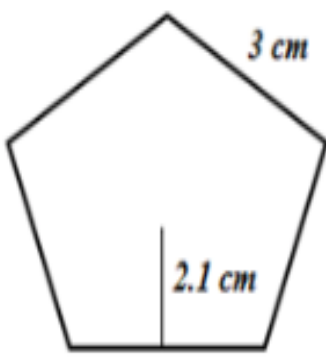
Topic 2.2 Task 10

A farmer has a plot of land in the form of a parallelogram whose base is 120 m. He needs to fence it. What is the length of the fencing he will need to surround his plot if the slanting side is 70 m long?

Topic 2.2 Task 11

In the following task s =side length and a =altitude which is the distance from the mid-point of one side to the centre of the figure. Use the given a to figure out the area of the triangles inside the figure.

Calculate the area and perimeter for each polygon

Heptagon	Hexagon	Pentagon
 <p>$a = 7.3 \text{ cm}$ $s = 7 \text{ cm}$</p>	 <p>$a = 5.2 \text{ cm}$ $s = 6 \text{ cm}$</p>	 <p>$a = 2.1 \text{ cm}$ $s = 3 \text{ cm}$</p>



Points to Remember

Regular polygons are named by the number of sides that they have, and can be divided into triangles that all meet at the centre point.

To find **perimeter** of a shape we add up the lengths of all the sides

To find the **area** of a shape we divide it into shapes that we know (mostly into parallelograms where $\text{Area} = \text{base} \times \text{height}$ and triangles where $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$)

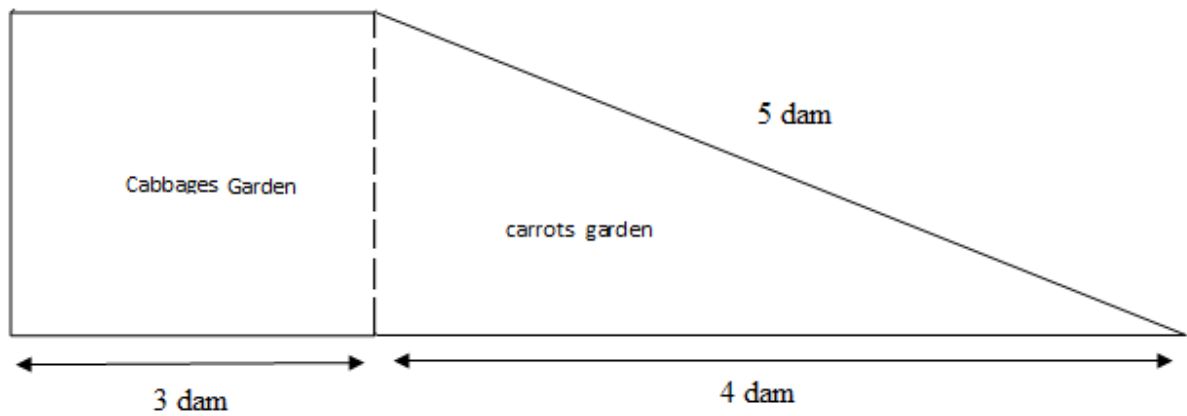


Formative Assessment

Work in pairs to solve the following problems and be prepared to explain your answers to the class.

A vegetable garden is divided into 2 parts. A squared part of cabbages and a triangular part of carrots as shown below:⁵

⁵ Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher's Manual and Learner's Book*. Waltham: Education Development Center) Education Development Center.



- What is the perimeter of the cabbage garden and that of the carrot garden?
- What kind of shape do those two sections of a vegetable garden make? Calculate the perimeter of that shape.
- Calculate the area of the entire garden area.

Topic 2.3: Calculate lengths and areas of similar shapes using their properties (Thales theorem)

Key Competencies:

Knowledge	Skills	Attitudes
1. Explain the Midpoint Theorem	1. Solve problems involving Midpoint Theorem	1. Collaborative
2. Explain Thales Theorem and its converse	2. Solve problems involving Thales Theorem	2. Accurate
3. Identify proportional properties of given figures	3. Calculate lengths of proportional sides of different figures	3. Focused on Detail

Getting Started: What do we know and where are we going?



Tasks:

Topic 2.3 Task 1

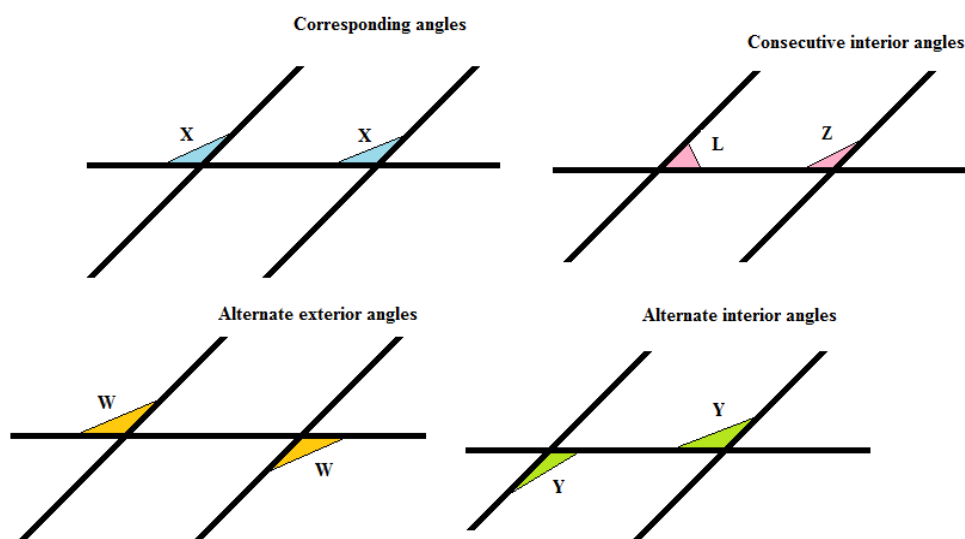
Describe and draw an example of each of the following angles:

- a) corresponding angles
- b) consecutive interior angles
- c) alternate exterior angles
- d) alternate interior angles

Topic 2.3 Task 2

Think about the pairs of angles that are formed when two parallel lines are cut by a transversal. These were named and described in Task 1.

Explain which of the pairs of angles are equal and which are supplementary:

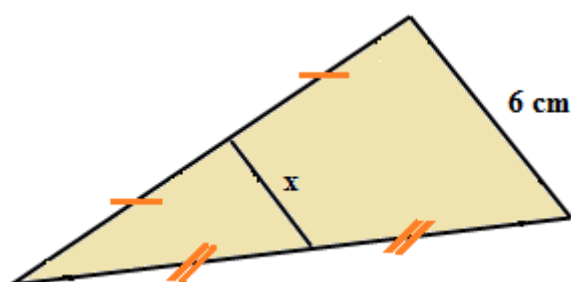


Topic 2.3 Task 3

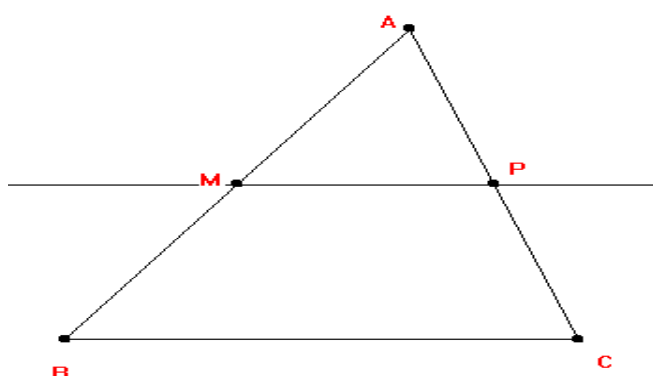
Midpoint Theorem:

The segment joining the mid-points of two sides of a triangle is parallel to the third side and that segment is half the length of the third side.

This is illustrated in the following diagram. Segment X bisects the two sides, and thus is parallel to the base side (which measures 6 cm) and is half as long as the base side of the bigger triangle. Segment X = $\frac{1}{2} \times 6 \text{ cm} = 3 \text{ cm}$.



Observe the triangle drawn below while keeping Midpoint Theorem in mind.



If the line through M and P bisects the two sides it intersects, and if Segment BC is 12 m long, tell two facts that we know about segment MP.



Problem Solving Activity

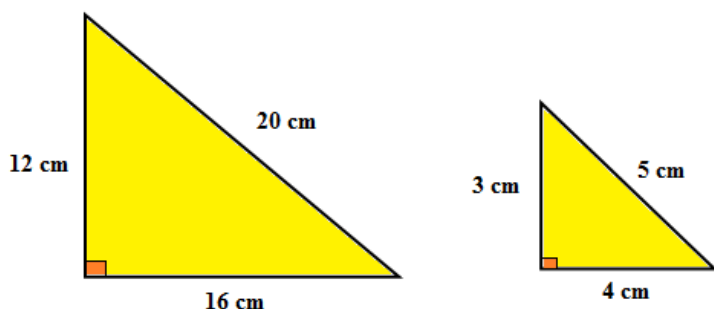


Tasks:

Topic 2.3 Task 4

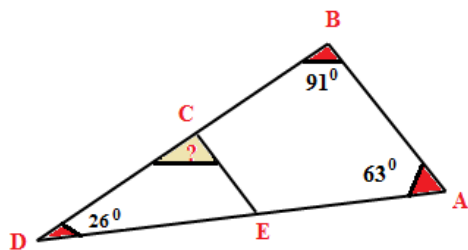
Carefully observe the two triangles given below.

- Using a ruler and a protractor, redraw the two right triangles in your notebook.
- State the pattern that you see when you compare the measures of their sides.
- Use a protractor to measure the angles of the two triangles.
- What do you notice about the measures of the angles of these two triangles?



Topic 2.3 Task 5

The figure below shows a large triangle with vertices A, B and D, and a small triangle with vertices C, D, and E. Point C is the mid-point of BD and point E is the mid-point of AD.



The two triangles in the figure above are similar triangles.

Observe the figure carefully and complete the following sentences.

Knowing that Angle D corresponds to angle D in both triangles, which angle corresponds to:

- Angle E?
- Angle C?
- What is the angle measure for the marked angle at C?
- If segment DE measures 8 cm, what is the measure of segment EA?
- If segment DB measures 15 cm, find the length of segment DC.



Guided Practice Activity

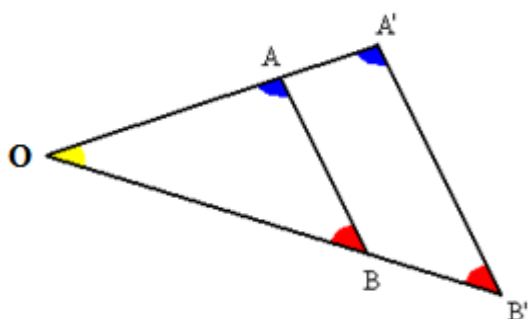
2.3 Key Facts

Similar figures are identical in shape (angles will be equal) but not necessarily the same size—the corresponding edges are proportional to each other but not the same length.

If two triangles are similar their angles are equal and their corresponding sides are proportional.

When two triangles are proportional, the reduced ratio of any pair of corresponding sides is called the '**scale factor**'. Use the scale factor to multiply the side lengths of the smaller figure by that scale factor to get the side lengths of the bigger figure

According to **Thales Theorem**, when two triangles have a common angle and they have parallel opposite sides then the two triangles are similar and will have proportional sides.

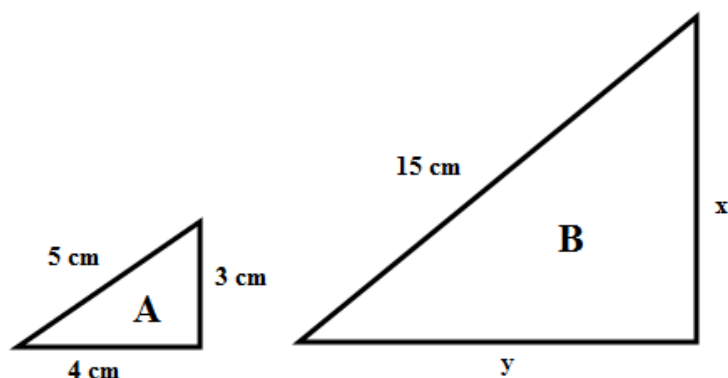


Task:

Topic 2.3 Task 6

Observe the two similar right triangles.

- Find the scale factor and then the value of X and Y.
- The perimeter and area of each triangle.

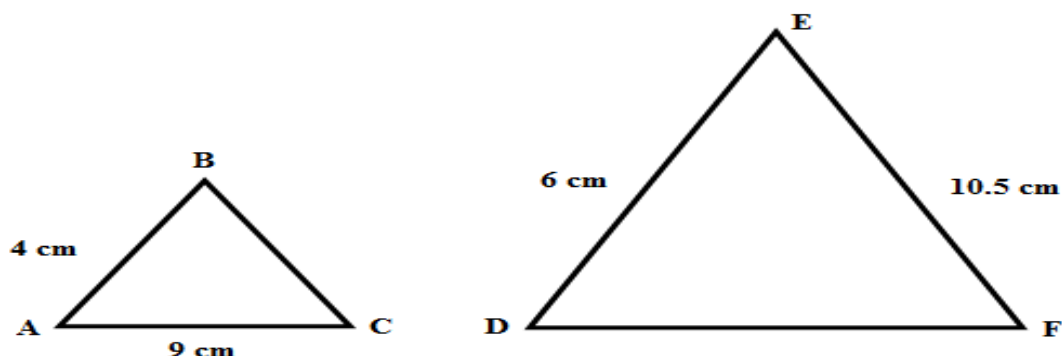


The scale factor is $15:5 = 3$

Topic 2.3 Task 7

Triangles ABC and DEF are similar.

- Work out the lengths of DF and BC and give the perimeter of both triangles.
- If the area of the small triangle is 36 cm^2 , what is the area of the bigger triangle?



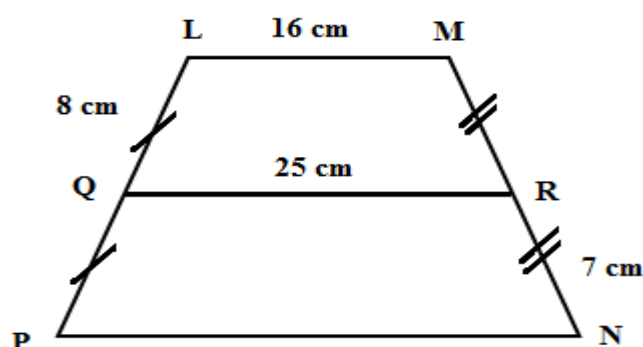
Application Activity



Task:

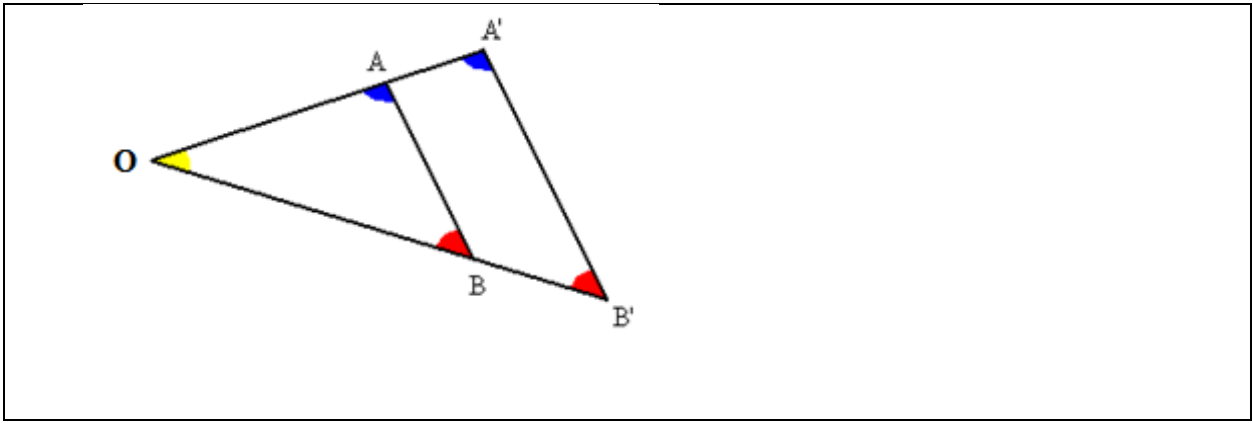
Topic 2.3 Task 8

Given that the length of the mid-segment QR = $\frac{1}{2} (LM + PN)$, find the perimeter of trapezium LMNP.



Points to Remember

- The midpoint theorem states that the segment joining two sides of a triangle at the midpoints of those sides is parallel to the third side and is half the length of the third side.
- Two triangles are similar when they have equal angles and proportional sides.
- Thales Theorem: When two triangles have a common angle and they have parallel opposite sides, they are in **Thales position**. Then they are similar triangles and have proportional sides.
- In this example, the two triangles share angle O, the parallel opposite sides are AB and A'B' so the sides of triangle OAB are proportional to the sides of triangle OA'B'.

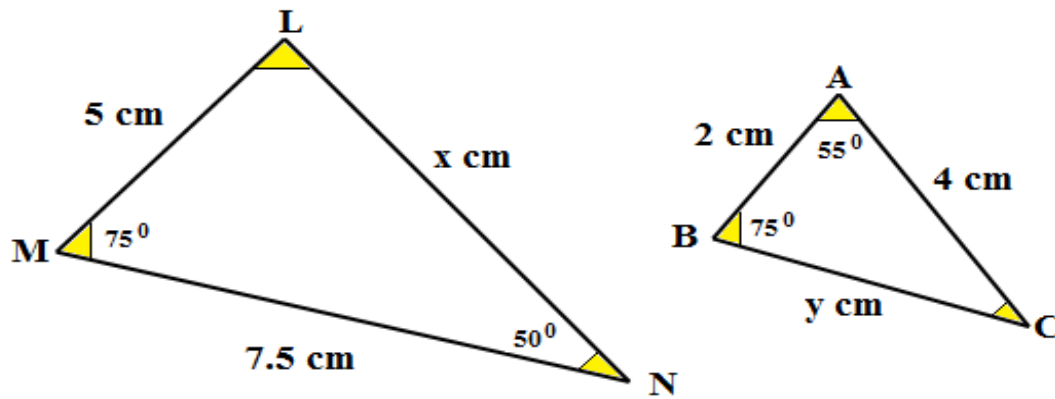




Formative Assessment

The triangle ABC is similar to triangle LMN. Find the:

- values of x and y
- perimeter of the 2 triangles
- missing angle in each of the triangles.



Topic 2.4: Construct mathematical arguments using the angle properties of parallel lines

Key Competencies:

Knowledge	Skills	Attitudes
1. Explain how to draw parallel and transversal lines	1. Draw parallel lines and transversals	1. Persistent
2. Describe properties of angles and of parallel lines	2. Analyse, compare and construct mathematical arguments related to different angles	2. Accurate
3. Provide logical arguments using properties of angles, shapes, and parallel lines	3. Solve missing angle problems including providing reasons for each step	3. Cooperative



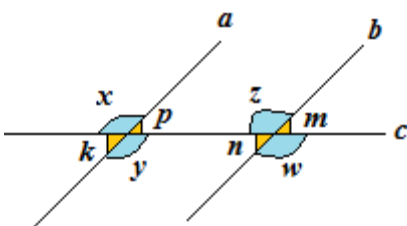
Getting Started: What do we know and where are we going?



Task:

Topic 2.4 Task 1

Copy the following figure into your notebooks then use a protractor to measure each of the angles. Record the angle measures on your drawing.



- Based on the dimensions of the measured angles, classify all angles as either acute or obtuse.
- Find four pairs of opposite angles.
- Find eight pairs of supplementary angles.



Problem Solving Activity



Task:

Topic 2.4 Task 2

There are two tasks below. Each pair will work on one task then teach the other pair in your group how to solve it. Review the pictures and the given information carefully. Respond to the question asked (find the missing angles)—being sure to provide reasons for each step.

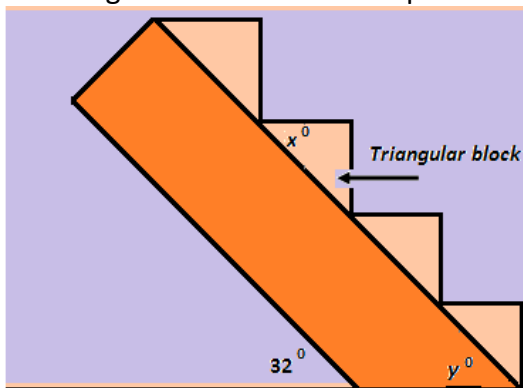
A.

Observe the picture below carefully. Each rung of the ladder is parallel to the rung directly above it. Explain why the top rung is parallel to the bottom rung.



B.

One way to build stairs is to attach triangular blocks to an angled support, as shown in the picture. The sides of the angled support are parallel. If the support makes a 32° angle with the floor, what must angle x° be so that the top of the step will be parallel to the floor? Explain your reasoning.



Guided Practice Activity

2.4 Key Facts

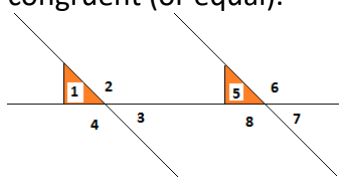
Two lines in a plane are **parallel** if they never intersect, when extended infinitely in both the direction. The distance between two parallel lines remains constant, never changes.

The symbol for denoting parallel lines is \parallel . If lines p and q are parallel to each other, we can write it as $p \parallel q$ and which is read as 'p is parallel to q'.

If two parallel lines are cut by a transversal, then corresponding angles, consecutive interior angles, alternate exterior angles and opposite angles are formed.

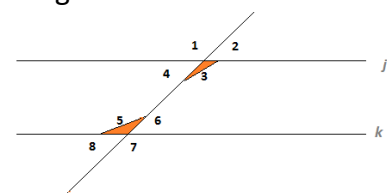
Angles that have the same measure are congruent or equal.

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent (or equal).



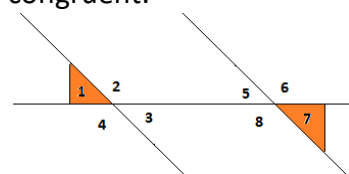
OR the converse which tells us: if two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.



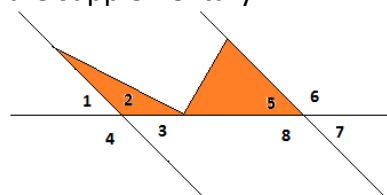
OR the converse which tells us: if two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.



OR the converse which tells us: if two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.



OR the converse which tells us: if two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.

In summary: If two parallel lines are cut by a transversal, then:
 The pairs of corresponding angles are congruent.
 The pairs of alternate interior angles are congruent.
 The pairs of alternate exterior angles are congruent.
 The pairs of consecutive interior angles are supplementary.

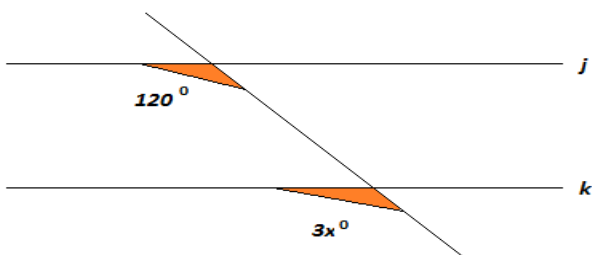


Task:

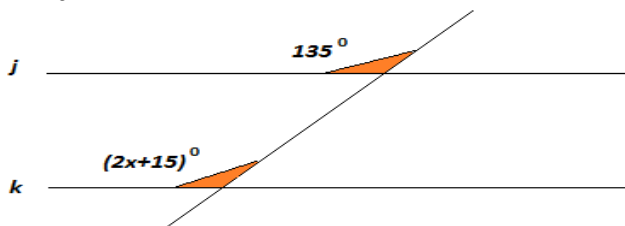
Topic 2.4 Task 3

Observe the angles made by the parallel and transversal lines. Using the information given, identify all the angles and their measures by finding the value for x .

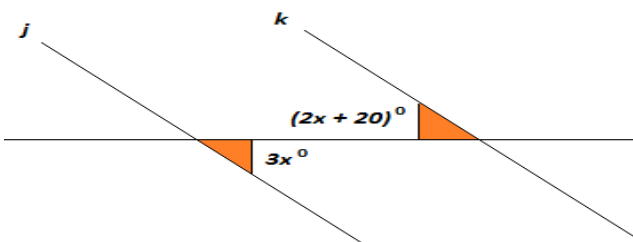
a.



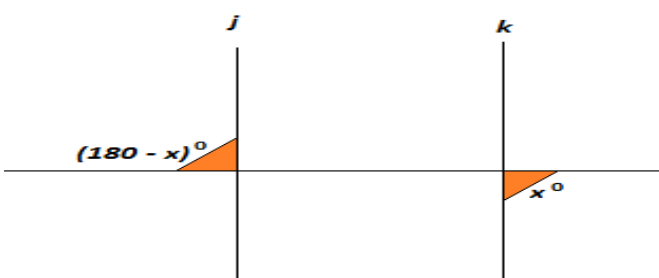
b.



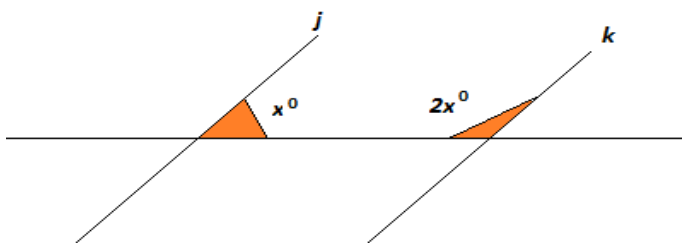
c.



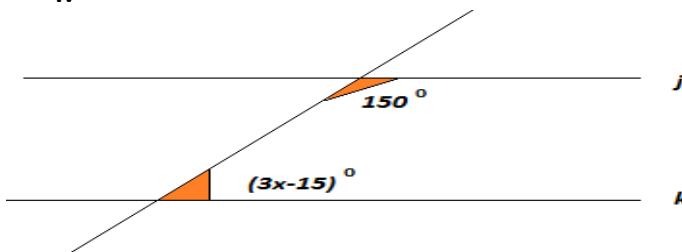
d.



e.



f.



Application Activity



Tasks:

Topic 2.4 Task 4

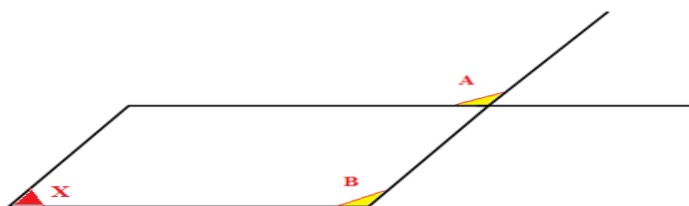
Draw two parallel lines and a transversal line and name all formed angles as follow (use different diagrams):

- Name the corresponding angles by the letter X.
- Name the alternate interior angles by the letter Y.
- Name the alternate exterior angles by the letter W.
- Name the consecutive interior angles by the letter L and Z.

Topic 2.4 Task 5

Observe the following figure. Angle X measures 70° .

- How many angles are show in the drawn figure?
- Find the measure of angles A and B.
- Identify 2 other angles which have the same measure as angle B. Give reasons to explain.





Points to Remember

If two parallel lines are cut by a transversal, then:

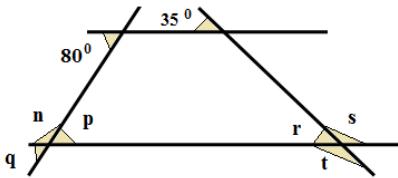
- ✓ The pairs of corresponding angles are congruent/ equal.
- ✓ The pairs of alternate interior angles are congruent/ equal.
- ✓ The pairs of alternate exterior angles are congruent / equal.
- ✓ The pairs of consecutive interior angles are supplementary.



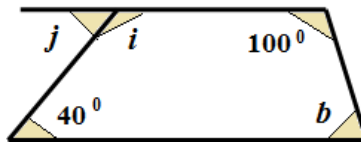
Formative Assessment

1. Find the measure of all angles marked with letters in the figures below, given that the two sides of the figure are parallel and other lines are transversal:

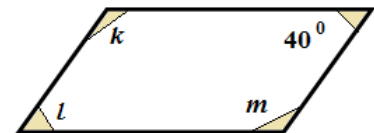
a)



b)



c)



2. Draw ☹, 😐, or 😊 in your notebook after you have completed the formative assessment to indicate how well you feel you have understood the lesson.

Topic 2.5: Calculate the surface area and volume of common geometrical solids using formulas

Key Competencies:

Knowledge	Skills	Attitudes
1. Describe types of solids: prism, pyramid, cylinder, cone, sphere	1. Use formulae to find volumes for different figures	1. Collaborative
2. Identify components of solids: Faces, Vertices, Edges	2. Represent figures using their Net and use the net to calculate surface areas	2. Accurate
3. Describe surface area and volume	3. Determine the number of faces (f), number of vertices (v), and number of edges (e) for a variety of solids	3. Creative

Getting Started: What do we know and where are we going?



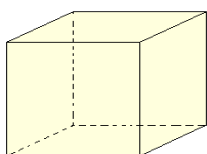
Task:

Topic 2.5 Task 1

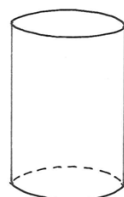
Observe the figures below. For each figure find the following:

a) the number faces b) the number of vertices c) the number of edges d) name the figure

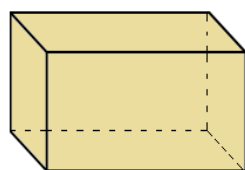
a)



b)



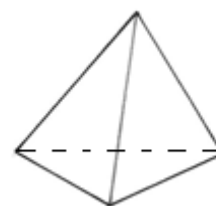
c)



d)



e)



Problem Solving Activity



Task:

Topic 2.5 Task 2: Group Investigation

You have been given an example of a solid figure.

- Count the number of faces, edges and vertices on your solid.
- Carefully measure each edge of the solid and use your measurements to calculate the area of each face.

- c. What do you notice about the relationship between the areas of the faces on your solid?
- d. If you wanted to cover your solid with clean paper, what is the total area of the paper you will need to cover the solid completely? This is called SURFACE AREA—that is the area of the entire surface of the solid figure.



Guided Practice Activity⁶

Demonstration: Trainer will demonstrate the following:

FIRST count the number of faces, edges and vertices of the box and identify it either as a cube or rectangular prism.

THEN cut open the box and lay it out flat on the table.



Tasks:

Topic 2.5 Task 3

Observe the box given to your group during the investigation. Count the number of Faces.

Cut open your box to make a net, confirm that the number of faces has remained the same.

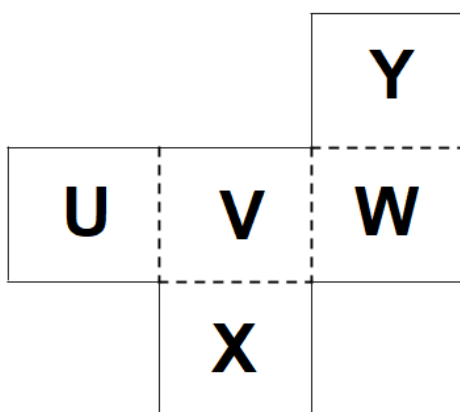
Sketch the net into your copy book.

Re-fold the box to make it into a 3-D figure again, note different groups may have different shaped nets even though their boxes may be similar—this is because of different cuts.

Topic 2.5 Task 4

A piece of paper is cut out and labelled as shown in the diagram.

- a. If it is folded along the dotted lines what kind of solid will it make?
- b. If the box is placed on a table so that the top of the box is upward, then the label at the bottom of the box is ____.



⁶ Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher’s Manual and Learner’s Book*. Waltham: Education Development Center) Education Development Center.

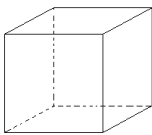
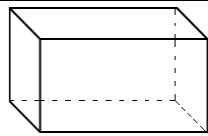


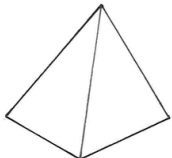
2.5 Key Facts

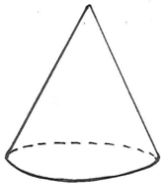
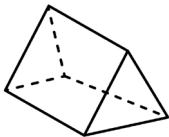
Solids in geometry are three-dimensional shapes: cubes, rectangular prisms, cylinders, spheres, cones, and pyramids.

A net of a solid is a 2-dimensional shape that can be folded to form the 3-dimensional solid. A solid may have different nets—depending how the figure is ‘cut’ to lay it flat.

Surface area of a solid is the sum of the areas of all the faces.

Volume is calculated using the formulae given below.

Volume and surface area of solids			
Figure of solid	Image	Volume (V)	Surface area (S)
Cube E—length of all edges		$V=E^3$, Volume = Length x width x depth	$S=6 \times E^2$, Because each face is a square the area of each face is E^2 and there are 6 faces
Rectangular Prism L—length W—width H--height		$V= L \times W \times H$ Volume = Length x width x height	$S= 2(L \times W) + 2(L \times H) + 2(W \times H)$ There are six faces, each face has an equal pair on the opposite side
Cylinder r- radius h- height		$V=\pi r^2 h$ This formula gives the area of the base (a circle) x height	$S=2 \pi r^2 + 2 \pi r h$ There are 3 faces: 2 circles and one rectangle The lateral side is a rectangle (if cut) with one side = h and the other side being the circumference of the circle
Sphere r-radius		$V = \frac{4}{3} \pi r^3$	$S=4\pi r^2$
Pyramid A-area of base h-height		$V = \frac{1}{3} Ah$ A=area of base	Surface area is calculated by finding the area of each face and adding all together

Cone r-radius of the circular base h-height		$V = \frac{1}{3}\pi r^2 h$	Complicated by slanted side which is not a regular shape
Prisms: Two parallel polygons are the bases of a prism, height is the distance between bases		Volume= Area of base x height	Surface area is calculated by finding area of each face and adding all together

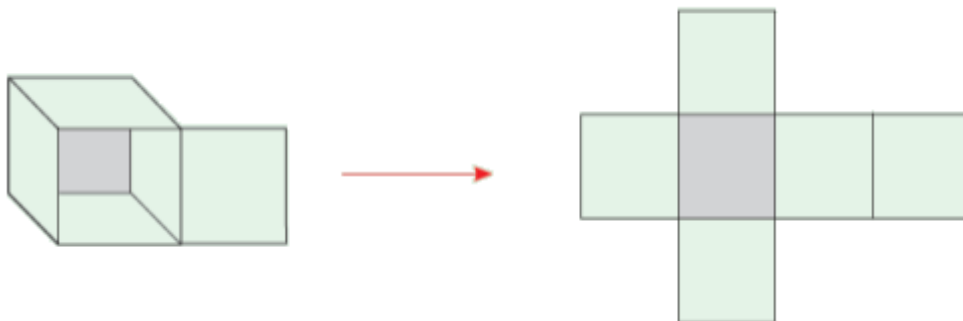
Topic 2.5 Task 5

With your group, determine the kind of solid you have as your example then use your measurements in the formula given in the Key Facts to determine the volume of your solid.

Topic 2.5 Task 6

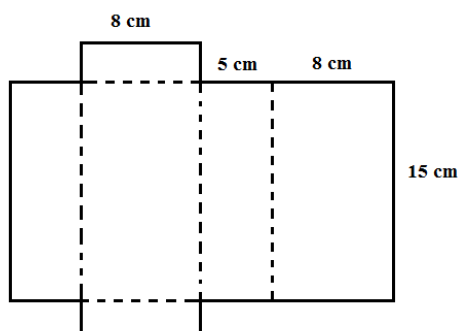
Figure 1:

The figure drawn below is a cube. Each edge measures 40 cm.



- Using properties of a square calculate the surface area.
- Using the correct formula, calculate the volume of the given cube.

Figure 2:



- a. Using properties of a square calculate the surface area.
- b. Using the correct formula, calculate the volume of the given cube.

Topic 2.5 Task 7

Calculate Surface Area and Volume for the solids described below:

- a. a sphere with radius 5 m
- b. a cone with height of 10 m and radius of 2 m (only find volume)
- c. a square-based pyramid with square edges of 5 cm, height of 4 cm, and triangular faces of area 10 cm^2



Application Activity⁷



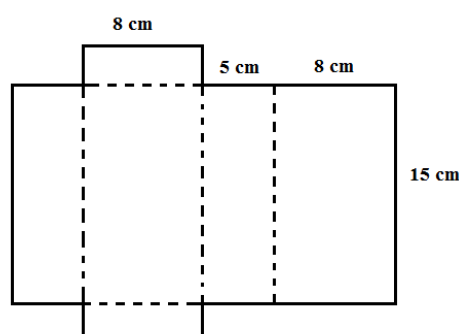
Tasks:

Topic 2.5 Task 8

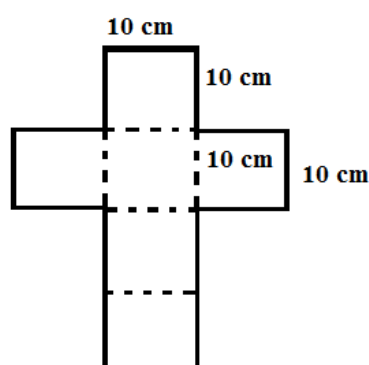
Draw the following nets on Manila paper. Cut the nets out. Fold them along the dotted lines.

- a. Which of the nets will make a cube?
- b. If the net does not make a cube, explain reasons why you think it does not.

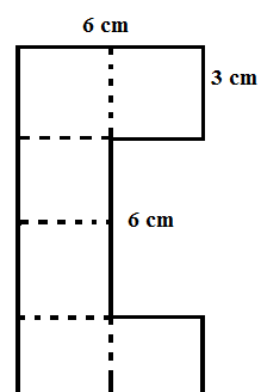
i)



ii)



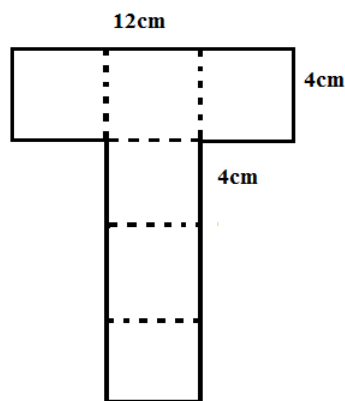
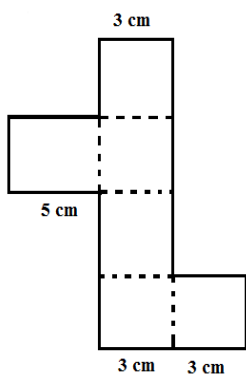
iii)



iv)

v)

⁷ Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher's Manual and Learner's Book*. Waltham: Education Development Center) Education Development Center.

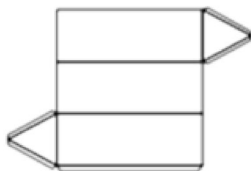


Topic 2.5 Task 9

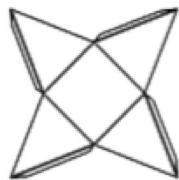
For each figure below, answer the following:

- List the 2-dimensional shapes you see in the net.
- What is the name of the solid given by the net in the figure?

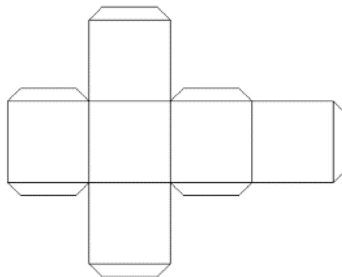
i)



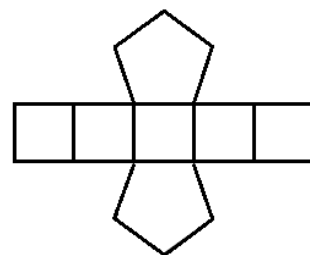
ii)



iii)



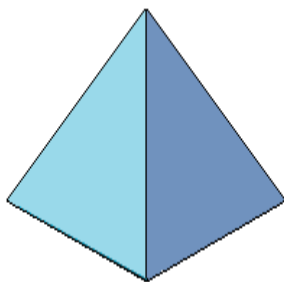
iv)



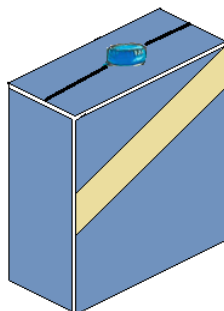
Topic 2.5 Task 10

Draw the nets of these three-dimensional shapes.

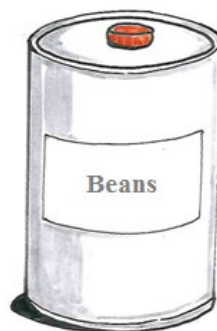
a)



b)



c)



Topic 2.5 Task 11

Find the surface area and the volume of a cylinder of radius 3cm and height 4cm.



Points to Remember

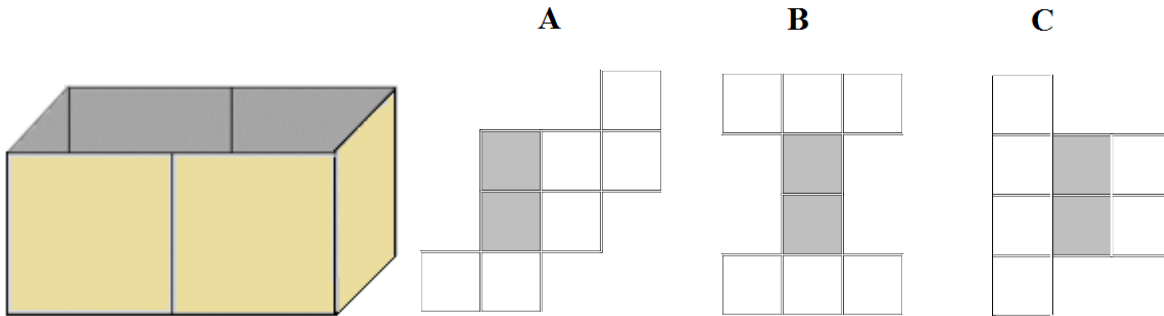
- Solids in geometry are three-dimensional shapes. Common examples are cubes, rectangular prisms, cylinders, spheres, cones, and pyramids.
- To find surface area we use a net of the solid which is a 2-dimensional shape that can be folded to form to become our 3-D solid.
- To find the volume of a solid we use the given formulae.



Formative Assessment⁸

1.

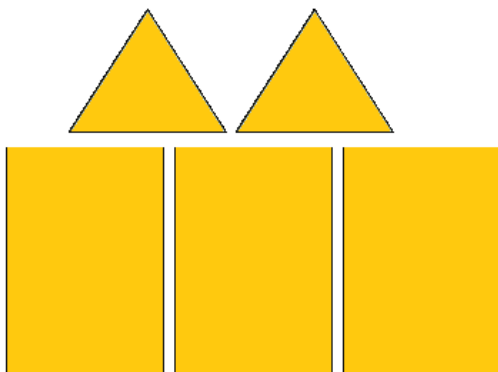
Here is an open box. Imagine folding each of the given nets. Which will fold together to make the box drawn below? (The grey squares show the base.)



2.

Aloys is making a 3D object using just the shapes below.

- Which object is he making?
- Once constructed (note that this is NOT a NET), give the number of faces, edges and vertices.

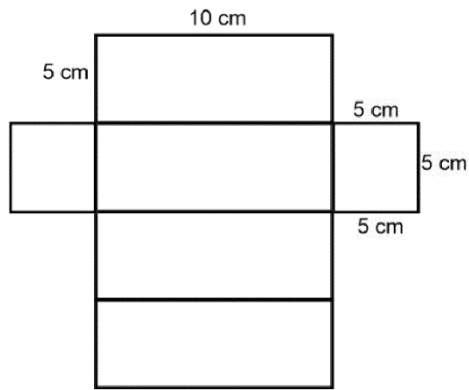


3.

Observe the net below carefully.

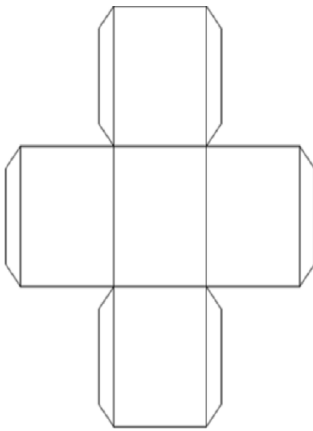
- Accurately draw this net.
- Find the area of each face then find the total surface area for this figure.
- Cut out and fold this net to create a rectangular prism.
- Calculate the volume of this rectangular prism.

⁸ Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher’s Manual and Learner’s Book*. Waltham: Education Development Center) Education Development Center.



4.

Alice thinks that this net will fold to create a cube. Do you agree with Alice? Explain your answer and if the solid is not a cube then give the name of the solid that is formed.



Self-Reflection

1. Draw ☹, ☺, or 😊 to indicate how well you feel you have understood concepts of geometry that were presented and practiced in this unit.
2. Take a few minutes and re-take the self-assessment that you took at the beginning of Unit 2. Now that you have completed the unit of study, use an X to indicate how well you have understood the concepts listed on the left side and see which areas you have improved.

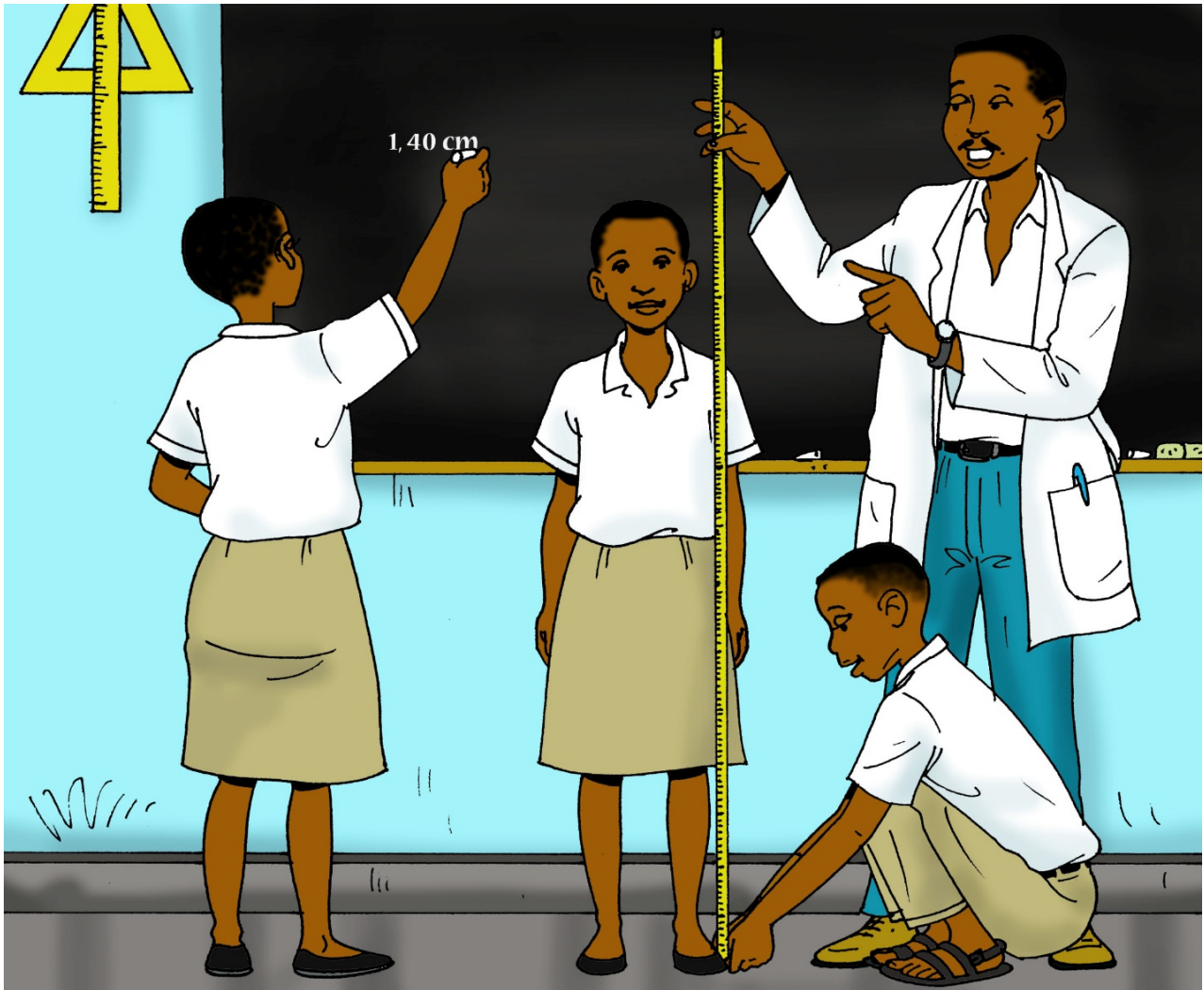
Note: this self-assessment is designed to help you identify your strengths and the areas where you need improvement, and to enable the trainer to focus support to where you most need it.

My experience	I don't have any experience doing this topic.	I know a little about this topic.	I have some experience doing this topic.	I have a lot of experience with this topic.	I am confident in my ability to do this topic.
Knowledge, skills and attitudes					
Sketch triangle, square, rectangle, hexagon, and pentagon in accordance with their properties					
Find the sum of interior angle measures of any regular polygon (triangle, square, pentagon, and hexagon, etc.)					
Calculate side lengths and areas of similar shapes					
Draw two parallel lines and a transversal, identify all angles that are equal, vertically opposite, corresponding, alternate, or supplementary					
Solve missing angle problems using the angle properties of parallel lines having constructed mathematical arguments					
Calculate the surface area and volume of a cube, rectangular and triangular prisms, pyramid, cylinder, cone and sphere					

3. Complete the following table to summarize and focus learning intentions for the future.

Areas of strength	Areas for improvement	Actions to be taken to improve
1.	1.	1.
2.	2.	2.

Unit 3: Apply Statistical Concepts and Calculations



Topics

- 3.1** Record and organize statistical data for analysis
- 3.2** Compute the measures of central tendency (Mean, Mode, and Median) using statistical data sets based on life situations
- 3.3** Present and communicate statistical data and analysis of results using mathematical representations (graphs, charts and tables)

Unit Summary:

Statistics is all around us. It would be difficult to go through a full week without using statistics. Imagine watching a football game where no one kept the score! The action itself might provide enough excitement to hold your attention for a while, but think of all the drama that would be lost if winning and losing were not an issue. Without statistics we could not make budgets, pay our taxes, enjoy games to their fullest, evaluate classroom performance, and many other things such as these.

In this book we will see how data can be recorded, organized, analysed, and presented in tables, pie charts, bar charts and line graphs. We will also learn how to find the mean, median and mode.

Self-Assessment: Unit 3

1. Look at the illustration at the beginning of this unit. Describe what you see. What do you think this unit will be about? What topics might be covered? Write three or four sentences in your notebook that describe what you expect to learn based on your observations.
2. Fill in the self-assessment below. There is no right or wrong way to answer this survey. It is for your own use to monitor your own learning. Think about what you know already: do you think you can do the tasks described? How well? Read the statements and put a check in column that best represents your situation.
3. At the end of this unit, we will take this survey again to see how you have improved.

My experience	I don't have any experience doing this topic.	I know a little about this topic.	I have some experience doing this topic.	I have a lot of experience with this topic.	I am confident in my ability to do this topic.
Knowledge, skills and attitudes					
Record and organize statistical data for analysis					
Compute the Mean of a data set from real situations					
Determine the Mode of a data set from real situations					
Find the Median of a data set from real situations					
Present and communicate statistical data and analysis of results using mathematical representations (graphs, charts and tables)					

Topic 3.1: Record and Organize Statistical Data for Analysis

Key Competencies:

Knowledge	Skills	Attitudes
1. Describe qualitative, quantitative, discrete and continuous data	1. Differentiate type of data	1. Persistent
2. Understand how to use frequency distribution tables for grouped and ungrouped data sets	2. Collect and organize data	2. Innovative
	3. Group data into classes, calculate frequencies, and make frequency distribution tables	3. Accurate

Getting Started: What do we know and where are we going?



Task:

Class activity:

List your age, number of siblings and favourite animal into the table on the board then copy the class data into a similar table in your notebook.

Age	Number of siblings	Favorited animal

- Describe what you see in the chart.
- What conclusions can you draw from the three data sets?



Activity1: Problem Solving



Task:

Topic 3.1 Task 1

The following data was gathered from one class of primary students.

Age	Tallies	Number of pupils
11 years	HHH	5
12 years	HHH HHH HHH ////	19
13 years	HHH ////	9
14 years	///	3
15 years	////	4
	Total:	40

- Explain why some tallies have cross marks while others are just vertical lines.
- Describe a relationship between the column with tallies and the column with numbers.
- What is the most common age for students in this class?
- What is the age difference between the oldest students and the youngest students?
- Consider the students in the 12-year-old group: are all the students in that age group EXACTLY the same age?



Guided Practice Activity



Tasks:

Topic 3.1 Task 2

In groups, organize the data from the blackboard into a table like the table below. When data is organized, write down anything interesting you see about trainees in this class.

Groups will show their table and present in front of the class the following:

- Explain the processes used to complete the table.
- Share two interesting facts learned from the organized data.

Sample data table:

Age (or # of siblings) (or favourite animal)	Tallies	Number of trainees
Add rows below if needed per data set		

3.1 Key Facts

Types of data:

- ▶ **Qualitative data:** this is the information that is not a number, rather it is a quality or a word (for example: data about favourite colours or favourite foods, or data about if you rode a moto, a bus, a car, or walked to school).
- ▶ **Quantitative data:** these are the data which can be expressed using numbers. It may be measured, counted or compared on a scale. Quantitative data can be broken down into two types namely; **discrete data** (only counting numbers) and **continuous data** (which may have decimals or fractions included). Examples include:
 - ▶ **Discrete data:** number of people in each car driving by, number of cars, number of animals...
 - ▶ **Continuous data:** These are data in an interval e.g. height, age, weight, time...

Data, which is not yet organized, processed, or ordered in any way, is called **raw data**.

The number of data points in each group of grouped data, or for each response for ungrouped data is called the **frequency**.

Mode: the most common data point (the data point with the highest frequency)

Range is the difference between the biggest data value and the smallest data value (range can be for a population, a sample, or even one group of data)

Data is gathered from a **population** (which is defined as a set of entities concerning which statistical inferences are to be drawn).

Data is often collected from a **sample** (portion) of the population because the population may be too large, too difficult to survey, too costly, etc.

- Population - A population is the group from which data are to be collected
- Sample - A sample is a subset of a population
- Class - a group/interval into which items of the same characteristics are put

Class limits or class boundaries - the extreme values of each class organized into the frequency table. There are two limits namely the lower-class limit (or boundary) where the class starts and the upper class limit (or boundary) where the class stops.

Note: Upper Class Boundary of one class could be the Lower Class Boundary of the next class, but it must be clear where each data point belongs so classes must not to overlap.

Topic 3.1 Task 3

A teacher recorded the age of her pupils.

Make an organized table. Use tallies to put the data points where they belong inside the table. Then answer the questions below the data set to help you analyse this data set.

14	11	12	12	13
13	14	15	11	12
13	12	12	15	12
12	13	15	12	13

14	12	12	12	11
12	13	15	12	12
13	13	12	12	11
12	12	13	11	12

- How many students are there in this class?
- How many of the pupils in this class are 12 years old?
- How old are the oldest pupils in that class?
- How old is the youngest pupil in this class?
- What is the age range for pupils in this class?
- What is the most common age for pupils in this class?
- What is the statistics term for the most common age? The data point with the highest frequency?



Application Activity



Tasks:

Topic 3.1 Task 4

Work with a partner to organize the data points (trainee heights in cm) from the board into a table. Then answer the following questions about your collected and organized data:

- What class (or group) has the most students?
- What vocabulary word is used to describe the class with the highest frequency?
- What is the range of the heights of trainees in this class?
- Our data is now grouped 'classes.' Guess what mid-class value is, and give an example.

Topic 3.1 Task 5

A trainee did a survey of the families in her community. She asked each family how many children were living in their household. The following table shows the results:

Number of children	Tally marks	Number of families
1	HHH	
2	HHH HHH HHH HHH HHH HHH HHH HHH	
3	HHH HHH HHH HHH HHH HHH HHH	
4	HHH HHH HHH //	
5	HHH HHH ///	

	Total	
--	-------	--

- Complete the table, the third column, which gives the frequency (number of families) for each data point (number of children in the household).
- How many families have five children living in their homes?
- Which number of children is the most common in this community??
- Which is the least likely number of children for a family in this community to have?
- How many families were surveyed? That is, what is the total frequency?

Topic 3.1 Task 6

Kamali sells soft drinks in his wholesale shop. The table below shows the types of drinks and the number he sold at his shop in one week.

Type of drink	A	B	C	D	E	F
Number of bottles sold	11000	9636	10800	12024	8842	9000

- Which type of drink is the most likely to be sold?
- Which type of drink had the least likely to be sold?
- What was the total number of bottles sold that week?



Points to Remember

- There are two main kinds of data: **Qualitative data (not numbers)** and **Quantitative data (numbers—and can either be discrete data (only counting numbers) or continuous data (which may have decimals or fractions included).**
- Data, which is not yet organized, processed, or ordered in any way, is called **raw data**.
- **Frequency** is the number of data points in each group, in a sample, or in the population.
- **Mode** is the most common data point (the data point with the highest frequency).
- **Range** is the difference between the highest data value and the lowest data value.
- The **Mid-Class Value** is the value of the term in the exact centre of any given class.



Formative Assessment

Work independently on the following tasks so you can self-assess your learning.

Formative Assessment

1.

The mass of 50 tomatoes (measured to the nearest g) were noted and shown in the table.

86	101	114	118	87	92	93	116	105
102	92	93	101	111	96	117	100	106
118	101	107	96	101	102	104	92	99
107	98	105	113	100	103	108	92	109
95	100	103	110	113	99	106	116	101
105	86	88	108	92				

- Construct a frequency distribution table, using equal class intervals of width 5 g, starting with the class 85 – 89 g, 90 – 94 g, ..., 115 – 119 g.
- Find the frequency for each group, and cite the modal class and the range of the data.

2.

The table below shows the number of days in which different temperatures were recorded in 60 days. Temperatures were rounded to the nearest whole number.

Temperature	Tally marks	Number of days
22° C	HHH HHH HHH	15
24° C	HHH HHH //	
25° C	HHH HHH	
26° C		9
28° C		13
	Total:	

- Copy and complete the table.
- How many days had a temperature of 26° C?
- Which temperature was recorded on the most number of days?
- Which temperature was recorded on the least number of days?
- Is this data qualitative or quantitative? Continuous or Discrete?

Topic 3.2: Determine the measures of central tendency (Mean, Mode and Median) using statistical data sets based on life situations

Key Competencies:

Knowledge	Skills	Attitudes
1. Explain how to make a frequency table, and how it can be used	1. Make frequency table of both grouped and ungrouped data	1. Creative
2. Explain how to calculate mean, mode, and median	2. Calculate the measures of central tendency for any given data set.	2. Accurate
	3. Discuss the meaning of the different measures of central tendency for real life situations	3. Focused on Detail



Getting Started: What do we know and where are we going?



Tasks:

Topic 3.2 Task 1

Brainstorm some times in real life when averages are used. List out in your notebook.

Topic 3.2 Task 2

Find the averages of the following three sets of numbers:

- Find the average temperature for the week. The 7 daily temperatures were: 21, 25, 24, 20, 19, 20, 21
- Find the average number of bananas on a hand of bananas sold at Simba Supermarket. 8 banana hands that were examined, and we counted the following bananas: 9, 13, 14, 11, 12, 13
- In a nearby village 5 families were raising goats. The families had the following numbers of goats each: 6, 7, 8, 9 and 11. On average, a family owns how many goats?

Vocabulary Activity:

- Write clear definitions for the following words based on what was learned in the Topic 3.1:
Frequency Range Mode (and Modal Class)
- We will learn about the following terms in this unit. Write a definition and check your responses as we move through the topic.
Mid-class value Mean Median



Problem Solving Activity



Task:

Topic 3.2 Task 3

Below are marks scored in a test:

30 50 80 45 90 65 75 30 80 85 10 65 50
50 15 85 50 65 75 10 90 55 50 100 85

- Re-write the scores, listed in order from the smallest to the biggest. Repeat any scores that occur more than once. The total number of scores in the new list will equal the number of scores shown in the original list above.
- Find the score that is in the exact centre of the ordered scores and underline it on your list.
- Label the score that is in the exact centre of the list as MEDIAN SCORE then, on the list, count how many scores are below the median AND how many scores are above. Are they equal?
- Which score appears most often?
- What is the correct term for the term that occurs most often?
- Find the average (which, in statistics, we call this number the MEAN).



Guided Practice Activity

3.2 Key Facts

The purpose of statistics is to **gather, review, analyse and draw conclusions** from data.

To organize data sets we order the **raw data** (usually from smallest to largest) and often organize the data into a data table which helps to review and begin analysis on the data.

Data is usually collected from a sample of a population. The number of data points collected is called the **FREQUENCY**. We often organize the data by grouping it into classes (intervals). The number of data points in each interval is the frequency of that class.

The difference between the biggest data value and the smallest data value is called the **RANGE**. We often find the range of the entire data set but we can also find the range of a class or an interval.

The data point that appears the most often, that is, the data point with the highest frequency is called the **MODE**. The mode tells us which data value is most likely to appear. When working with grouped data, the class that has the highest frequency is called the **MODAL CLASS**.

Once we have put our data set into ascending order (from smallest to largest values) we can then count the data points and find the data point at the exact centre of the set. The data point in the exact centre of our ordered data is called the **MEDIAN**. This data value

tells us an approximate measure for the value at the centre of our data set, that is, half the values will be smaller and half the values will be bigger.

Note: if the frequency is an even number then there will be two numbers at the centre. In this case, add those two numbers and divide by 2 to find a single value for MEDIAN.

In statistics, the average value for a data set is called the **MEAN**. We can find this point by adding all the data values together then dividing by the number of values.

The Mode, Median and Mean are called the “**values of central tendency**” because they tell us which number is most likely to appear (mode) which number is as likely to have a value higher as a value lower (median) and the average value (mean).



Tasks:

Topic 3.2 Task 4

The following table shows the marks obtained by students of form 3 in a math test.

Mark	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39
No. of students	5	8	12	18	14	11	6	3

- Compare this data set to the data set of test scores in Task 3.
- Find the range, mean, median and mode of the data presented in the table above.

Topic 3.2 Task 5

A different group of students in form 3 took the same math test as was taken in Task 4. Their results were slightly different and are shown in the table below.

Mark	Frequency F	Mid class value, X	F x X
4-8	2	$(4 + 8) / 2 = 6$	$2 \times 6 = 12$
9-13	4	$(9 + 13) / 2 = 11$	$4 \times 11 = \dots$
14-18	7	$(14 + 18) / 2 = \dots$	\dots
19-23	14	\dots	\dots
24-28	8	\dots	\dots
29-33	5	\dots	\dots

- Complete the table.
- Identify the modal class and give an approximate value for the mode.
- Find the range of the data and the median class and an estimate for the median value.
- Use your completed table to calculate the mean of this class.

- e. Consider the measures of central tendency that you have calculated here in Task 2 and those that were calculated for Task 1 and draw a conclusion about which class has stronger students based on your analysis.



Application Activity



Task:

Topic 3.2 Task 6

The 30 members of a certain music band were asked how many instruments each could play. The results are set out in the frequency distribution table shown below:

Number of instruments, x	1	2	3	4	5	6
Frequency, f	11	10	5	3	1	0

- a. Complete the table below:

Number of Instruments played by each student X	Frequency F	$F \times X$
1		
2		
3		
4		
5		
6		
Totals:		

- b. Determine the mode for this data set.
 c. Identify the median value for this data set.
 d. Calculate the mean for this data set.
 e. Find the range of this data set.



Points to Remember

There are three main statistical measures, which attempt to locate a typical value of a data set. These are:

1. **Mean**—or average value of the data points
2. **Mode**—or the value that appears most often (also a modal class: highest frequency)
3. **Median**—the data value that has the same number of data values lower and higher

These values are called the **measures of central tendency** of a set of data.

When we are only given grouped data, we can only estimate these values and we rely on the mid-class value as the representative value for all data points in that class.



Formative Assessment

Work independently then check your work with a peer. Be sure to show methods.

1.

Find the median and the mean for the data sets below:

- a. 20, 30, 16, 28, 42
- b. 60, 50, 75, 56, 66, 80

2.

The mass of 31 boys is measured to the nearest kg and noted in the following table:

Mass (kg)	60 - 64	65 - 69	70 - 74	75 - 79	80 - 84	85 - 89
Frequency	2	4	10	9	5	1

Use the data provided in the table to answer the following questions:

- a. Use the given data to complete the following table:

Mass	Frequency F	Mid class value X	F x X
60 - 64	2	$(60 + 64)/2 = 62$	$2 \times 62 = 124$
65 - 69	4	$(65 + 69)/2 = 67$	$4 \times 67 = \dots$
70 - 74	10	$(70 + 74)/2 = \dots$	\dots
75 - 79	9	\dots	\dots
80 - 84	\dots	\dots	\dots
85 - 89	\dots	\dots	\dots
Total	\dots	Total	\dots

- b. Find the modal class.
- c. Calculate the range of this data set.
- d. Identify the approximate median value.
- e. Calculate the mean.

Topic 3.3: Present and Communicate statistical data and analysis using mathematical representations (graphs, charts, and tables)

Key Competencies:

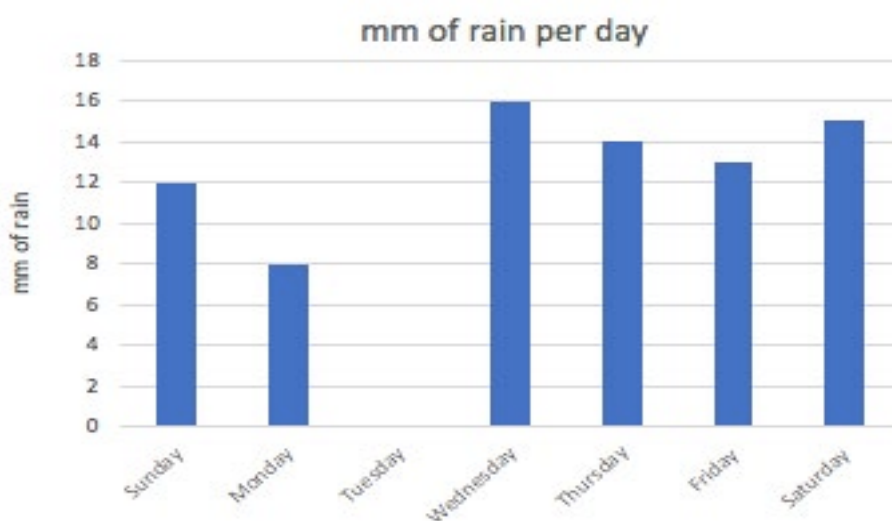
Knowledge	Skills	Attitudes
1. Differentiate types of statistical data	1. Collect and represent data	1. Team work
2. Explain the concept of mean, mode, median for ungrouped data	2. Calculate frequency, range, and mean (or average)	2. Persistence
3. Read and analyse data presented in charts, tables, and graphs	3. Create different types of statistical data presentation (polygons, bar chart, pie chart, tables)	3. Accuracy

Getting Started: What do we know and where are we going?



Task:

Topic 3.3 Task 1



- Observe the above bar chart carefully. Brainstorm. Write down your observations and ideas.
- Which day had the most rain? The least?
- Calculate the mean rainfall during the week using information displayed on the chart.



Problem Solving Activity



Task:

Topic 3.3 Task 2

A class of L2 students has recorded their ages. The ages all fall between 15 and 22 years old. The number of students at each age is recorded in the table below:

Age	Number of trainees
15	2
16	4
17	5
18	4
19	7
20	3
21	5
22	3

Using the data provided in the table above:

- Display this data in a bar chart with clear axis labels so we can visualize the situation.
- Calculate the range of this data.
- Find the median value for this data set.
- Identify the mode of this data set—and explain how the bar chart make this task easy.
- Calculate the mean of this data set (remember you will need to take the $F \times X$ for each age).



Guided Practice Activity

3.3 Key Facts

Statistical data can be presented using a variety of visual representations, namely:

- Bar charts or histograms
- Frequency polygons
- Line graph
- Pie charts or circle graphs
- Cumulative frequency curve or ogive graphs.

A **histogram** is a special type of **bar graph**. It has the following properties:

- The bars are joined at the class boundaries
- The bars are vertical
- The bars are rectangles drawn to represent the frequency for the class covered.

A **frequency polygon** may be superimposed on histogram by joining the mid points of the tops of the rectangles. NOTE: frequency polygon can also be drawn without drawing a histogram if we plot the frequency (y-axis) against the mid-class value (x-axis).

A **line graph** plots two data points against one another, for example rainfall per day.

A **pie chart** is a circular display of data. Each part of the circle (or pie) represents one data value and the size of that portion is a fraction (frequency of that data point / total frequency). Commonly, a fraction determined by frequency is changed to percentage to determine the relative size of the portion of the circle representing a data value.

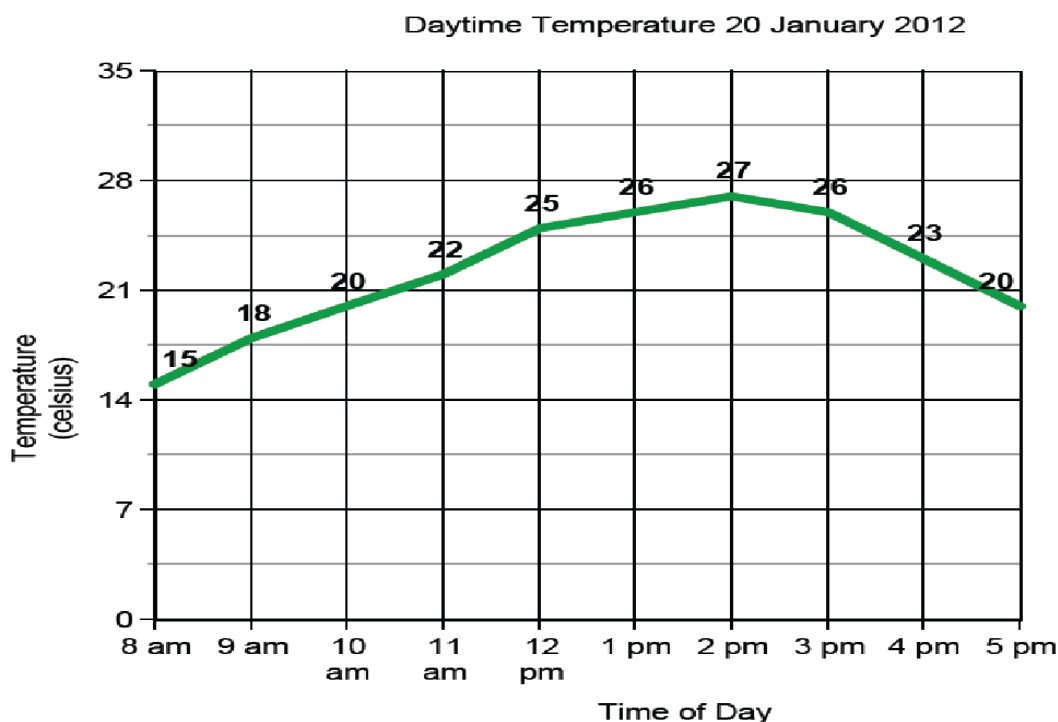
A **cumulative frequency curve** is a graph of the cumulative frequency (y-axis) plotted against the upper-class boundaries. This graph is sometimes called an **Ogive graph**.



Tasks:

Topic 3.3 Task 3

The Line graph below shows the temperature during some hours during the day on 20 January 2012.⁹



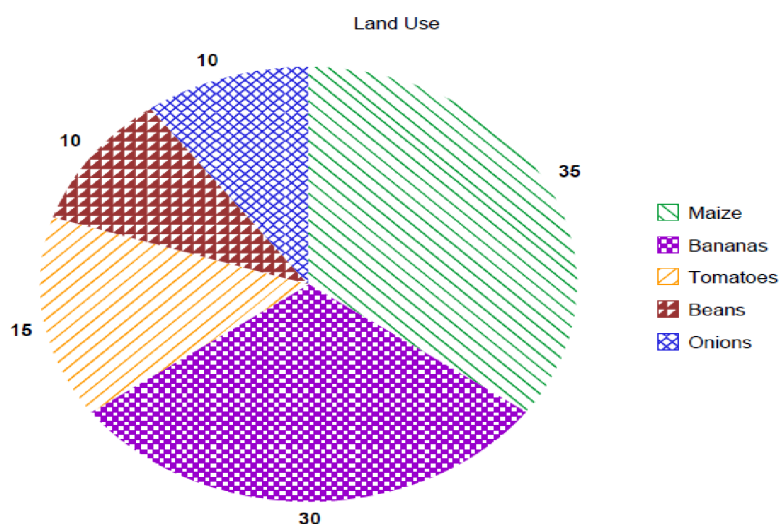
- What type of data is presented in the table?
- What was the temperature at 12 noon?
- When was it the hottest? The coldest?
- What is the range of temperatures that were measured?
- Find the average temperature during the 10 hours that temperatures were recorded.

Topic 3.3 Task 4

Below you will see a method of representing data called a pie chart. The entire circle represents all the gathered data, and each portion represents one of the possible categories for data.¹⁰

⁹ Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher’s Manual and Learner’s Book*. Waltham: Education Development Center) Education Development Center.

¹⁰ Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher’s Manual and Learner’s Book*. Waltham: Education Development Center) Education Development Center.



In this case, the pie chart represents use of agricultural land in a rural village. Crops are divided into 5 possible categories by what is planned: maize, bananas, tomatoes, beans, or onions.

The gathered data is the following:

Crop planted	Maize	Bananas	Tomatoes	Beans	Onions
Number of Hectares	36	30	16	10	10

- How many hectares were surveyed for land use?
- What information does this pie chart tell you?
- What percentage of land is used for growing maize? Beans? Onions? Bananas?
- Which crop uses the most land?
- Which crops together make up 50% of the land?

Topic 3.3 Task 5

The following frequency table shows marks earned by students in L2 in a Mathematics exercise:

Mark	1	2	3	4	5	6	7	8	9	10
Number of students	3	4	2	8	6	12	5	2	5	3

Data Value (Mark on exercise)	Frequency (number of students)	Cumulative Frequency
1	3	3
2	4	$3 + 4 = 7$
3	2	$7 + 2 = 9$
4	8	$9 + 8 = 17$
5	6	$17 + 6 = 23$
6	12	$23 + 12 = 35$
7	5	$35 + 5 = 40$
8	2	$40 + 2 = 42$

9	5	$42 + 5 = 47$
10	3	$47 + 3 = 50$
Total:	50	

Complete:

- Graph using Data Value as the x axis and Cumulative Frequency as the y axis
- The mode
- The median mark for this class



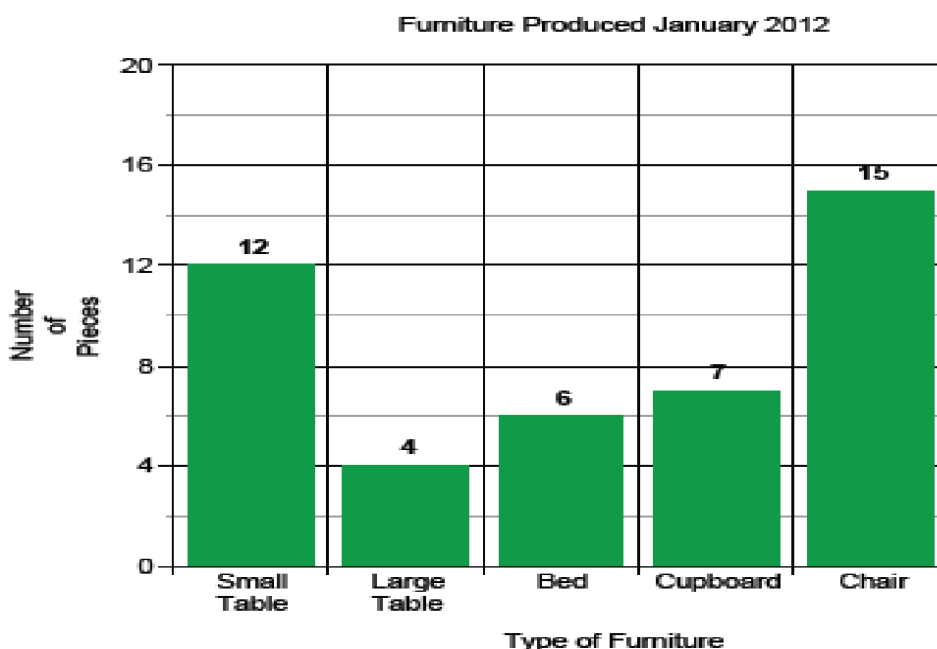
Application Activity



Tasks:

Topic 3.3 Task 6

The following chart shows how many different types of furniture a factory produced during one month¹¹:



- What type of data is presented in the table?
- Why would a carpenter want to keep track of the number of pieces of furniture he made in a month?
- How many beds did the carpenter make? Chairs? Small tables?
- The carpenter made 7 pieces of which type of furniture?
- List the types of furniture produced, in order of the most to least number of pieces.

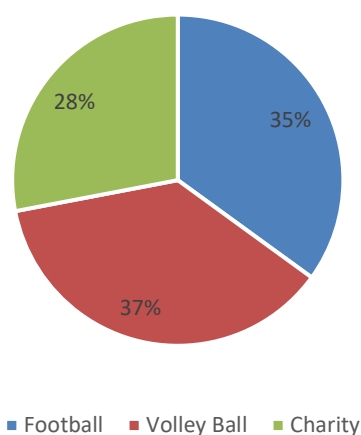
Topic 3.3 Task 7

¹¹ Education Development Center (2012). *Akazi Kanoze: Youth Livelihoods Project –Accelerated Learning Program: Mathematics Part 1: P3/P4 Review and Foundation for P5/P6 Concepts Teacher's Manual and Learner's Book*. Waltham: Education Development Center) Education Development Center.

The following table and pie chart show the number of students involved in extracurricular activities.

No	Extracurricular Activity	Number of Student	Percentage
1	Football	15	35%
2	Volley Ball	16	37%
3	Charity	12	28%

Extra curricular activities (%)



- What information does the pie chart tell you?
- Which extracurricular activity is the most popular?
- How might the administration at a school use this information?

Topic 3.3 Task 8

At home, after school today:

- Identify ways in which people around you use data or statistics to help their lives.
- Look in books, magazines, or other written materials for tables, graphs, or charts that represent some sort of statistical data.



Points to Remember

- Data can be presented using a variety of visual representations. Those representations can be used to understand data more clearly.
- A **pie chart** is best used for qualitative data (categories rather than numbers)
- A **histogram** is best used for continuous, quantitative data.
- Bar charts** can display either qualitative or quantitative data.

- A **line graph** plots two data points against one another, for example rainfall per day.



Formative Assessment

You will have 15 minutes to work independently on the assessment below. The purpose is for you to self-assess your understanding of the content of this Topic.

Formative Assessment

1.

In our village, there is a man called Kagabo who is an exemplary farmer. He has lots of animals and grows different types of crops. He divided his land into four plots as follows:

- 10 ha for bananas plants
- 5 ha for pineapples
- 5 ha for maize and
- 30 ha for animals

- a. Determine what percentage of the whole plot of land is used for each different purpose.
- b. Use the percentages to sketch a pie chart for a visual representation of Kagabo's land use.

2.

Kagabo decided to record the milk production of his 12 cows. He found his cows provided the following litres of milk on Monday: 8, 3, 4, 5, 3, 10, 4, 3, 4, 4, 10, 4

- a. Organize the data first in ascending order then into a frequency table.
- b. What is the total amount of milk produced that day by Kagabo's cows?
- c. What is the average number of litres produced by each of Kagabo's cows?
- d. What is the most common amount of milk for a cow to provide?

3.

The table below shows the summary records of milk produced daily for the whole week.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Litres	62	60	55	50	60	65	53

- a. Present the data on a graph, reporting on X-Axis the days and on Y-Axis the litres.
- b. How many litres were milked on Thursday?
- c. How many litres were milked on Friday?
- d. Which day had the highest number of litres and how many?
- e. Which day had the lowest number of litres and how many?
- f. What was the total amount of milk collected during that week?
- g. Calculate the average production per day.

- h. If 1 litre of milk costs 200 Frw, how much money did Kagabo get within that week?



Self-Reflection

1. Draw ☹, 😐, or 😊 to indicate how well you feel you have understood concepts presented in Unit 3.
2. Take a few minutes and re-take the self-assessment that you took at the beginning of Unit 3. Now that you have completed the unit of study, use an X to indicate how well you have understood the concepts listed on the left side and see which areas you have improved.

Note: this self-assessment is designed to help you identify your strengths and the areas where you need improvement, and to enable the trainer to focus support to where you most need it.

My experience	I don't have any experience doing this topic.	I know a little about this topic.	I have some experience doing this topic.	I have a lot of experience with this topic.	I am confident in my ability to do this topic.
Knowledge, skills and attitudes					
Record and organize statistical data for analysis					
Compute the Mean of a data set from real situations					
Determine the Mode of a data set from real situations					
Find the Median of a data set from real situations					
Present and communicate statistical data and analysis of results using mathematical representations (graphs, charts and tables)					

3. Complete the following table to summarize and focus learning intentions for the future.

Areas of strength	Areas for improvement	Actions to be taken to improve
1.	1.	1.
2.	2.	2.

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